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MEASURING PUBLIC PREFERENTIAL POLARIZATION

Ugur OZDEMIR
Ali Ilhsan OZKES

Cahier n° 2014-06
Measuring Public Preferential Polarization

Ugur Ozdemir∗
Wallis Institute
University of Rochester

Ali Ihsan Ozkes†
Department of Economics
Ecole Polytechnique

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Abstract

We adapt an axiomatically derived measure of polarization due to Esteban and Ray (1994) to measure polarization of political preferences. Previous work used different measures such as variance, kurtosis, Cronbach’s alpha, median distance to median and the mean distance between groups. Yet, none of these measures are theoretically connected to a notion of polarization. Although the initiation of the current one is in the lieu of income inequality measurement, it is conceptually suitable for preferential polarization as well. This paper offers a methodology for that purpose. The second contribution of the paper is that we use the Aldrich-McKelvey Scaling to correct for differential-item functioning in estimating ideal points of the individuals. We use the American National Election Survey Data for years between 1984-2008 to implement the theory offered in the paper. Our findings suggest that there is not a statistically significant increasing trend in polarization in this time period in many issue dimensions but there is an upward trend in the latent ideology dimension which is significant during the 1990s.

∗W. Allen Wallis Institute of Political Economy, University of Rochester. uozdemir@ur.rochester.edu.
†Murat Sertel Center for Advanced Economic Studies and Department of Economics, Istanbul Bilgi University and Department of Economics, École Polytechnique. ali-ihsan.ozkes@polytechnique.edu.
1 Introduction

There is a never ending public and academic debate on the trend of public polarization in the U.S. for the last couple of decades. Although politicians were never exempt from it  

Obama’s victory speech is particularly relevant for he reveals his corner on the issue by taking side with those who claim that the public is not polarized but the elites are. In the academic dimension, there are two major camps on the issue: those who forefront an increase in the polarization and those who hold that if anything happened it is not polarization but sorting.

In this paper we argue that the measures used in the literature are not theoretically connected to a notion of polarization and most of the disagreement arises from this very fact. We adapt an axiomatically derived measure due to (Esteban and Ray, 1994), which originates in the measurement of income inequality but is nevertheless conceptually suitable for measuring preferential polarization. Before going into detail as to why we preferred this method and how we proceed let us first clarify a bit more the subject matter.

Aside from the conceptual discussion of what polarization is, we should first make clear polarization of whom and what we will be analyzing. As it is prevalent in the literature, either public (or mass) or elite polarization is to be measured. In another dimension we have to choose between partisan (marked with the distance between party affiliates) and preferential (or attitudinal, opinion) polarization. Hence, we might summarize the issues with the following matrix.

<table>
<thead>
<tr>
<th>Elite Partisan</th>
<th>Elite Preferential</th>
</tr>
</thead>
<tbody>
<tr>
<td>Public Partisan</td>
<td>Public Preferential</td>
</tr>
</tbody>
</table>

This paper is an attempt at providing insights into the public preferential polarization in the United States since 1980s. For that purpose, we introduce the use of a method for measuring polarization of the public opinion in the form of responses to survey questions while discussing the conceptualization and assumptions that lie behind our methods.

Citizens do not have a House-like institution with devices such as roll call votes to directly reveal their political ideologies intensively in a dynamic manner. Also, it is not always the best

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1 In fact, as in the example of the influential “culture war speech” by Patrick Buchanan in 1992 Republican National Convention, politicians have embraced the issue deeply.

2 Sorting refers to the situation where party affiliations of individuals are getting more aligned with ideologies; more and more liberals vote for democrats and vice versa. Today these camps are mainly represented by Alan Abromowitz and colleagues and Morris Fiorina and colleagues respectively.

3 The methodology followed here is readily applicable for elite preferential polarization as well. We do this in an ongoing project.
way to extract ideologies from the votes of the public for offices (presidential, house or senate) simply due to representation issues, as argued in (Fiorina and Abrams, 2008). Hence it comes to public opinion surveys where large numbers of individuals are asked about many issues that are thought to be salient and representative regarding ideological preferences.

Aside from the measure, there are two further methodological apparatuses we employ in the paper. First, we use a scaling due to (Aldrich and Mc Kelvey, 1977) to correct for differential-item functioning that arises when respondents interpret issue scales like the standard seven-point liberal-conservative scale differently and distort their placements of the stimuli and themselves in estimating ideal points of the individuals. Second, we use the scaling due to (Poole, 1998) in estimating the latent ideological dimension of respondents.

We delve into the American National Election Survey data for years between 1982-2008 to implement the framework developed in the paper. Our findings suggest a significant increase in the polarization in the latent ideological dimension during the 1990s. Although there is an upward trend during the 2000s, we are unable to classify this as significant. The trends in the separate issue areas show some significant changes but do not present a clear pattern.

The organization of the paper is as follows. In Section 2 we explore the previous literature on the subject and expose some measures of polarization utilized before. Section 3 is devoted to introduction of measures while Section 4 is devoted to the methodology and data. Section 5 comments on the results. We conclude afterwards in section 6 while addressing some limitations and further research interests.

2 Literature review

Although the concept of polarization has a clear intuitive meaning, it is not as clear when it comes to formally defining and measuring it. (DiMaggio et al., 1996) conceptualize polarization process as the motion in the opinions toward the poles of a distribution. A large body of research, in light of this view, hence searches for clusters around poles, or simply bi-modality, in distributions representing public preferences. In fact what we see in (Fiorina et al., 2005), (Fiorina et al., 2008), (Fiorina and Abrams, 2008), (Fiorina and Abrams, 2010) along side (DiMaggio et al., 1996) (updated in (Evans, 2003)) is this kind of an approach: commenting on the variation in distribution of preferences over years in form of a decrease in the center and increase in the extreme ends or comparing variances, kurtosis or simply the weights of extreme category responses. As strongly emphasized in (Fiorina and Levendusky, 2006), no polarization is observed by these works for the last couple of decades; instead, voters aligned better: correlation between policy views and partisan identification increased.

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4 In (Abramowitz and Saunders, 2006), (Abramowitz, 2006) and (Jacobson, 2006) also we see utilization of standard deviation variation as an indicator of polarization.

5 (Fiorina and Abrams, 2008), however, acknowledges that the activists (in fact, campaign-active partisans) are polarized since 1970s.

6 See also, inter alia, (Davis and Robinson, 1997), (Baker, 2005), (Layman and Carsey, 2002) and (Levendusky, 2009).
But, let us look at what (Downey and Huffman, 2001) noted, illustrated in Figure 1 below. Here we end up in a trimodal distribution from a normal-like distribution when half-way masses on both sides move in equal weights to middle and extreme points. In this case kurtosis and variance stay silent. 

Figure 1: Kurtosis and variance cannot tell the difference.

Next, consider the following Table 1 reproduced from (Fiorina et al., 2008), which shows the change from 1984 to 2004 in percentages of respondents that chose each point on the scales for six items, for illustration of another example of an approach in the same vein.

<table>
<thead>
<tr>
<th>Item 1</th>
<th>6</th>
<th>2</th>
<th>3</th>
<th>0(-9)</th>
<th>0</th>
<th>-2</th>
<th>-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Item 2</td>
<td>5</td>
<td>4</td>
<td>5</td>
<td>-3(-5)</td>
<td>-3</td>
<td>-3</td>
<td>-2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Item 3</th>
<th>0</th>
<th>-2</th>
<th>-5</th>
<th>-5(-7)</th>
<th>-1</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Item 4</td>
<td>-5</td>
<td>-4</td>
<td>-3</td>
<td>-5(-4)</td>
<td>8</td>
<td>4</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 1: Percentage changes from 1984 to 2004 in positions for six items in the American National Election Studies. Numbers in parentheses are changes when “Don’t Know”s are treated as moderates.

The changes in variances are insignificant due to (Downey and Huffman, 2001) who provide also evidence on how prevalent trimodal distributions, for example, in General Social Survey data.
stands for the fact that the percentage of respondents who chose the first conservative position from center decreased by one, from 1984 to 2004, while ”2” in the row means the percentage of respondents that chose the second liberal position from center for “Item 1” increased by 2. The authors argue that polarization can only be claimed to be observed for the fifth item. In items 1 and 2 they see a shift towards left and in items 3 and 4 towards right that leads exclusion of these from the polarized in their view. In item 6, they see no change and the only case they see polarization is item 5 where a movement from center to extremes are observed. However if the moves are from the high supported side to the low so that the side with low approaches to the side with high in size, we would be inclined to see polarization.\footnote{Not that we suggest increase in polarization for that particular data, but the point is that this method might lead to inappropriate conclusions.} Consider the following case, in Figure 2, where a right to left shift induces a distribution with two equal masses. The latter situation seems clearly more compelling to be tagged as more polarized.

![Figure 2: A left shift.](image)

Although not as severe as (Hunter, 1992) depicts, some prominent authors claimed an increasing trend of polarization among American public for recent decades by employing another set of measures. Leaving behind phenomena such as partisan, geographical or religion-based polarization -that are argued sometimes wrongly in place of public preferential polarization- we move forward to cast an eye on two prevalent methods that can be found especially in Alan Abramowitz and his colleagues’ works.

The first of these methods includes a comparison of some sort of correlation between issues. For a better exposition see Figure 3 where each respondent $i, j$ and $k$ picks positions on seven different issues, from extremely liberal to extremely conservative, in the same manner. To keep everything comparable with (Abramowitz and Saunders, 2008), where we gathered the following method, we depict one of the issues (the issue number 1) as a four-scale issue.\footnote{The two positions with bold markers in between the extreme points are to denote interim positions for the four-scaled issue.}
Respondent $i$ has four extremely liberal positions, two centrist positions and one extremely conservative position. Respondent $j$ has four slightly liberal positions, one liberal position, one centrist position and one slightly conservative position. Respondent $k$ has five extremely conservative positions and two extremely liberal positions. The method proposes to label respondents with 0 or 1 as absolute value difference between liberal and conservative positions with low, respondents with 2 or 3 as absolute value difference with moderate and respondents with 4 or beyond with high in polarizedness scale. It advances on by comparing the proportions of highly polarized respondents among years. So in a year with only $j$ type respondents, we will have only highly polarized profile hence high polarization while in a year with any combinations of $i$ and $k$ (even when half are of $i$ type and half are of $k$) we will have a moderate profile.\textsuperscript{10}

One other prevalent indicator used in some papers such as (Fiorina and Abrams, 2010), (Abramowitz, 2010), (Abramowitz, 2006), (Jacobson, 2006), (Abramowitz and Saunders, 2005) and (Abramowitz and Saunders, 2006) is the change in the ratio of the weights on the extremist responses to the weights on the centrist. However, let’s consider the situation similar to the distribution A in Figure 1 with the only difference that the middle mass is now 100 instead of 200. If we similarly dissolve the masses in between extremes and center into both sides equally, the ratio of the extreme masses to the middle mass doesn’t change whereas this change, we argue, should increase polarization.

For another review of methods in the literature, we refer the reader to (Hill and Tausanovitch, 2014) where provided an analysis of measures used previously, related especially to the application to the data from American National Election Studies.\textsuperscript{11} Other interesting reviews may include, among others, (Hetherington, 2009), (Prior, 2013), (Fiorina and Abrams, 2008), (Nivola and Brady, 2006) and (McCarty et al., 2006).

We now turn to a novel approach to the measurement of polarization, borrowed from the

\textsuperscript{10}These methods are also applied to some omnibus scales that are prone to some rescaling issues. We provide, in the Appendix B, examples of rescalings found in the aforementioned literature.

\textsuperscript{11}The authors come up with a list of ten different measures employed.
3 Measuring Preferential Polarization

Polarization is usually seen as an intensifying disagreement in preferences (or ideologies). Naturally, assessment of it depends highly on the context. In American politics, polarization is generally seen as a separation of politics into liberal and conservative camps, remarked in (McCarty et al., 2006). At the congress and elites level, academic literature reflects more or less a consensus on that it has increased. On the contrary, the literature is rather polarized on if the polarization of public opinion has increased (or took place, if taken as a process) or not.

Obviously, most of the debate until today rests on the fact that there does not exist a unique definition of polarization, especially for the case of public preferences. Following the work of Joan Esteban and Debraj Ray that is initiated as a study on income polarization, we argue that the (public preferential) polarization can be seen as the aggregation of pairwise antagonisms within society. The antagonism between two individuals can be seen as a function of the distance between positions (alienation) and increasing with the support of the held positions (identification). (Esteban and Ray, 1994) characterized first a class of functions of this feature with some plausible axioms. This class of measures, basically, consists of those that sum up all pairwise alienations, weighted in a certain way with the population shares of the positions of the components.

To illustrate, suppose we have $N$ individuals and $y \in \mathbb{R}^N$ is the vector of positions. For each such vector $y$ let there be $m(y) \in \mathbb{N}$ positions with nonzero support. Let $M(y) = (M_1(y), ..., M_{m(y)}(y)) \in \mathbb{R}^{m(y)}$ be the vector of those positions. Furthermore, let $\phi(y) = (\phi_1(y), ..., \phi_{m(y)}(y)) \in \mathbb{R}_{++}^{m(y)}$ be the vector of frequencies for each reported position, hence $\sum_{j \in \{1, ..., m(y)\}} \phi_j(y) = 1$, for all $y$. The class of functions given by

$$P(y, \alpha) = K \sum_{i=1}^{m(y)} \sum_{j=1}^{m(y)} [\phi_j(y)(\phi_i(y))]^{1+\alpha} |M_i(y) - M_j(y)|$$

where $\alpha \in [1, \alpha^*]$ with $\alpha^*$ being approximately 1.6 is the only class within the description above that satisfies the aforementioned axioms. We think this measure is conceptually better suited for measuring public polarization for the cases where the distribution can be, or originally is, described with clusters on a line. To illustrate, we discuss some properties along with examples we introduced before.

The function has a value zero whenever there is full consensus in the society. It is hence the
case also when we have all of the respondents have the same type \((i, j\) or \(k)\) in the clustering example above. Naturally, the case where half is type \(i\) and half is type \(k\) is more polarized than the consensus case for any plausible reduction of dimensions. For left (or right) shift discussion of Figure \(2\) above, consider the case where \(p\) portion of the society hold a leftist position and the rest \(1 - p = p + \epsilon\) hold a rightist position. The polarization, if the distance is \(d\) and \(\alpha = 1\), in this case is: 
\[
p^2(p + \epsilon)d + p(p + \epsilon)^2d = dp(p + \epsilon)[2p + \epsilon].
\]
When \(\epsilon\) is equally shared by both positions however, the new measure is:
\[
d(p + \frac{\epsilon}{2})^2[2p + \epsilon].
\]
The difference, \(\epsilon^2/4\), is always positive.

Finally, the kurtosis-variance example goes as follows: The distribution \(A\) has value 0.264 while distribution \(C\) has almost double, 0.448. Thus, we established that the measure gives the desired comparisons in the discussions above.

This is not the first paper to propose the use of this measure for polarization in politics. \[\text{Clark, 2009}\] employs it to measure ideological polarization on the Supreme Court while \[\text{Oosterwaal and Torenvlied, 2010}\] compares the trend in polarization in the Netherlands measured by \(P\). However, as it will be elaborated further below, we will be dealing with data generated from large surveys that describe the distributions of preferences with (estimated) density functions. This very fact brings the need of a measure applicable to such distributions and the version of the measure to be utilized in this paper, thus, is due to \[\text{Duclos et al., 2004}\] where the authors provide a natural extension. The class of functions, (denoted by DER from now on)
\[
P_\alpha(f) \equiv \int \int f(x)^{1+\alpha} f(y)|x - y|dxdy
\]
with \(\alpha \in [0.25, 1]\), defined on all continuous densities in \(\mathbb{R}_+\) is shown to be characterized by a set of axioms in the same vein as before. A detailed analysis is provided in the Appendix C.

## 4 Data and the estimation methodology

We use the ANES data for the election years between 1984 and 2008 for our empirical analysis. There are seven questions we are studying:\[15\]

1. Liberal/conservative, with scale 1 (extremely liberal) to 7 (extremely conservative)
2. Government aid to blacks, with scale 1 (help) to 7 (no help)
3. Defense spending, with scale 1 (greatly decrease) to 7 (greatly increase)
4. Jobs and living standards, with scale 1 (provide) to 7 (let go)
5. Government services and spending, with scale 1 (few provision) to 7 (more provision)
6. Health insurance, with scale 1 (govt plan) to 7 (private plan)
7. Abortion, with scale 1 (never permit) to 4 (always permit)

These are chosen following the earlier literature \[\text{Abramowitz and Stone, 2006, Abramowitz and Saunders, 2008, Abrams and Fiorina, 2012, Fiorina and Abrams, 2008, Fiorina et al., 2005}\].

\[15\]The exact wording of these questions are given in Appendix D.
Figure 4: The Outline of the Empirical Strategy

Figure 4 summarizes our empirical approach. Basically there are two different procedures we follow. In the first one, we use the Aldrich-McKelvey scaling in order to correct for the differential item functioning bias and measure polarization in separate issues using these ideal points. Note that A-M technique produces ideal points on a continuous scale hence we are able to estimate the distribution confidently with DER. In the second one, we initially derive a single policy dimension from the underlying seven issues using the Poole’s scaling and estimate the polarization using this “aggregate” distribution.

In what follows we are going to elaborate these different steps involved in our analysis.

4.1 Estimation of the DER Measure

The estimator for the function in (2) à la (Duclos et al., 2004) is given by:

\[
\pi_\alpha(f) = \frac{\sum_{i=1}^{n} w_i f(y_i)^\alpha \cdot a(y_i)}{\sum_{i=1}^{n} w_i}
\]  

(3)

where \(w_i\)s are the weights given to the positions, \(f(.)\) is the density function of the ideal point distribution, \(y_i\)s are empirical quantile for percentiles between \((i - 1)/n\) and \(i/n\), \(\alpha \in [0.25, 1]\) and

\[
a(y_i) = \mu + y_i \left( \frac{2 \sum_{j=1}^{i} w_j - w_i}{\sum_{j=1}^{n} w_i} - 1 \right) - \left( \frac{2 \sum_{j=1}^{i-1} w_j y_j + w_i y_i}{\sum_{i=1}^{n} w_i} \right).
\]

(4)

If the weights are taken to be equal, the function \(\pi_\alpha\) reduces to

\[
\pi_\alpha(f) = \frac{1}{n} \sum_{i=1}^{n} f(y_i)^\alpha \cdot a(y_i)
\]

(5)

where

\[
a(y_i) = \mu - y_i + \frac{2}{n} [y_i (r(i) - 1) - \sum_{j=1}^{i-1} y_j].
\]

(6)
The following normalization leads to the final version of the function which is used to estimate polarization:

\[ P_\alpha(f) = \frac{1}{2\mu^{1-\alpha}}\pi_\alpha(f). \quad (7) \]

Hence, in order to find the polarization index, we need to have the empirical distribution of individual ideal points. We find this distribution using the Gaussian kernel density estimation:

\[ \hat{f}(y) \equiv n^{-1} \sum_{i=1}^{n} K_h(y - y_i) \quad (8) \]

\[ K_h(z) \equiv h^{-1}K(z/h) \quad (9) \]

\[ K(u) = (2\pi)^{-0.5} \exp^{-0.5u^2} \quad (10) \]

where \( h \) is the bandwidth for kernel function and it is chosen so as to minimize the mean squared error.

The meaning of \( \alpha \) needs some further elaboration since it will be critical in interpreting our results. As it was discussed earlier, there are two forces which determine the polarization level: alienation and identification. An individual located at point \( x \) feels alienation vis-a-vis another located at \( y \), and this alienation increases with the distance between these individuals, \(|x - y|\). However, for this alienation to be translated into polarization, the individual must - to a greater or a lesser degree- identify with the rest of the society. An individual located at ideal point \( x \) experience a sense of identification and this is given by \( f(x) \).

The weight of the density function on the measure increases with \( \alpha \) as seen in the functional forms in (5) and (7), which would mean that the weight assigned to identification increases. In terms of what we see from the empirical distributions, the shadow of peaks and multi-modality kicks in as \( \alpha \) gets bigger. Note that there is always a “trade-off” between the height of the distribution and the width of the distribution since the area underneath any density function is constant and equal to 1. The former is related to identification and the latter with increasing distances, hence alienation. The interplay between these two forces is the most critical dynamic behind the DER measure, and \( \alpha \) parametrizes the relative weights of these two forces.

Finally, the standard errors are computed by bootstrapping and all analyses are carried out with the statistical computing environment R.

4.2 Aldrich-McKelvey Scaling

A-M Scaling (Aldrich and McKelvey, 1977) is a solution offered for the differential item functioning problem in estimating the positions of political stimuli and survey respondents along a latent policy dimension from issue scale data. This problem arises in all cases where the respondents interpret issue scales differently and distort their placements of the stimuli and themselves. The method treats placements as linear distortions of the “true” positions of the stimuli. By estimating

\footnote{We have tried other kernels for robustness, but results do not change.}

16
each respondents perceptual distortion parameters, it is possible to recover the locations of the stimuli as well as the respondents.

Let $z_{ij}$ be the perceived location of the political stimulus $j$ (party or candidate) by individual $i$. The A-M model assumes that the individuals reports a noisy linear transformation of the true location of stimulus, $z_j$:

$$z_{ij} = \alpha_i + \beta_i z_j - u_{ij}$$

where $\alpha_i$ is the shift term, $\beta_i$ is the stretch term, and $u_{ij}$ satisfies the usual Gauss-Markov assumptions for error terms. So in cases where you not only have the self-placements of the voters on the issue scale but also their perceptual placements of different political stimuli such as political parties and candidates, A-M scaling offers a way to correct for the bias due to interpersonal differences in interpreting the scale.

Aldrich and McKelvey estimate these parameters using a Lagrange multiplier. We use the maximum-likelihood based method for A-M scaling available in the R package basicspace (Poole et al., 2012). In fact, very recently, a Bayesian implementation has been introduced in (Armstrong et al., 2014). There are two main advantages of this Bayesian approach: it can handle missing data and it allows for idiosyncratic error variances. The latter is an important improvement since the assumption that respondents are assumed to have an unequal likelihood of reporting an incorrect ordering of the stimuli is quite unrealistic given the variation in respondents’ political sophistication. We will be replicating our results using this Bayesian methodology in the next version of the paper.

### 4.3 Poole’s Scaling

This is a scaling procedure, introduced in (Poole, 1998), for estimating the latent (unobservable) dimensions underlying a set of manifest (observable) variables. We will be using this technique to derive the underlain ideological positions of the respondents that are, we believe, observable through their self-placements on different issues. In contrast to more prevalent dimension reduction techniques such as factor analysis which work with a covariance matrix computed from the data matrix, it analyzes the data matrix directly without any intermediary transformations of the original data. This allows for the method to handle missing observations which is an important problem especially with the survey data. (Poole, 1998) provides Monte Carlo experimentation to show that the procedure performs well, in deed, in reproducing the missing elements of a matrix even when the level of missing data is high.

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17 Zero in expectation, homoscedasticity and independence.

18 In effect this is a method to obtain an Eckart-Young lower-rank approximation matrix of a matrix with missing entries.
5 Observations on the results

The plots which summarize our findings are given in the Appendix A, Figures 5-8. These are the polarization estimates with %95 confidence intervals around them. Our main finding is depicted in Figure 5a. There is an upward trend in polarization for the primary ideology dimension after 1992. The increase is significant\textsuperscript{19} between years 1992 and 1996 and between 1996 and 2000. Although the trend continues up until year 2008, these increases are not significant. We also observe a decrease in polarization from year 1988 to 1992. Hence our results suggest that 1990s was the decade in which public polarization increased significantly. One other observation is that the polarization differences get more evident as $\alpha$ gets smaller, or as the relative weight of alienation increases. This is due to a “release” in the distribution which increases alienation, as can be seen in Figure 9a.

We are not going discuss the results for each and every issue-year pair. Instead we will make some particular observations which we think are interesting for these issues.

The polarization on the blacks issue also increases from 2000 to 2004 and from 2004 to 2008, but this time for $\alpha = 1$ and $\alpha = 0.5$. The reason why the polarization also increases for $\alpha = 0.5$ is that the multi-modality is not as severe and peaks are not as high in 2008 and the distances are greater in 2004 compared to the job issue as depicted in Figure 10a. One rather interesting observation about this alienation-identification framework is that the polarization on the blacks issue increases significantly for $\alpha = 0.25$ and decreases significantly for $\alpha = 1$ from 1984 to 1988. This is a nice illustration how the interaction between alienation and identification can yield significantly different results under different values of $\alpha$.

The polarization on the defense spending issue has an upward trend from 1992 until 2000 for $\alpha = 0.25$ and $\alpha = 0.5$. This is in accord with the fact that American public was quite divided on this issue along the partisan lines starting with the First Gulf War in 1990. This divide continued with other military operations US participated such as the Bosnian and Kosovo wars. It is also interesting to note that the polarization in this issue decreases significantly from 2000 to 2004. This can be attributed to the increased external threat awareness and nationalistic sentiments which led to a convergence of opinions on this issue after the September 11 terrorist attacks.

The polarization on the job creation issue increases from 2004 to 2008 only for $\alpha = 1$, i.e, when the weight of identification is highest. As it can be seen from the Figure 9b, the multimodal (peaked) structure of the distribution derives this result. This observation is quite consistent with the fact that the public became more polarized about the extent to which government should intervene an economy in recession starting with the 2000s. A similar pattern is visible for the government services issue which supports this explanation.

The polarization on the abortion issue makes a significant peak in year 2000. The fact that, together with gay/lesbian rights, it was particularly an important issue in the 2000 presidential election concurs this observation. Although the debate on abortion is still alive, it never gained that momentum it had in between Al Gore and George W. Bush.

\textsuperscript{19}Unless otherwise noted, we refer to a significance at the 0.05 level.
6 Conclusion

In this paper we have suggested a methodology to measure public polarization and implemented it for the election years in the U.S. from 1984 to 2008. Pointing out the need for a theoretically supported measure in the literature, we adapted a measure à la (Duclos et al., 2004) for this purpose. With its intuitive axiomatic support, DER is a reliable measure to help us understand the dynamics of preferential polarization. Rather than using some summary statistic, it uses the distribution of ideal points in the society as a whole. By parametrizing the relative weight it puts on alienation versus identification, it provides a nice micro-behavioral foundation. In fact, identifying how the interaction between alienation and identification lead to changes in polarization can be an important step towards having a more nuanced understanding of the nature of polarization in different issues. The two other tools we employed were the A-M scaling (Aldrich and McKelvey, 1977) to correct for the differential item-functioning and Poole’s scaling (Poole, 1998) to derive the latent policy dimension.

Our empirical findings show that although the US polarization measured on the underlying latent ideology dimension has an upward trend after 1992, this is only significant during the 1990s. There is not a common pattern when we do an issue-wise analysis. Although there are significant changes between some time periods, issues do not seem to have a shared pattern. This discrepancy between the aggregated ideology dimension and its components is probably due to the fact that individuals are changing their opinions on different issues but becoming more consistent about their choices on different issues overall. This certainly needs a more careful analysis of the particular issues and time periods.

6.1 Limitations of the Framework and Future Work

We study polarization on a unidimensional policy space. However, there is a robust finding in the spatial voting literature that a two dimensional policy space is needed in order to capture the ideological spectrum (Benoit and Laver, 2006). This suggests that developing a polarization measure for multidimensional spaces will be a significant contribution.

We used the same question (issues) to measure polarization every year. Even if this approach is convenient in the sense that it makes comparisons more meaningful, it implicitly assumes that the salient issues do not change from one year to another. This is certainly a restrictive assumption. We could try to identify different issues for different years depending on their saliency. One way to do this would be to run a logit model using the vote choice of respondents as the dependent variable and choose those issues which seem to have significant effect.

We might as well consider giving more weight to those individuals who have higher engagement in the political issues. If public polarization is the sum of all binary antagonisms, then it can be argued that the more “activist” the individuals are the more their effect should be on the level of polarization. As we discussed earlier, the DER measure is general enough to handle these weights.
ANES includes questions that can be used to determine the activism level of the respondents.\footnote{See \cite{Schofield2011}.}

Using alternative dimensions for alienation and identification seems to be another promising direction for future study. One might argue, for instance, that people identify themselves with their income group but the alienation takes place on the liberal-conservative scale.

We can employ the estimated polarization measure as a dependent variable and investigate the causes of polarization and use it as an explanatory variable in order to understand its consequences. The latter is particularly important in that it is related to the fundamental question of why we care about polarization in the first place. After all, the consequences of polarization “are not entirely clear and may include some beneficial as well as detrimental consequences.” as discussed in \cite{Epstein2007}. There have been some work on the consequences of elite polarization on voter turnout \cite{Crepaz1990}, on foreign policy \cite{McCormick1990} and on the judicial system \cite{Binder2000} but the effects of public polarization has not been studied\footnote{One exception is \cite{Abramowitz2006}. This paper however uses proxy measures for polarization, like the ratio of the extreme voters.}. The absence of a well-defined continuous measure was probably one of the reasons why this has been the case.
References


Appendices

A Results

Figure 5: Ideology.

Figure 6: Issues.
Figure 7: Issues.

(a) Jobs Creation Issue

(b) Govt Services Spending

Figure 8: Abortion Issue
Density curves for the latent ideology

(a) Latent ideology 1996-2008

Density curves for the job issue

(b) Jobs issue 2004-2008

Figure 9: Latent ideology and jobs issue.

Density curves for blacks issue

(a) 2004-2008

Density curves for the blacks issue

(b) 1984-1988

Figure 10: Blacks issue comparisons.
B  Rescaling and several other measures

The following analysis of polarization with a rescaling method is provided to make sample of the approaches in aggregating issue-wise data.

(Abramowitz, 2006) employs the scaling method depicted in Figure 11 in the following way. First, the responses from the survey questions are collapsed into three categories (liberal, moderate and conservative) with groupings, respectively, 1 – 3, 4 and 5 – 7 for questions with seven scales. For questions with four scale, the groupings were 1-2, 3 and 4. Second this new scale then combined into a fifteen point liberal-conservative scale, i.e. a respondent with liberal position on all issues got -7. Then, this fifteen point scale collapsed again into a 5 point scale: \((-7, -5)\) as consistent liberals, \((-4, -2)\) as moderate liberals, \((-1, +1)\) as inconsistents etc. Finally, on this 5 point scale, the percentage at the central position, position 3, and positions 1 and 5 is compared over years. A decrease at position 3, for instance, indicated an increase in polarization.

(Abramowitz and Saunders, 2008) also use the average correlation between responses to different issues and see the values 0.20 in 1980s, 0.26 in 1990s and 0.32 in 2004 as a proof of increase in ideological thinking, hence in polarization.

Another measure in the same paper is introduced by first arguing that the ones whose opinions matter first are the ones that are well informed and politically engaged. Political engagement is measured with interest (in political campaign), knowledge (an aggregate of 10 questions including accurately placing candidates on abortion and liberal/conservative scales) and participation. Then comparing the weight of individuals with high stances (on an omnibus scale where responses to 16 questions are collapsed, combined and recoded), for example, among the low and high knowledge individuals, it is shown that polarization is more among the high knowledge individuals.

Finally, the authors also create an overall index of political engagement by combining the three measures of interest, knowledge and participation to eventually show that (although the means are the same) the standard deviation of high engagement group being the double of the low engagement group on the omnibus scale is a strong indicator of the high polarization among the population.

Figure 11: The rescaling method employed in (Abramowitz, 2006).

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high engagement group. Furthermore, they compare the number of respondents on the extreme (left and right) points on the scale for each group.

In another very influential work, (Alesina et al., 1999), we see the median distance to median (MDM) as a measure for polarization where authors argue that polarization in the form of preference variations among ethnic groups is the underlying reason for public good underprovision.

![Figure 12: A case for median distance to median.](image)

Figure 12 presents two distributions, the uni-modal distribution is the standard Normal distribution and the bimodal distribution is a mixture of two Normal distributions with means $-0.8$ and $0.8$ and a common variance of $0.2$. Hence MDM for the bimodal distribution is smaller than the unimodal distribution, leading to conclude under the analysis of (Alesina et al., 1999) that it is less polarized. But this would contradict an intuitive approach that considers heavy concentrations as an ingredient of polarization. In fact MDM is just a proxy for the variance of the distribution and its implications regarding to polarization is no more that the implications of the variance, that of which we demonstrated in the above text as a bad measure of polarization.

C Axiomatic analysis of the DER Measure

The overarching idea of the DER measure is to aggregate pairwise antagonisms in the society. For the sake of illustration, suppose we have a distribution of preferences that can be described by a function $f$ defined on $\mathbb{R}_+$. Assuming that the distance between any two point on $\mathbb{R}_+$ represents the alienation between two individuals with preferences represented with those points, the antagonism, as argued in (Duclos et al., 2004), can be seen as a function of the alienation as well as the identification which would be a function of the density at the point. So the effective antagonism of $x$ towards $y$ under $f$ can be represented by a function $T(f(x), |x - y|)$. It is assumed that $T$ is
increasing in $|x - y|$ and that $T(\cdot, 0) = T(0, \cdot)$. The polarization then would be (proportional to) the sum of all effective antagonisms:

$$P(f) = \int \int T(f(x), |x - y|) f(x)f(y) dxdy.$$  

Within this large class of functions, the analysis advances on searching for subclasses that satisfy certain plausible axioms. Before getting into the presentations of axioms, we need to define one central item of the analysis, namely, a squeeze. A squeeze is a sort of mean-preserving reduction in the spread of a distribution. More specifically, a $\lambda$-squeeze, used in what follows, of $f$ is a transformation such that:

$$f^\lambda(x) \equiv \frac{1}{\lambda} f\left(\frac{x - [1 - \lambda]\mu}{\lambda}\right).$$

Any $\lambda$-squeeze collapses the density inwards towards the global mean and the following properties can be proved. (i) For each $\lambda \in (0, 1)$, $f^\lambda$ is a density function. (ii) For each $\lambda \in (0, 1)$, $f^\lambda$ shares the same mean with $f$. (iii) $0 < \lambda < \lambda' < 1$ implies that $f^\lambda$ second-order stochastically dominates $f^{\lambda'}$. Finally, (iv) as $\lambda \downarrow 0$, $f^\lambda$ converges weakly to the degenerate measure granting all weight to $\mu$.

Given this, a measure in the above class is shown in (Duclos et al., 2004) to satisfy the following four axioms, DER1-4, if and only if it is proportional to

$$\mathcal{P}_\alpha(f) \equiv \int \int f(x)^{1+\alpha} f(y)|x - y|dydx$$

for some $\alpha \in [0.25, 1]$.

**DER1** If a distribution is composed of a single basic density, then a squeeze of that density cannot increase polarization.

---

**Figure 13:** A squeeze should not increase polarization.

**DER2** If a symmetric distribution is composed of three basic densities with the same root and mutually disjoint supports, then a symmetric squeeze of the side densities cannot reduce polarization.

---

**Figure 14:** A symmetric double squeeze should not decrease polarization.
Consider a symmetric distribution composed of four basic densities with the same root and mutually disjoint supports, as in Figure 4. Slide the two middle densities to the side as shown (keeping all supports disjoint). Then polarization must go up.

Figure 15: A symmetric outward slide should increase polarization.

If $P(F) \geq P(G)$ and $p > 0$, then $P(pF) \geq P(pG)$, where $pF$ and $pG$ represent (identical) population scalings of $F$ and $G$, respectively.

### D Data

The wording of the questions we used in the empirical analysis change slightly in different years. We quote here some representative version. The questions are the same in cases where respondents locate themselves and the political stimuli (parties and candidates).

1. **Liberal-Conservative Scale**
   
   We hear a lot of talk these days about liberals and conservatives. Here is a seven-point scale on which the political views that people might hold are arranged from extremely liberal to extremely conservative.

   Q. Where would you place yourself on this scale, or haven’t you thought much about this?

   1. extremely liberal
   2. liberal
   3. slightly liberal
   4. moderate, middle of road
   5. slightly conservative
   6. conservative
   7. extremely conservative
   8. Don’t know
   9. NA
   0. Haven’t thought much

2. **Aid to Blacks**

   Some people feel that the government in Washington should make every effort to improve the social and economic position of blacks. Others feel that the government should not make any special effort to help blacks because they should help themselves.

   Q. Where would you place yourself on this scale, or haven’t you thought much about this?

   1. government should help blacks
   2.
3. **Defense Spending**

Some people believe that we should spend much less money for defense. Others feel that defense spending should be greatly increased.

Q. Where would you place yourself on this scale, or haven’t you thought much about this?
1. greatly decrease defense spending
2. 
3. 
4. 
5. 
6. 
7. greatly increase defense spending
8. Don’t know
9. NA
0. Haven’t thought much

4. **Jobs and Living Standards**

Some people feel the government in Washington should see to it that every person has a job and a good standard of living. Others think the government should just let each person get ahead on his own.

Q. Where would you place yourself on this scale or haven’t you thought much about this?
1. government see to a job and good standard of living
2. 
3. 
4. 
5. 
6. 
7. Government let each person get ahead on own
8. Don’t know
9. NA
0. Haven’t thought much

5. **Health Insurance**
There is much concern about the rapid rise in medical and hospital costs. Some people feel there should be a government insurance plan which would cover all medical and hospital expenses for everyone. Others feel that all medical expenses should be paid by individuals, and through private insurance plans like blue cross or other company-paid plans.

Q. Where would you place yourself on this scale, or haven’t you thought much about this?

1. government insurance plan
2.
3.
4.
5.
6.

7. private insurance plan
8. Don’t know
9. NA
0. Haven’t thought much

6. Abortion

There has been some discussion about abortion during recent years.

Q. Which one of the opinions on this page best agrees with your view? You can just tell me the number of the opinion you choose.

1. By law, abortion should never be permitted.
2. the law should permit abortion only in case of rape, incest or when the woman’s life is in danger.
3. The law should permit abortion for reasons other than rape, incest, or danger to the woman’s life, but only after the need for the abortion has been clearly established.
4. By law, a woman should always be able to obtain an abortion as a matter of personal choice.
7. Other
8. Don’t know
9. NA