Anisotropies of the Lower and Upper Critical Fields in MgB2 Single Crystals
L. Lyard, P. Szabo, Thierry Klein, J. Marcus, C. Marcenat, K. H. Kim, B.W. Kang, H. S. Lee, S. I. Lee

To cite this version:

HAL Id: hal-00959095
https://hal.archives-ouvertes.fr/hal-00959095
Submitted on 13 Mar 2014

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L’archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d’enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.
Anisotropies of the Lower and Upper Critical Fields in MgB$_2$ Single Crystals

L. Lyard, 1 P. Szabó, 1,2 T. Klein, 1,3 J. Marcus, 1 C. Marcenat, 4 K. H. Kim, 5 B. W. Kang, 5 H. S. Lee, 5 and S. I. Lee 5

1 Laboratoire d’Etudes des Propriétés Electroniques des Solides, CNRS, BP 166, 38042 Grenoble CEDEX 9, France
2 Centre of Low Temperature Physics IEP SAS & FS UPJŠ, Watsonova 47, 043 53 Košice, Slovakia
3 Institut Universitaire de France and Université Joseph Fourier, BP 53, 38041 Grenoble CEDEX 9, France
4 Commissariat à l’Energie Atomique-Grenoble, Département de Recherche Fondamentale sur la Matière Condensée, SPSMS, 17 rue des Martyrs, 38054 Grenoble CEDEX 9, France
5 NVCRCIS and Department of Physics, Pohang University of Science and Technology, Pohang 790-784, Republic of Korea

(Received 10 July 2003; published 2 February 2004)

The temperature dependence of the upper ($H_c^2$) and lower ($H_c^1$) critical fields has been deduced from Hall probe magnetization measurements of high quality MgB$_2$ single crystals along the two main crystallographic directions. We show that $\Gamma_{H_c^1} = H_{c2||ab}/H_{c2||c}$ and $\Gamma_{H_c^2} = H_{c3||c}/H_{c1||ab}$ differ significantly at low temperature (being ~5 and ~1, respectively) and have opposite temperature dependencies. We suggest that MgB$_2$ can be described by a single field dependent anisotropy parameter $\gamma(H)$ ($=\lambda_c/\lambda_{ab} = \xi_{ab}/\xi_c$) that increases from $\Gamma_{H_c^1}$ at low field to $\Gamma_{H_c^2}$ at high field.

PACS numbers: 74.25.Op, 74.25.Dw

MgB$_2$ belongs to an original class of superconductors in which the electronic system consists of two types of carriers — derived from boron $\pi$ and $\sigma$ orbitals — with two superconducting energy gaps [1]. The coexistence of these two gaps [2] with different anisotropies then gives rise to very peculiar physical properties [1,3,4]. Among these, the strong temperature dependence of the anisotropy of the upper critical field $\Gamma_{H_c^2} = H_{c2||ab}/H_{c2||c}$ is now well established [5,6] ($H_{c2||ab}$ and $H_{c2||c}$ being the upper critical fields for magnetic fields parallel to the $ab$ planes and $c$ direction, respectively). However, the anisotropy of the lower critical field is still to be clarified. Caplin et al. [7] suggested that $\Gamma_{H_c^1} = H_{c1||c}/H_{c1||ab} \sim 2$, independent of temperature, but found a temperature independent $\Gamma_{H_c^2}$ value (above 25 K) in striking contrast with other measurements [5,6]. On the other hand, Zehetmayer et al. [8] found that $\Gamma_{H_c^2}$ decreased with temperature assuming, however, that $\Gamma_{\lambda} (=\lambda_c/\lambda_{ab}$, where $\lambda$ is the penetration depth) is equal to $\Gamma_{H_c^2}$.

In this Letter, we present magnetization measurements performed on a high quality single crystal [9] ($T_c \approx 36.5$ K) with flat and shiny surfaces of typical dimensions: $100 \times 100 \times 25 \mu$m$^3$. The largest surface (i.e., the $ab$ planes) of the sample has been placed either parallel to or perpendicular to the surface of a Hall probe in order to measure the magnetization for $H \parallel c$ and $H \parallel ab$, respectively [see sketches in Fig. 2 (below)]. We show that $H_{c1}$ is almost isotropic at low temperature and that, in contrast to $\Gamma_{H_c^2}$, $\Gamma_{H_c^1}$ increases with $T$. This increase is in good agreement with recent calculations of the anisotropy of $\lambda$ in the weakly coupled two bands superconductor [3].

Typical magnetization loops at $T = 5$ K and $T = 20$ K are presented in Fig. 1 for $H \parallel c$. The alignment of the external field with the main crystallographic axis has been obtained by slightly rotating the ensemble in order to get the maximum and minimum $H_{c2}$ values for $H \parallel ab$ and $H \parallel c$, respectively. In the Bean model (i.e., assuming that the hysteresis mainly arises from bulk pinning), the half-width of the loop is related to the critical current density $J$ through $Jd = (M_{up} - M_{down})/2$, where $M_{up}$ and $M_{down}$ are the magnetization for increasing and decreasing magnetic fields, respectively, and $d$ is a characteristic length scale on the order of the sample dimension. The deduced magnetic field dependence of $Jd$ is displayed in the inset of Fig. 2(b) together with the values deduced from ac-susceptibility measurements [5]. Taking $d \sim 50 \mu$m, $J \sim 10^{-13}$ A/cm$^2$ at low $T$ and low $H$ with very similar values obtained from the two measurements in the common magnetic field range. Given such small $J$ values (being several orders of magnitude smaller than the ones obtained in thin films [10]), the reversible magnetization can be easily obtained assuming that $\nu_H(Jd) < 1$.
absolutely value of $H_{c2}$ might be overestimated by a factor on the order of $\sqrt{w/t} \sim 2$ due to geometrical barriers. Similarly, for $H \parallel ab$, the effect of Bean-Livingston (BL) [13] barriers could lead to an overestimation of $H_{c2}[\text{lab}]$ up to a factor $\kappa / \ln(\kappa)$ (where $\kappa$ is the Ginzburg-Landau parameter). This overestimation is difficult to estimate in MgB$_2$ as $\kappa$ may be field dependent (see below), but BL barriers are expected to give rise to very asymmetric magnetization loops ($M$ being close to zero in the descending branch of the loop; see, for instance [14]), which we did not observe for our samples.

In the following, we have thus assumed that pinning mainly arises from bulk defects and that the lower critical magnetic field is equal to the first penetration field. $H_{c1}$ could then be easily determined from the well-defined minima in the $M_{\text{rev}}(H)$ curves (see arrows in the inset of Fig. 1). The temperature dependence of $H_{c1}[\text{lab}]$ and $H_{c1}[\text{ab}]$ is displayed in Fig. 3. The $H_{c1}$ values determined this way are about 5% larger than those estimated from the point where the reversible magnetization deviates from linearity, but we did not observe any significant difference in the temperature dependence of $H_{c1}$ deduced from either of the two criteria. It is important to note that, even though the absolute values of $H_{c1}$ depend on demagnetization effects, possible surface barriers, and/or on the determination criterion, the $T$ and $H$ dependence of the anisotropy parameters discussed below, and therefore our conclusion, is not affected by such uncertainties.

The temperature dependence of $H_{c2}$ defined as the onset of the diamagnetic response at $M_{\text{rev}}(H_{c2}) = 0$ is also presented (see inset of Fig. 3). For $H \parallel ab$, the values deduced from our magnetization measurements above

$M_{\text{rev}} = (M_{\text{up}} + M_{\text{down}})/2$. Typical curves are displayed in Fig. 2 for $T > 26$ K for fields along the two main crystallographic directions.

For $H \parallel c$, i.e., perpendicular to the platelet, important demagnetizing effects come into play and $H$ has been rescaled to $H = H_0 - N_c M_{\text{rev}}$, where $H_0$ is the external field and $N_c$ is the demagnetizing factor. $N_c$ can be estimated assuming that the sample is an ellipsoid of thickness $t = 25 \mu m$ and width $w = 100 \mu m$ giving $N_c = 1 - \pi t/2w = 0.6$ [as has then been set to $N_{ab} = (1 - N_c)/2$. This value is consistent with the fact that, after correction for the demagnetization effect, the magnetization curves present a nearly vertical slope for $H = H_{c1}$ [11] (see inset of Fig. 1; the y axis has been slightly rescaled in order to get a $-1$ slope in the Meissner state to account for the fact that the sample did not completely cover the surface of the Hall probe, especially for $H \parallel ab$). Note that, as pointed out by Zeldov et al. [12], the

![Graph](image)

**Fig. 2.** Reversible magnetization as a function of the internal field $H$ for $H \parallel c$ (a) and $H \parallel ab$ (b) at various temperatures. The schematic drawings show the geometry used for both directions. In the inset of (a), $d\mu_0 M_{\text{rev}}/d\ln(H)$ as a function of $H$ at 28 K in both directions, and (b) critical current density as a function of the applied field $H_0$ deduced from the width of the magnetization loop (open circles) and from ac-susceptibility measurement (closed circles [5]). A peak effect is clearly visible at low temperature close to $H_{c2}$.

![Graph](image)

**Fig. 3.** Temperature dependence of the lower critical field for $H \parallel c$ and $H \parallel ab$ determined from the minimum in the $M_{\text{rev}}(H)$ curves (see Fig. 2). The error bars using this determination criterion are on the order of the dimension of the symbols. In the inset: $H_{c2}$ vs $T$ for the same directions. The dotted line is an extrapolation below 26 K deduced from our previous magnetotransport measurements [5].
26 K (open circles) have been extrapolated down to 5 K (dotted line) using our previous high field magnetotransport data performed on another sample coming from the same batch [5] (those previous \( H_{c2} \) values were about 20% larger in both directions and have been rescaled to match our data in the common temperature range). As previously observed [5,6,8], close to \( T_c \), the temperature dependence of \( H_{c2} \) is almost linear for \( H \parallel c \) and presents a positive curvature for \( H \parallel ab \) leading to a strong decrease of \( \Gamma_{H_{c2}} \) for \( T \rightarrow T_c \) (see Fig. 4, open circles).

The situation is very different for the lower critical field which is almost linear for \( H \parallel ab \) and reveals a negative curvature for \( H \parallel c \) at high temperature. This unusual negative curvature close to \( T_c \) has been previously observed in thin films and polycrystalline samples [15] and can be explained by the two-band Ginzburg-Landau theory [16]. As shown in Fig. 3, \( H_{c1||ab} \approx H_{c2||c} \) at low temperature (i.e., \( \Gamma_{H_{c1}} \approx 1 \ll \Gamma_{H_{c2}} \)) and the negative curvature for \( H_{c1||c} \) leads to an increase of \( \Gamma_{H_{c1}} \) with temperature (see Fig. 4, solid circles). This behavior is in contrast with that previously obtained by Caplin et al. [7], who suggested that \( \Gamma_{H_{c1}} \approx 2 \) down to low temperature and that \( \Gamma_{H_{c1}} \approx \Gamma_{H_{c2}} \) at \( T > 25 \) K. Our result is also in apparent contrast with the Zehetmayer et al. [8] measurements (\( \Gamma_{H_{c1}}(0) \approx 3 \)). This discrepancy will be discussed below.

These temperature dependencies can be compared to those predicted for the anisotropy of the coherence length \( \xi = \xi_{ab}/\xi_c \) and penetration depth \( \lambda = \lambda_c/\lambda_{ab} \). “Classical” superconductors can be characterized by one field independent anisotropy parameter: \( \gamma = \xi_{ab}/\xi_c(= \Gamma_{H_{c2}}) = \lambda_c/\lambda_{ab} \), whatever the anisotropy of the superconducting gap. However, it has been suggested that \( \Gamma_{H_{c2}} \) could differ considerably from \( \Gamma_{H_{c1}} \) at low temperature in MgB\(_2\) due to the presence of two superconducting energy gaps with different anisotropies [3]. Indeed, at low temperature the anisotropy of the upper critical field is mainly related to the anisotropy of the Fermi velocities over the quasi-2D \( \sigma \) sheet: \( \Gamma_{H_{c2}} = \sqrt{(v_{ab}^2)/(v_c^2)} \approx 6 \). \( \Gamma_{H_{c1}} \) then decreases with temperature as the influence of the small, nearly isotropic gap increases due to thermal mixing of the two gaps [3,4]. On the other hand, the anisotropy of \( \lambda \) (when deduced from low field measurements) is related to the anisotropy of the Fermi velocities over the whole Fermi surface which is expected to be of the order of 1.1 in good agreement with recent neutron scattering data [17] and scanning tunneling imaging [18]. The two anisotropies finally merge at \( T_c \) as they are then determined by the same “mass tensor” [3].

In the London model, the penetration depth for \( H \parallel c \) can in principle be deduced from \( H_{c1} \) through [11]: \( \mu_0 H_{c1} = (\Phi_0/4\pi \lambda_{ab}^2)[\ln(\lambda_{ab}/\xi_{ab}) + c] \), where \( c \approx 0.5 \) in isotropic superconductors. Apart from the uncertainty in \( c \), another difficulty arises in MgB\(_2\) as the superconducting parameters here depend on the value of the magnetic field. Indeed, in a classical system \( \xi \) can be obtained from the upper critical field (\( \mu_0 H_{c2||c} = \Phi_0/2 \pi \xi_{ab}^2 \)), which here leads to \( \xi_{ab}(0) \approx 10 \) nm and then to \( \lambda_{ab}(0) \approx 60 \) nm (taking \( c = 0.5 \)). However, the radius of the vortex core deduced from tunneling measurements at low field (i.e., below the “upper critical field” \( H_{c2}^\ast \)) [19] is actually much larger (i.e., \( \xi \approx 50 \) nm) than the one obtained from \( H_{c2} \). The condition \( \lambda/\xi \gg 1 \) would not be valid any longer, and the use of this equation in MgB\(_2\) should thus be considered carefully.

On the other hand, still assuming that the London model for type II superconductors is valid, \( \lambda \) can be estimated from the slope of the magnetization curve: \( 1/\lambda^2 = (8 \pi/\Phi_0)d(\mu_0 M_{rev})/d \ln(H) \) [11]. As shown in the upper inset of Fig. 2, at low field, this slope rapidly decreases for increasing fields and saturates above some characteristic field \( H^*(T) \). Such a decrease of \( d(\mu_0 M_{rev})/d \ln(H) \) is expected in type II superconductors as the slope of the \( M_{rev}(H) \) curve is expected to be vertical for \( H = H_{c1} \) [11]. However, it is important to note that here it occurs for the same \( H^* \) value in both directions despite very different \( H_{c2} \) values. \( H^* \) can thus probably be associated with the rapid filling of the small gap (i.e., of the \( \pi \) band) observed in point-contact spectroscopy experiments [20] in good agreement with specific heat [21] and tunneling [19] measurements (\( H^* \approx H_{c2}^\ast \)). \( \lambda \) thus rapidly increases with increasing fields [22] reaching \( \lambda_{ab} \approx 70 \) nm at 5 K for \( H > H^* \approx 0.5 \) T. This increase is related to the lowering of the superfluid density (\( \approx 1/\lambda^2 \)) as the superconductivity in the \( \pi \) band is destroyed [23]. Our “high field” \( \lambda_{ab}(5 \) K) estimation is larger than the \( \lambda \) value previously estimated in single crystals by Caplin et al. [7] (~43 nm) but lower than estimates obtained

![FIG. 4. Temperature dependence of the anisotropy parameters \( \Gamma_{H_{c2}} \) and \( \Gamma_{H_{c1}} \). The dotted line is an extrapolation of \( \Gamma_{H_{c1}} \) at low temperature deduced from our previous magnetotransport measurements [5].](image-url)
for thin films and polycrystalline samples from rf [24] techniques (∼100 nm; for a review, see [15]). However, this value is comparable to the one deduced from μSR measurements ∼85 nm [25] (which have been performed above $H_{c2}^\parallel$) as well as the one obtained by Zehetmayer et al. [8] (∼82 nm) from high field magnetization measurements. On the other hand, the $\xi_{ab}(0)$ values obtained from tunneling measurements at low field (∼50 nm [19]) are much larger than those deduced from $H_{c2}^\perp$ (∼10 nm) suggesting that $\xi_{ab}$ decreases with field. This would imply that $\kappa_{ab}$ rapidly increases, ranging from ≈1 at low field to ≈7 above $H^*$. The estimation of $\lambda_c$ from $H_{c1}$ for $H \parallel ab$ is even more difficult as it is not obvious how the logarithmic term is affected by the anisotropy of the system. However, it has been suggested by Balatskii et al. [26] that for uniaxial superconductors this term does not depend on the orientation between the field and the c axis which would directly lead to $\Gamma_{H_{c1}} = \Gamma_{\lambda} \sim 1$ at low temperature in good agreement with previous experimental data [23,18]. Assuming that $\Gamma_{H_{c1}} = \Gamma_{\lambda}$, the increase of $\Gamma_{H_{c1}}$ with temperature is then in good agreement with theoretical predictions for the weakly coupled two bands superconductor [3]. Note that we did not observe any significant change in the temperature dependence of $\Gamma_{\lambda}$, whatever the choice of the logarithmic correction. The choice of this correction and the uncertainties on the absolute value of $H_{c1}(0)$ indeed lead to some uncertainty on $\Gamma_{\lambda}(0)$. However, using the fact that $\Gamma_{\lambda} \rightarrow \Gamma_{H_{c1}}$ close to $T_c$, we can estimate that $\Gamma_{\lambda}(0) < 1.5$.

$\Gamma_{\lambda}$ can also be estimated through $\Gamma_{\lambda} = [dM_{rev}/d \ln(H \parallel c)]/[dM_{rev}/d \ln(H \parallel ab)]$. As shown in the upper inset of Fig. 2, these derivatives are very similar in both directions for low magnetic fields confirming that the “low field” $\Gamma_{\lambda}$ value remains of the order of 1 up to $T = 28$ K (below 25 K, the $H_{c2}^\perp$ value for $H \parallel ab$ was larger than our maximum field and we could thus not deduce $M_{rev}$ very accurately at high field). However, it is important to note that this ratio rapidly increases with a field approaching ∼3 above $H_{c2}^\perp$. This high field $\Gamma_{\lambda}$ value is then on the order of $\Gamma_{H_{c1}} \approx 3.7$ at 28 K. Indeed, above $H_{c2}^\perp$ the anisotropy of $\lambda$ is no more given by that of the whole Fermi surface and the two anisotropy parameters are thus expected to merge. A similar increase of $\Gamma_{\lambda}$ with field has also been observed in [23] at low temperature, and $\Gamma_{\lambda}$ is thus very different from $\Gamma_{H_{c1}}$ above $H_{c2}^\perp$. This high field ratio is then consistent with the one obtained by [8] from high field torque measurements. More details on the field dependence of the anisotropy parameters will be given elsewhere.

In conclusion, we have shown that $\Gamma_{H_{c1}}$ and $\Gamma_{H_{c2}}$ not only differ in absolute value at low temperature but also have opposite temperature dependencies in MgB$_2$. $\Gamma_{H_{c1}}$ is ≈$\Gamma_{H_{c1}}$ at low field but increases for $H > H_{c2}^\perp$. This suggests that MgB$_2$ could be described by an unique field dependent anisotropy parameter $\gamma(H) = \Gamma_{\lambda} = \Gamma_{\lambda}$ rising from $\Gamma_{H_{c1}}$ at low field to $\Gamma_{H_{c2}}$ at high field.

The work of P. Szabó was partially supported by the Slovak Science and Technology Assistance Agency under Contract No. APVT-51-020102. This work in Korea was supported by the Ministry of Science and Technology of Korea through the Creative Research Initiative Program.

[22] The “low field” $\lambda$ value is expected to be very sensitive to interband scattering ranging from $\lambda_{ab}(0) = 100$ nm in dirty systems to $\lambda_{ab}^{dir}(0) \approx 40$ nm for clean samples [A. A. Golubov et al., Phys. Rev. B 66, 054524 (2002)].