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Towards the coupling of a discontinuous Galerkin method with a MoM for analysis of susceptibility of planar circuits

Caroline Girard1,2, Stéphane Lanteri2, Ronan Perrussel1, and Nathalie Raveu1

1Université de Toulouse ; INPT, UPS ; CNRS ; LAPLACE ; ENSEEIHT, Toulouse, France
2NACHOS project-team, INRIA Sophia Antipolis-Méditerranée, Sophia Antipolis, France

We aim at coupling a method of moments, the Wave Concept Iterative Procedure, and the Hybridizable Discontinuous Galerkin method to study electromagnetic susceptibility of innovative planar circuits in 3D. Hybridizing the Wave Concept Iterative Procedure with volumic methods like the Frequency Domain Transmission Line Matrix method, the Finite Element Method and the Hybridizable Discontinuous Galerkin method in 2D is a first step for the validation of the proposed coupling technique. The considered problem is Maxwell’s equations in the frequency domain. Three test cases in 2D and a preliminary result in 3D are provided.

Index Terms—Microwave propagation, method of moments, finite element methods.

I. INTRODUCTION

The Wave Concept Iterative Procedure (WCIP) [1] is a method adapted to the study of microwave circuits, solving Maxwell’s equations in guided and stratified structures. Nevertheless, it cannot characterize circuits with dielectric inhomogeneities [2]. This has naturally led to the issue of hybridization of the WCIP with volumic methods such as the finite element method (FEM), the hybridized discontinuous Galerkin method (HDG) [3] or a method based on transmission line theory, the Frequency Domain Transmission Line Matrix method (FDTLM). A hybridization with the FDTLM has already been achieved in [4]. The advantage of the HDG method lies in its flexibility with regards to the type of mesh used for the discretization of the volumic part (it can be unstructured, hybrid, non-conforming) and in its adaptivity in the polynomial approximation order; this interpolation order can be chosen lower near the interface (if the field is not regular because of discontinuities) and higher some elements further.

This work is concerned with circuit modeling in the high frequency range. It describes the hybridization of numerical methods in the frequency domain to study the electromagnetic susceptibility of planar circuits. We aim at detecting potential perturbations induced in a circuit by an external electromagnetic source. In this purpose, the planar circuit is illuminated by a wave and we calculate the electric field and potential perturbations induced in a circuit by an external net magnetic susceptibility of planar circuits. We aim at detecting

...
where $Y_{1,n}^{\text{TE}}$ corresponds to a mode of order $n$ admittance injected in domain 1, $Z_0$ the free space impedance ($377 \Omega$), $a$ the distance between metallic slabs ($a = 1.27 \text{cm}$), $k_0$ the wave number in vacuum at frequency $f_0 = 16 \text{GHz}$, which gives $k_0 = 335 \text{rad/m}$. Where $\psi = 10 \times 10^{10} \text{rad/s}$ and $\mu_0 = 1.26 \times 10^{-6} \text{H.m}^{-1}$. The transmission operator $S$ between both domains satisfies

$$
\begin{pmatrix}
A_1^{(k)} \\
A_2^{(k)}
\end{pmatrix} = S 
\begin{pmatrix}
B_1^{(k)} \\
B_2^{(k)}
\end{pmatrix} = \begin{pmatrix}
S_{11} & S_{12} \\
S_{21} & S_{22}
\end{pmatrix} 
\begin{pmatrix}
B_1^{(k)} \\
B_2^{(k)}
\end{pmatrix},
$$

(3)

where $S_{11}$ and $S_{12}$ are N-sized matrices ($N$ being the number of segments on $\Sigma$), respectively equal to -1 and 0 on metal segments and respectively equal to 0 and 1 on insulator segments. For instance, when there is no metal, $S_{11}$ is filled with zeros and $S_{12}$ is equal to the identity matrix.

Consequently, the linear system to be solved is

$$
\begin{pmatrix}
I & S^W \\
S^W & 0 \\
0 & S^W \\
\end{pmatrix} \begin{pmatrix}
B_1 \\
B_2
\end{pmatrix} = \begin{pmatrix}
B_0 \\
0
\end{pmatrix},
$$

(4)

where $I$ is the identity matrix, $S^W$ a matrix for the discretization by the WCIP, defined by

$$
S^W = \text{FMT}^{-1} \Gamma \text{FMT}.
$$

(5)

The coupling of the WCIP with a volumic method is obtained by replacing $S^W_2$ by a matrix $S^F_2$ for the discretization by a volumic method. Matrix $S^F_2$ has to characterize the relation between $B_2$ and $A_2$. In the hybridization setting, the wave $A_2$ is introduced as a source term in the weak formulation. In 2D, TE and TM modes are uncoupled, which explains that waves are only along $y$-axis, according to Fig. 1 orientation. The corresponding weak formulation for the FEM is given by

$$
\int_{D_2} \nabla E_{2y} \cdot \nabla w ds - k_0^2 \int_{D_2} E_{2y} w ds + jk_0 \int_{\Sigma} E_{2y} w dl = 2jk_0 \sqrt{\mu_0} \int_{\Sigma} A_{2x} w dl,
$$

(6)

where $D_2$ corresponds to domain 2, $E_{2y}$ is the electric field component along $y$-axis, $w$ stands for a test function and $A_{2x}$ is the outgoing wave component along $y$-axis. The insertion of the source term for the HDG method is given by writing a conservativity condition [3] adding a specific term on $\Sigma$

$$
\int_{\partial \Sigma} n \times (H_{2x} x + H_{2z} z) \text{d}l - \int_{\partial \Sigma} \tau(E_{2y} - \lambda_h) \text{d}l + \int_{\Sigma} \frac{1}{Z_0} \lambda_h w dl = \frac{2}{\sqrt{\mu_0}} \int_{\Sigma} A_{2x} \text{d}l,
$$

(7)

where $\Sigma$ is the triangulation of domain 2, $\tau$ a stabilization parameter equal to $1/Z_0$ in our examples and $\lambda_h$ the hybrid variable introduced in HDG. It is a continuous variable defined at the interface of the elements, which represents the tangential electric field: $\lambda_h = E_{2y}/Z_0$. In FEM, field $E_{2y}$ is continuous at the frontiers between two elements whereas in HDG it is not. Indeed, fields $E_{2y}$, $H_{2x}$ and $H_{2z}$ are calculated independently in each element and called, as a result, local fields. HDG solution is achieved calculating a hybrid variable $\lambda_h$ on all interfaces with (7) (after having eliminated local fields $E_{2y}$, $H_{2x}$ and $H_{2z}$ with Maxwell’s equations [3]) and then local fields are deduced. Incoming wave component $B_{2y}$ can be calculated according to

$$
B_{2y} = \frac{1}{2j \sqrt{Z_0}} \left( E_{2y} | - \frac{Z_0}{j \mu_0} \frac{\partial E_{2y}}{\partial z} \right),
$$

(8)

or

$$
B_{2y} = \frac{1}{\sqrt{Z_0}} E_{2y} | - A_{2x},
$$

(9)

due to the wave definitions (recalled in [4]). Equation (8) was mentioned in [6], but it appeared that equation (9) provided more accurate results and improved the convergence. $E_{2y} |$ was identified to variable $\lambda_h$ on $\Sigma$ to perform $B_{2y}$ calculation.

In the linear system (4), matrices are never explicitly built; (4) of size $2N$ is solved by a restarted GMRES method [7]. Iterative solution is stopped when the norm of the residue has been divided by $10^{6}$. A comparison with the BICGSTAB method [8] is also discussed in the next section.

### III. Numerical results

Three test cases are considered in 2D: diffraction of a guided mode in vacuum on a perfect sheet (Fig. 2a), diffraction of a guided mode on a microstrip line in vacuum and diffraction of a guided mode on a microstrip line printed on an inhomogeneous substrate ($\epsilon_r, 1 = 1$ and $\epsilon_r, 2 = 5$) (Fig. 2b). Some preliminary results are also given for a 3D configuration.

#### A. Diffraction of a guided mode on a perfect sheet

The example of Fig. 2a is studied with $H = 1.27 \text{cm}$ and $a = 1.27 \text{cm}$ at $16 \text{GHz}$. The source wave, $B_0$, corresponds to the TE$_1$ mode. Domains 1 and 2 are vacuum. Analytical expressions for electric and magnetic fields being known, the relative discretization error in $L_2$-norm, defined by

$$
\text{Error}_{\text{analytical}} = \sqrt{\frac{\text{max} |E_{y, \text{analytical}}|}{\int_0^a |E_{y, \text{analytical}} - E_{y, \text{numerical}}|^2 dx}}^{1/2}
$$

(10)

is provided in Fig. 3 for the E-field, where mesh step is the edge length.

The TE$_1$ mode expression is given by

$$
B_{0y} = \frac{\sqrt{Z_0}}{Z_{1e,1} + Z_0} f_1(x) \text{ where } Z_{1e,1} = \sqrt{\frac{j \mu_0}{a}} \sqrt{\frac{\pi}{a} - k_0^2},
$$

(11)

and where $f_1(x) = \sqrt{\frac{2}{a}} \sin \left( \frac{\pi}{a} x \right)$.
The FEM is implemented with quadrangular elements and first order approximation (FEM-Q1) and HDG with triangular elements and first order approximation (HDG-P1). The HDG-P1 method provides better results as far as relative error is concerned (see Fig. 3). It is noteworthy that the HDG discretization results in more degrees of freedom. Convergence order is defined by

$$\text{order} = \frac{\log(\text{Error}_h_1) - \log(\text{Error}_h_2)}{\log(h_1) - \log(h_2)}$$ (12)

where Error_1 and Error_2 correspond to the relative discretisation errors in L2-norm respectively for mesh step h_1 and for refined mesh step h_2. Here, the convergence order of the three methods is 2, which means that they converge in h^2, h denoting the mesh step. A comparison between hybrid methods using HDG-P0, HDG-P1 and HDG-P2 [3] in domain 2 was also performed for the E-field. This comparison shows that convergence order is 1 with HDG-P0, 2 with HDG-P1 and also 2 with HDG-P2 because WCIP limits convergence order, but relative error on this example is improved with HDG-P2.

**B. Diffraction of a guided mode on a microstrip line**

A microstrip line is inserted on the surface Σ (see Fig. 2b). It is centered and the metal proportion compared to air is 50%. Since domains 1 and 2 are vacuum, dielectric permittivities are ε_1 = 1 and ε_2 = 1. In this case, an analytical solution is not available and therefore, the chosen reference is the solution obtained with the WCIP alone, meshing the domain with N = 2^15 where N is the number of segments on Σ. We inject the TE_1 mode on the microstrip line and we calculate the relative error on the E-field and the J-current compared to the WCIP reference. Relative errors on electric field and current are respectively summarized in Tables I-IV. Mesh size represents the edge length ratio of the rectangles compared to the initial mesh. For instance, 1/2 means that the step size is twice smaller than the initial step size in both axes. Initial mesh is characterized by a step size of 794µm in both directions x and z.

We observe that convergence orders are respectively 1 and 0.5 for E-field and J-current (order reduction coming from the discontinuity between metal and dielectric) in TE case whatever method used in domain 2, with very close relative discretization errors between hybrid methods. Furthermore, convergence orders are the same between hybrid methods WCIP/HDG-P0 and WCIP/HDG-P1, probably because of the low regularity of the solution. These results motivate the polynomial adaptivity when there are metal and dielectric discontinuities at the interface.

**C. Diffraction of a guided mode on a microstrip line printed on an inhomogeneous substrate**

On Fig. 2b, a microstrip line is printed on a substrate of permittivity ε_2, surrounded by two layers of permittivity ε_1. We take here ε_2 = 5 and ε_1 = 1, i.e. vacuum. Our reference is obtained meshing hybrid method WCIP/FEM-Q1 with N = 2^{10}. Relative discretization errors are provided in Table V and VI. For the E-field, convergence order is close to 1 and for the J-current, it is between 0.5 and 1.

In order to observe the influence of GMRES on the number of iterations needed to solve the linear system (4), a compari-
son with another iterative method (BICGSTAB [8]) is achieved in Table VII. We can conclude that GMRES requires less iterations, in particular for fine meshes.

D. Diffraction of a guided mode on a perfect sheet with WCIP 2D/HDG 3D

We present preliminary results which correspond to the test case already considered in subsection III-A but now in 3D. Here TE and TM modes are coupled. Furthermore, the FMT is quite different from the 2D case but the linear system (4) is unchanged. Geometry of domain 2 is therefore a cube of length 2.27 cm meshed in a structured way with tetrahedra in domain 1. Σ is a surface meshed with rectangles that are called Kuhn faces.

TABLE V

<table>
<thead>
<tr>
<th>Mesh size (x10^{-1})</th>
<th>WCIP/FEM-Q₁</th>
<th>WCIP/FDTLM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative error</td>
<td>Order</td>
<td>Relative error</td>
</tr>
<tr>
<td>1</td>
<td>2.2x10^{-2}</td>
<td>-</td>
</tr>
<tr>
<td>1/2</td>
<td>1.25x10^{-2}</td>
<td>0.8380</td>
</tr>
<tr>
<td>1/4</td>
<td>6.50x10^{-3}</td>
<td>0.9388</td>
</tr>
<tr>
<td>1/8</td>
<td>3.26x10^{-3}</td>
<td>0.9959</td>
</tr>
<tr>
<td>1/16</td>
<td>1.50x10^{-3}</td>
<td>1.1164</td>
</tr>
</tbody>
</table>

D. Diffraction of a guided mode on a perfect sheet with WCIP 2D/HDG 3D

We present preliminary results which correspond to the test case already considered in subsection III-A but now in 3D. Here TE and TM modes are coupled. Furthermore, the FMT is quite different from the 2D case but the linear system (4) is unchanged. Geometry of domain 2 is therefore a cube of length 2.27 cm meshed in a structured way with tetrahedra in domain 1. Σ is a surface meshed with rectangles that are called Kuhn faces.

TABLE VI

<table>
<thead>
<tr>
<th>Mesh size (x10^{-2})</th>
<th>WCIP/FEM-Q₁</th>
<th>WCIP/FDTLM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative error</td>
<td>Order</td>
<td>Relative error</td>
</tr>
<tr>
<td>1</td>
<td>2.11x10^{-2}</td>
<td>-</td>
</tr>
<tr>
<td>1/2</td>
<td>1.43x10^{-2}</td>
<td>0.5615</td>
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<td>1/4</td>
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<td>1/8</td>
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<td>0.6879</td>
</tr>
<tr>
<td>1/16</td>
<td>3.31x10^{-3}</td>
<td>0.8137</td>
</tr>
</tbody>
</table>

D. Diffraction of a guided mode on a perfect sheet with WCIP 2D/HDG 3D

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TABLE VII

<table>
<thead>
<tr>
<th>Mesh size (x10^{-2})</th>
<th>WCIP/FEM-Q₁</th>
<th>WCIP/FDTLM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative error</td>
<td>Order</td>
<td>Relative error</td>
</tr>
<tr>
<td>1</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>1/2</td>
<td>27</td>
<td>30</td>
</tr>
<tr>
<td>1/4</td>
<td>48</td>
<td>51</td>
</tr>
<tr>
<td>1/8</td>
<td>68</td>
<td>80</td>
</tr>
<tr>
<td>1/16</td>
<td>84</td>
<td>147</td>
</tr>
</tbody>
</table>

D. Diffraction of a guided mode on a perfect sheet with WCIP 2D/HDG 3D

We present preliminary results which correspond to the test case already considered in subsection III-A but now in 3D. Here TE and TM modes are coupled. Furthermore, the FMT is quite different from the 2D case but the linear system (4) is unchanged. Geometry of domain 2 is therefore a cube of length 2.27 cm meshed in a structured way with tetrahedra in domain 1. Σ is a surface meshed with rectangles that are called Kuhn faces.

IV. CONCLUSION

In this work, we have realized a feasibility study of the hybridization technique between the WCIP and other volumic methods. A convergence order of 2 has been emphasized in a canonical case whatever the hybrid method (FEM-Q₁, HDG or FDTLM) and using HDG-P₂ does not improve convergence order. The insertion of a microstrip line between both domains is also relevant, because the 3 methods provide similar results, namely a convergence order of 1 for E-field and an order of 0.5 for electric current for a TE₁ mode in excitation. We manage to tackle the intrinsic inhomogeneity problem of the WCIP comparing hybrid methods WCIP/FEM-Q₁ and WCIP/FDTLM. The test case was a microstrip line printed on an inhomogeneous substrate. Convergence orders of 1 for E-field and between 0.5 and 1 for J-current were found. It constitutes the basis for 3D work. Consequently, we implemented the same procedure for the hybridization between WCIP-2D and HDG-3D, with a preliminary validation considering the vacuum case. It yielded a convergence order is between 2 and 3, which is in accordance with expectations from 2D results. These results are rather promising for considering more complex 3D cases, namely calculations on electric currents on planar circuits enforcing electronic specific functions.

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