On the microlocal modelling of thermoelastic periodic composites

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Introduction

In the recent formulations of various homogenized models for periodic material structures two main lines of modelling can be distinguished. The first line is based on rather intuitive physical assumptions and lead to various engineering theories of composite materials. An excelent example of this approach is given in a book by R. JONES [4], where also a list of references can be found. The second line of approach takes into account some asymptotic theorems of analysis and is resumed in books by A. BENSOUSSAN, J. L. LIONS, G. PAPANICOLAOU [2], E. SANCHEZ-PALENCIA [16] or N. S. BAHVALOV, G. P. PANASENKO [1].

A certain alternative approach to the modelling of periodic composites has been proposed lately in [19, 21, 23, 24 - 26, 28, 29], where the concepts of the nonstandard analysis combined with some postulated a priori heuristic assumptions resulted in new homogenized models of the composites under consideration. These models, in the case of thermoelastic periodic composites, are governed by systems of equations for unknown macrodeformations, macrotemperatures and for certain extra unknowns called microlocal parameters. The microlocal parameters (kinematical and thermal) make it possible to evaluate not only mean but also local values of deformation and temperature gradients and hence stresses and heat fluxes in every material component of the composite. That is why the homogenized models obtained in [19, 21, 23, 24 - 26, 28, 29] can be referred to as the models with microlocal parameters.

The first aim of the paper is to discuss the general form of the homogenized models with microlocal parameters for the thermoelastic periodic composites and then to pass to the linearized form of equations. After that various applications of the derived equations as well as some special problems are analysed.

1. Fundamentals of the Microlocal Modelling

We start with the basic concepts and governing equations of the nonlinear thermoelasticity. By B^0 we define the regular region in R^3 occupied by a thermoelastic body in its natural state. In B_0 we introduce the curvilinear coordinates $X = (X^\alpha) \in \Omega_R$(1), given by a smooth invertible mapping $\varkappa: \Omega_R \to B_0$. The deformed state of a body and the absolute temperature field at a time instant t (here and in the sequal $t \in [t_0, t_f]$) will be given by the mappings $x = \chi(X, t)$, $\Theta = \Theta(X, t)$, $X \in \Omega_R$, respectively.

Let us also define $A(X) \equiv (\nabla\varkappa(X))^{-1}$, $J(X) \equiv \det\nabla\varkappa(X)$, $L(X, t) \equiv 0.5 \times$ $\times [\nabla\chi^T(X, t)\nabla\chi(X, t) - \nabla\varkappa^T(X)\nabla\varkappa(X)]$ and $g(X, t) \equiv \nabla\Theta(X, t)$. We shall assume that there are known: 1) mass density $\varrho_0(X)$, $X \in \Omega_R$ of a body in the natural state B^0, 2) free energy scalar function $\varphi = \varphi(X, A^T(X)L(X, t)A(X), \Theta(X, t))$, $X \in \Omega_R$, 3) heat flux vector function $H_0(X, \Theta(X, t), A^T(X)g(X, t))$, $X \in \Omega_R$, related to B_0. Moreover, let $b(X, \chi(X, t), t)$, $\alpha(X, \chi(X, t), \Theta(X, t,), t)$, $X \in \Omega_R$, be the body forces and the heat adsorption, respectively. Under the aforementioned notations, the governing equations of the nonlinear thermoelasticity can be assumed in the form

$$
\begin{aligned}
&\operatorname{Div} T_R(X, t) + J(X)\varrho_0(X)b(X, \chi(X, t), t) = J(X)\varrho_0(X)\ddot{\chi}(X, t), \\
&\operatorname{Div} h_R(X, t) + J(X)\varrho_0(X)\alpha(X, \chi(X, t), \Theta(X, t), t) = J(X)\Theta(X, t)\dot{\eta}(X, t),
\end{aligned} \tag{1.1}
$$

where

$$
\begin{aligned}
T_R(X, t) &= J(X)\varrho_0(X)\nabla\chi(X, t)\frac{\partial\varphi}{\partial L(X, t)}, \\
h_R(X, t) &= J(X)A(X)H_0(X, \Theta(X, t), A^T(X)g(X, t)), \\
\eta(X, t) &= -\frac{\partial\varphi}{\partial\Theta(X, t)},
\end{aligned} \tag{1.2}
$$

with $\chi(X, t)$, $\Theta(X, t)$, $X \in \Omega_R$, $t \in [t_0, t_f]$, as the basic unknowns.

Now we shall pass to problems in which the thermoelastic body under consideration constitutes a certain periodic composite. To this aim we introduce the known triple $(\delta^1, \delta^2, \delta^3)$ of real positive numbers and define $\Delta \equiv (0, \delta^1) \times (0, \delta^2) \times (0, \delta^3)$. In order to treat Eqs. (1.1), (1.2) as the governing equations of a termoelastic composite we shall assume that the functions $\varrho_0(\cdot)$, $\varphi(\cdot, E, \Theta)$, $H_0(\cdot, \Theta, h)$ in Eqs. (1.1), (1.2) for every $E \in R^{(3\times 3)}$(2) and $h \in R^3$ are Δ-periodic. Moreover, setting $\delta \equiv \sqrt{(\delta^1)^2 + (\delta^2)^2 + (\delta^3)^2}$, we also assume that the length parameter δ is sufficiently small as related to the smallest characteristic length dimension of the region Ω_R. It follows that the number of the periodicity cells $\Delta(Z) := \{X \in \Omega_R | X = Z + Y, Y \in \Delta\}$, $Z = (Z^\alpha)$, $Z^\alpha = \nu\delta^\alpha$, $\nu = 0, \pm 1, \pm 2, \ldots$, in the body under consideration is very large and hence Eqs. (1.1), (1.2) become too complicated to be successfully applied for such periodic composites as the basis for the analysis of engineering problems. Thus the question arises how to pass from Eqs. (1.1), (1.2) to the equations for a certain "homogenized" model of the Δ-periodic composite under consideration. The answer to this question is not unique. As it is known (cf. Introduction), there are many ways in which various "homogenized" models can be formulated. Here we are to restrict ourselves to the method of microlocal modelling proposed in [23, 28]. The microlocal modelling of Δ-periodic thermoelastic composites is based on the two following heuristic assumptions. First, it is assumed that the macroscopic response of the composites under consideration is independent of δ as δ tends to zero from its actual (real) value, provided all other entities in Eqs. (1.1), (1.2) (and in the pertinent initial and boundary conditions) as well as parameters $k^\alpha \equiv \delta^\alpha/\delta$, $\alpha = 1, 2, 3$, are held constant. This assumption is referred to as the fine periodicity assumption.

(1) Indices α, β, γ as well as i, j, k, l run over the sequence 1, 2, 3; summation convention holds.

(2) Symbol $R^{(3\times 3)}$ stands for a set of all symmetric 3×3 matrices the elements of which are real numbers.

Secondly, we look for approximate solutions $\tilde{\chi}(\cdot)$, $\tilde{\Theta}(\cdot)$ of Eqs. (1.1), (1.2) in special classes of functions which are called microlocal approximations. Every class of microlocal approximations is determined by the n-tuple of what are called the shape functions $l_a(\cdot)\colon R^3 \to R$, $a = 1, \dots, n$ (3). They are Δ-periodic continuous functions which have the continuous first derivatives except possibly at a finite number of points, lines or surfaces in every periodicity cell. Moreover, the shape functions are assumed to be oscillating (integrals from $l_a(\cdot)$ over Δ are equal to zero) and their dependence on the parameter δ can be given by the fromula

$$l_a(Y) = \delta\lambda_a\left(\frac{Y}{\delta}\right), \quad a = 1, \dots, n,$$

where $\lambda_a(\cdot)$, $a = 1, \dots, n$ are certain Δ/δ-periodic functions; mind that $\Delta/\delta = (0, k^1) \times (0, k^2) \times (0, k^3)$. Define a lattice $\mathscr{L}$ of points in R^3 setting $\mathscr{L} \equiv \{Z \in R^3;\ Z^{\alpha} = \nu\delta^{\alpha},\ \nu = 0, \pm 1, \pm 2, \dots;\ \alpha = 1, 2, 3\}$. Under the aforementioned notations every class of microlocal approximations can be given by the conditions (4)

$$\begin{aligned} \tilde{\chi}(X, t) &= \psi(Z, t) + l_a(Y) q^a(Z, t) + O(\delta; X, t), \\ \tilde{\Theta}(X, t) &= \vartheta(Z, t) + l_a(Y) \pi^a(Z, t) + O(\delta; X, t), \end{aligned} \tag{1.3}$$

and

$$\begin{aligned} &\psi(X, t) = \psi(Z, t) + O(\delta; X, t), \quad \vartheta(X, t) = \vartheta(Z, t) + O(\delta; X, t), \\ &q^a(X, t) = q^a(Z, t) + O(\delta; X, t), \pi^a(X, t) = \pi^a(Z, t) + O(\delta; X, t), \quad a = 1, \dots, n, \end{aligned} \tag{1.4}$$

which have to hold for every $X \in Z + Y \in \Omega_R$, $Y \in \bar{\Delta}$, $Z \in \mathscr{L} \cap \Omega_R$ and $t \in [t_0, t_f]$. Here $\psi(\cdot, t)$, $\vartheta(\cdot, t)$, $q^a(\cdot, t)$, $\pi^a(\cdot, t)$, $a = 1, \dots, n$, are sufficiently regular unknown fields defined on $\bar{\Omega}_R$ for every $t \in [t_0, t_f]$. In particular, $\psi(\cdot, t)$, $q^a(\cdot, t)$ are vector fields called macrodeformations and kinematical microlocal parameters, respectively. Similarly, $\vartheta(\cdot, t)$, $\pi^a(\cdot, t)$ are scalar fields called macrotemperature and thermal microlocal parameters, respectively. The fine periodicity assumption implies that Eqs. (1.3), (1.4) also hold as δ tends to zero from its actual value. Let us observe that the terms in Eqs. (1.3) involving $l_a(Y)$ are of order $O(\delta)$ but their material gradients (which involve terms $\Lambda_a(Y) \equiv \nabla l_a(Y) \in O(1)$) are of order $O(1)$. Thus, from Eqs. (1.3), (1.4) we can derive the following formulae

$$\begin{aligned} \chi(X, t) &= \psi(X, t) + O(\delta; X, t), \\ \nabla\chi(X, t) &= \nabla\psi(X, t) + \Lambda_a(X) \otimes q^a(X, t) + O(\delta; X, t), \\ \tilde{\Theta}(X, t) &= \vartheta(X, t) + O(\delta; X, t), \\ \nabla\tilde{\Theta}(X, t) &= \nabla\vartheta(X, t) + \Lambda_a(X) \otimes \pi^a(X, t) + O(\delta; X, t). \end{aligned} \tag{1.5}$$

Eqs. (1.5) represent (together with Eqs. (1.4)) what is called the microlocal approximation assumption. Taking into account also the aforementioned fine periodicity assumption we can neglect terms $O(\delta; X, t)$ while evaluating the values of the RHS of Eqs. (1.5). It has to be emphasized that the choice of the shape functions $l_a(\cdot)$, $a = 1, \dots, n$, and hence the postulated class of the microlocal approximation has to be based on the physical

(3) Indices a, b run over $1, \dots, n$; summation convention holds.

(4) Symbol $L(\delta; X, t)$ stands for an arbitrary (sufficiently regular) function depending on δ, $\delta > 0$, and defined on $\Omega_R \times [t_0, t_f]$, which together with all its material and time derivates takes the values of order $O(\delta)$. Obviously, symbols $O(\delta; X, t)$ in different formulae denote the values of different functions.

character of the problem, namely on the material structure of a typical periodicity cell of the composite under consideration. Let us also observe that the validity of Eqs. (1.4) (with terms $O(\delta; X, t)$ neglected as suffciently small) can be verified only after obtaining the solutions to the boundary-value problem under consideration (for the fixed actual value of the parameter δ).

2. Nonlinear Thermoelasticity with Microlocal Parameters

By the use of the concepts of nonstandard analysis it is possible to pass from Eqs. (1.1), (1.2) to the system of equations for $\psi(\cdot)$, $\vartheta(\cdot)$, $q^a(\cdot)$ and $\pi^a(\cdot)$, [28]. Here we shall discuss only the final form of the resulting equations. To this aim we firstly introduce the strain measures $E(X, t) \in R^{(3\times 3)}$, $F^a(X, t) \in R^3$, $G^{ab}(X, t) \in R$ by means of the equations

$$
\begin{aligned}
E(X, t) &= 0.5[\nabla\psi^T(X, t)\nabla\psi(X, t) - \nabla\varkappa^T(X)\nabla\varkappa(X)],\\
F^a(X, t) &= \nabla\psi^T(X, t)q^a(X, t),\\
G^{ab}(X, t) &= 0.5q^a(X, t)q^b(X, t).
\end{aligned}
\tag{2.1}
$$

We also define $F \equiv (F^a)$, $G \equiv (G^{ab})$, $\pi \equiv (\pi^a)$.

Let

$$
\langle\varphi(\cdot)\rangle \equiv \frac{1}{\mathrm{vol}\Delta}\int_{\Delta}\varphi(Y)dY^1dY^2dY^3
\tag{2.2}
$$

holds for any Δ-periodic integrable function $\varphi(\cdot)$. Now we introduce the following homogenized material functions

$$
\begin{aligned}
&\tilde{\varrho}_0 \equiv \langle\varrho_0(\cdot)\rangle,\\
&\tilde{\varphi}(E, F, G, \vartheta; A) \equiv \langle\varrho_0(\cdot)\varphi(\cdot, A^T[E+0.5(F^a\otimes\Lambda_a+\Lambda_a\otimes F^a)+\\
&\qquad\qquad +G^{ab}\Lambda_a\otimes\Lambda_b]A, \vartheta)\rangle\tilde{\varrho}_0^{-1},\\
&H_0(\vartheta, \nabla\vartheta, \pi; A) \equiv \langle H_0(\cdot, \vartheta, A^T(\nabla\vartheta+\Lambda_a\pi^a)),\\
&G_{a0}(\vartheta, \nabla\vartheta, \pi; A) \equiv \langle H_0(\cdot, \vartheta, A^T(\nabla\vartheta+\Lambda_b\pi^b))\otimes\Lambda_a(\cdot)\rangle.
\end{aligned}
\tag{2.3}
$$

Then it can be shown, [28], that the governing equations of the homogenized model of the thermoelastic composites under consideration are given by the strain relations (2.1) by the generalized equations of motion and generalized heat conduction equations

$$
\begin{aligned}
&\mathrm{Div}\,\tilde{T}_R(X, t)+\varrho_0 J(X)b(X, \psi(X, t), t) = \varrho_0 J(X)\ddot{\psi}(X, t),\\
&S_{aR}(X, t) = 0, \quad a = 1, \ldots, n,\\
&\mathrm{Div}\,\tilde{h}_R(X, t)+\varrho_0 J(X)\alpha(X, \psi(X, t), \vartheta(X, t), t) = \tilde{\varrho}_0 J(X)\vartheta(X, t)\dot{\tilde{\eta}}(X, t),\\
&g_{aR}(X, t) = 0, \quad a = 1, \ldots, n,
\end{aligned}
\tag{2.4}
$$

and by the homogenized constitutive equations

$$
\tilde{T}_R(X, t) = \tilde{\varrho}_0 J(X)\left[\nabla\psi(X, t)\frac{\partial\tilde{\varphi}}{\partial E(X, t)}+q^a(X, t)\otimes\frac{\partial\tilde{\varphi}}{\partial F^a(X, t)}\right],
$$

$$S_{aR}(X,t) = \tilde{\varrho}_0 J(X)\left[\nabla\psi(X,t)\frac{\partial\tilde{\varphi}}{\partial F^a(X,t)} + q^b(X,t)\frac{\partial\tilde{\varphi}}{\partial G^{ab}(X,t)}\right], \quad a = 1, \dots, n,$$

$$\text{(2.5)} \quad \tilde{h}_R(X,t) = J(X)A(X)H_0(\vartheta, \nabla\vartheta, \pi; A),$$

$$g_{aR}(X,t) = J(X)A(X)G_{a0}(\vartheta, \nabla\vartheta, \pi; A), \quad a = 1, \dots, n,$$

$$\tilde{\eta}(X,t) = -\frac{\partial\tilde{\varphi}}{\partial\vartheta(X,t)}.$$

It is easy to see that Eqs. (2.1), (2.4), (2.5) lead to the system of equations for macrodeformations $\psi(\cdot, t)$, macrotemperatures $\vartheta(\cdot, t)$ and microlocal parameters $q^a(\cdot, t)$, $\pi^a(\cdot, t)$, $a = 1, \dots, n$, $t \in [t_0, t_f]$, as the basic unknown fields defined on Ω_R. Eqs. (2.1), (2.4), (2.5) are governing field equations of what is called the nonlinear thermoelasticity with microlocal parameters. It can easily be observed that for the microlocal parameters we obtain (from Eqs. $(2.4)_{2,4}$ and $(2.5)_{2,4}$) the system of algebraic equations. It can also be shown that if in the microlocal approximation (1.3) we introduce the extra postulate setting $q_k^a(X,t) = 0$, $\pi^b(X,t) = 0$, for $(X,t) \in \Omega_R \times [t_0, t_f]$ and for some values of indices a, b, k, then equations $S_a^k(X,t) = 0$, $q_b(X,t) = 0$ for the pertinent values of a, b, k do not hold, i.e. they drop out from Eqs. (2.4).

After obtaining solution $\psi(\cdot)$, $\vartheta(\cdot)$, $q^a(\cdot)$, $\pi^a(\cdot)$, $a = 1, \dots, n$, to the boundary-value problem for Eqs. (2.1), (2.4), (2.5), we have to verify conditions (1.4). If the terms $O(\delta; X, t)$ in Eqs. (1.4) cannot be neglected then we have to pass to another class of microlocal approximation (1.3), by introducing a new sequence of the shape functions. If otherwise, then deformations, temperatures and their gradients in the composite body can be calculated from Eqs. (1.5) after neglecting terms $O(\delta; X, t)$; after that we can evaluate local values of stresses and heat fluxes in the composite via Eqs. (1.2).

Summing up, we mention the main characteristic features of the microlocal modelling: 1. The method of modelling is very general, i.e. it can be applied to various materials, to the large deformations and it can lead to various homogenized models by the different choices of shape functions (which is a crucial point of the modelling). 2. The homogenized equations are easy to obtain and their form is not very complicated as compared with that of the homogeneous thermoelastic materials; in particular, equations for the microlocal parameters are not differential but algebraic. 3. The adaptive refinement of a model is possible by introducing new sequences of the shape functions. For more detailed discussion of the microlocal modelling (including also the modelling of thermo-unelastic composites) the reader is referred to [13, 19, 21, 28, 29].

3. Linearized Equations of the Thermoelasticity with Microlocal Parameters

Let $w(X,t) = \psi(X,t) - \varkappa(X)$, $\Theta(X,t) \equiv \vartheta(X,t) - \vartheta_0$, ϑ_0 being a certain reference temperature and let us define

$$A_{i_1 \dots i_m}^{\alpha_1 \dots \alpha_m}(X) \equiv A_{i_1}^{\alpha_1}(X) \dots A_{i_m}^{\alpha_m}(X), \quad m = 1, 2, \dots.$$

Then after the linearization with respect to $w(X,t)$, $\vartheta(X,t)$, $q^a(X,t)$, $\pi^a(X,t)$, the constitutive equations (2.5) take the form

$$\tilde{T}^{\alpha\beta}(X,t) = A^{\alpha\beta\gamma\delta}_{ijkl}(X)[\langle C^{ijkl}\rangle E_{\gamma\delta}(X,t) - \langle C^{ijkl} l_{a,\gamma}\rangle q^a_\delta(X,t)] + A^{\alpha\beta}_{ij}(X)\langle B^{ij}\rangle \vartheta(X,t),$$
$$S^\alpha_a(X,t) = A^{\alpha\beta\gamma\delta}_{ijkl}(X)[\langle C^{ijkl} l_{a,\beta} l_{b,\gamma}\rangle q^b_\delta(X,t) + \langle C^{ijkl} l_{a,\beta}\rangle E_{\gamma\delta}(X,t)\rangle] + A^{\alpha\beta}_{ij}(X)\langle B^{ij} l_{a,\beta}\rangle \vartheta(X,t),$$
$$(3.1)\quad \tilde{h}^\alpha(X,t) = A^{\alpha\beta}_{ij}(X)[\langle K^{ij}_0\rangle \vartheta_{,\beta}(X,t) + \langle K^{ij}_0 l_{a,\beta}\rangle \pi^a(X,t)],$$
$$g_a(X,t) = A^{\alpha\beta}_{ij}(X)[\langle K^{ij}_0 l_{a,\alpha}\rangle \vartheta_{,\beta}(X,t) + \langle K^{ij}_0 l_{a,\alpha} l_{b,\beta}\rangle \pi^b(X,t)],$$
$$\tilde{\eta}(X,t) = -\langle s\rangle \tilde{\varrho}^{-1}\vartheta_0 \vartheta(X,t) - A^{\alpha\beta}_{ij}(X)[\langle B^{ij}\rangle \tilde{\varrho}^{-1} E_{\alpha\beta}(X,t) + \langle B^{ij} l_{a,\alpha}\rangle \tilde{\varrho}^{-1} q^a_\beta(X,t)],$$

where the following linearized strain measures have been introduced

$$(3.2)\quad \begin{aligned} E_{\alpha\beta}(X,t) &\equiv 0.5[\varkappa^k_{,\alpha}(X) w_{k,\beta}(X,t) + \varkappa^k_{,\beta}(X) w_{k,\alpha}(X,t)],\\ q^a_\alpha(X,t) &\equiv \varkappa^k_{,\alpha}(X) q^a_k(X,t), \end{aligned}$$

and where $C^{ijkl}(X)$, $B^{ij}(X)$, $K^{ij}_0(X)$, $s(X)$, $X \in \Omega_R$, stand for the well-known thermoelastic material modulae of the linear thermoelasticity. After introducing the extra notations

$$\tilde{b}^\alpha(X,t) \equiv \tilde{\varrho}_0 b^\alpha(X,0,t), \quad \tilde{\alpha}(X,t) \equiv \tilde{\varrho}_0 \alpha(X,0,0,t),$$

the linearized form of Eqs. (2.4) will be represented by[5]

$$(3.3)\quad \begin{aligned} &\tilde{T}^{\alpha\beta}|_\beta(X,t) + \tilde{b}^\alpha(X,t) = \tilde{\varrho}\partial_t\partial_t \ddot{w}^\alpha(X,t),\\ &S^\alpha_a(X,t) = 0,\\ &\tilde{h}^\alpha|_\alpha(X,t) + \tilde{\alpha}(X,t) = \tilde{\varrho}\vartheta_0 \partial_t \tilde{\eta}(X,t),\\ &g_a(X,t) = 0. \end{aligned}$$

Eqs. (3.1), (3.2), (3.3) constitute the general form of the governing relations of the linearized thermoelasticity with microlocal parameters, [28].

Consider now composites consisting only of the isotropic components. Then

$$C^{ijkl}(X) = \lambda(X)\delta^{ij}\delta^{kl} + \mu(X)(\delta^{ik}\delta^{jl} + \delta^{il}\delta^{jk}),$$
$$B^{ij}(X) = \beta(X)\delta^{ij}, \quad K^{ij}_0(X) = k(X)\delta^{ij},$$

where $\lambda(X)$, $\mu(X)$ are Lamé moduli, $\beta(X)\Big/\left(\lambda(X) + \frac{2}{3}\mu(X)\right)$ denotes the coefficient of the volume expansion, $k(X)$ is the coefficient of the thermal conductivity. Moreover, setting $G^{\alpha\beta}(X) = A^\alpha_i(X) A^\beta_j(X)\delta^{ij}$ and taking into account that $\lambda(\cdot)$, $\mu(\cdot)$, $\beta(\cdot)$, $k(\cdot)$ are Δ-periodic functions, we obtain from Eqs. (3.1)

$$\tilde{T}^{\alpha\beta}(X,t) = G^{\alpha\beta}(X)G^{\gamma\delta}(X)[\langle\lambda\rangle E_{\gamma\delta}(X,t) + \langle\lambda l_{a,\gamma}\rangle q^a_\delta(X,t)] + [G^{\alpha\gamma}(X)G^{\beta\delta}(X) + G^{\alpha\delta}(X)G^{\beta\gamma}(X)][\langle\mu\rangle E_{\gamma\delta}(X,t) + \langle\mu l_{a,\delta}\rangle q^a_\gamma(X,t)] + G^{\alpha\beta}(X)\langle\beta\rangle\vartheta(X,t),$$

$$(3.4)\quad S^\alpha_a(X,t) = G^{\alpha\beta}(X,t)G^{\gamma\delta}(X)[\langle\lambda l_{a,\beta} l_{b,\gamma}\rangle q^b_\delta(X,t) + \langle\lambda l_{a,\beta}\rangle E_{\gamma\delta}(X,t)] + [G^{\alpha\gamma}(X)G^{\beta\delta}(X) + G^{\alpha\delta}(X)G^{\beta\gamma}(X)][\langle\mu l_{a,\beta} l_{b,\gamma}\rangle q^b_\delta(X,t) + \langle\mu l_{a,\beta}\rangle E^{\gamma\delta}(X,t)] + G^{\alpha\beta}(X)\langle\beta l_{a,\beta}\rangle\vartheta(X,t),$$

---

(5) The vertical line stands for covariant differentiation in the known metric $G_{\alpha\beta}(X) = \varkappa^k_{,\alpha}(X)\varkappa^l_{,\beta}(X)\delta_{kl}$.

$$\begin{aligned} \tilde{h}^{\alpha}(X,t) &= G^{\alpha\beta}(X)[\langle k\rangle \vartheta_{,\beta}(X,t)+\langle kl_{a,\beta}\rangle \pi^a(X,t)], \\ g_a(X,t) &= G^{\alpha\beta}(X)[\langle kl_{a,\alpha}\rangle \vartheta_{,\beta}(X,t)+\langle kl_{a,\alpha}l_{b,\beta}\rangle \pi^b(X,t)], \\ \tilde{\varrho}\tilde{\eta}(X,t) &= -\vartheta_0^{-1}\langle s\rangle \vartheta(X,t)-G^{\alpha\beta}(X)[\langle \beta\rangle E_{\alpha\beta}(X,t)+\langle \beta l_{a,\alpha}\rangle q_\beta^a(X,t)]. \end{aligned} \tag{3.4 [cont.]}$$

Eqs. (3.2), (3.3) and (3.4) constitute the governing equations of the linearized thermoelasticity with microlocal parameters for the case in which isotropic materials are components of the composite under consideration.

After obtaining solution $w(\cdot,t)$, $\vartheta(\cdot,t)$, $q^a(\cdot,t)$, $\pi^a(\cdot,t)$ of the pertinent boundary value problem for Eqs. (3.3), (3.4) and (3.2) we have to verify conditions (1.4) in which in the place of $\psi(\cdot,t)$ we should substitute $w(\cdot,t)$. The discussion of application of the obtained solution and the subsequent proceeding is the same as that given in Sect. 2 for the case of nonlinear thermoelasticity with microlocal parameters.

Let us assume that the curvilinear reference material coordinates $X=(X^\alpha)$ coincide in B_0 with the Cartesian material coordinates $x=(x_i)$, setting $\varkappa = id$, $\varkappa^k_{\alpha}$, $(X)=\delta^k_\alpha$, $A^\alpha_k(X)=\delta^\alpha_k$, $\Omega_R = B_0$. Then from Eqs. (3.2) we obtain

$$\begin{aligned} E_{ij}(x,t) &= 0.5[w_{i,j}(x,t)+w_{j,i}(x,t)], \\ q^a_\alpha(x,t) &= \delta^i_\alpha q^a_i(x,t). \end{aligned} \tag{3.5}$$

At the same time Eqs. (4.3) can be rewritten in the form

$$\begin{aligned} &\tilde{T}_{ij,j}(x,t)+\tilde{b}_i(x,t) = \tilde{\varrho}\partial_t\partial_t w_i(x,t), \\ &S_{ia}(x,t)=0, \\ &\tilde{h}_{i,i}(x,t)+\tilde{a}(x,t)=\tilde{\varrho}\vartheta_0\partial_t\tilde{\eta}(x,t), \\ &g_a(x,t)=0 \end{aligned} \tag{3.6}$$

and constitutive equations (3.4) yield

$$\begin{aligned} \tilde{T}_{ij}(x,t) &= \langle\lambda\rangle\delta_{ij}E_{kk}(x,t)+2\langle\mu\rangle E_{ij}(x,t)+\langle\lambda l_{a,k}\rangle\delta_{ij}q^a_k(x,t)+ \\ &\quad +[\langle\mu l_{a,i}\rangle q^a_j(x,t)+\langle\mu l_{a,j}\rangle q^a_i(x,t)]+\langle\beta\rangle\delta_{ij}\vartheta(x,t), \\ S_{ai}(x,t) &= \langle(\lambda+\mu)l_{a,i}l_{b,k}\rangle q^b_k(x,t)+\langle\mu l_{a,k}l_{b,k}\rangle q^b_i(x,t)+ \\ &\quad +\langle\lambda l_{a,i}\rangle E_{kk}(x,t)+2\langle\mu l_{a,k}\rangle E_{ki}+\langle\beta l_{a,i}\rangle\vartheta(x,t), \\ \tilde{\varrho}\tilde{\eta}(x,t) &= -\vartheta_0^{-1}\langle s\rangle\vartheta(x,t)-\langle\beta\rangle E_{kk}(x,t)-\langle\beta l_{a,k}\rangle q^a_k(x,t), \\ \tilde{h}_i(x,t) &= \langle k\rangle\vartheta_{,i}(x,t)+\langle kl_{a,i}\rangle\pi^a(x,t), \\ g_a(x,t) &= \langle kl_{a,i}\rangle\vartheta_{,i}(x,t)+\langle kl_{a,i}l_{b,i}\rangle\pi^b(x,t). \end{aligned} \tag{3.7}$$

Eqs. (3.5), (3.6) and (3.7) are the governing relations of the linearized thermoelasticity with microlocal parameters for the case of rectilinear periodicity and isotropic materials

4. Multilayered Periodic Thermoelastic Composites

We shall pass to the periodic multilayered (laminated) bodies in which the material properties (related to the undeformed state B_0) are δ-periodic in the direction of the x_1-material coordinate. Thus, every basic unit of a composite has a thickness δ and is assumed to be composed of $n+1$ homogeneous isotropic thermoelastic layers. Let ϱ_j,

λ_j, μ_j, β_j, k_j, $j = 1, \ldots, n+1$, be material constants of the subsequent layers and δ_j be the thickness of the j-th layer. Then Eqs. (3.5), (3.6), (3.7) yield

$$\begin{aligned}(4.1)\quad &\langle\lambda+\mu\rangle w_{k,ki}(x,t)+\langle\mu\rangle w_{i,kk}(x,t)+\langle\mu l_{a,1}\rangle\delta_i^1 q^a_{k,k}(x,t)+\langle\lambda l_{a,1}\rangle q^a_{1,i}(x,t)+\\ &\qquad+\langle\mu l_{a,1}\rangle q^a_{i,i}(x,t)+\langle\beta\rangle\vartheta_{,i}(x,t)+\tilde{b}_i(x,t)=\tilde{\varrho}\partial_t\partial_t w_i(x,t),\\ &\langle k\rangle\vartheta_{,ii}(x,t)+\langle kl_{a,1}\rangle\pi^a_{,1}(x,t)+\tilde{\alpha}(x,t)+\langle s\rangle\partial_t\vartheta(x,t)+\\ &\qquad+\vartheta_0\langle\beta\rangle\partial_t w_{i,i}(x,t)+\vartheta_0\langle\beta l_{a,1}\rangle\partial_t q^a_1(x,t)=0,\\ &\langle(\lambda+\mu)l_{a,1}l_{b,1}\rangle\delta_i^1 q^b_1(x,t)+\langle\mu l_{a,1}l_{b,1}\rangle q^b_i(x,t)=-\langle\lambda l_{a,1}\rangle\delta_i^1 w_{k,k}(x,t)-\\ &\qquad-\langle\mu l_{a,1}\rangle(w_{i,1}(x,t)+w_{1,i}(x,t))-\langle\beta l_{a,1}\rangle\delta_i^1\vartheta(x,t),\\ &\langle kl_{a,1}l_{b,1}\rangle\pi^b(x,t)=-\langle kl_{a,1}\rangle\vartheta_{,1}(x,t).\end{aligned}$$

Here the shape functions $l_a(\cdot)$ depend only on x_1-coordinate.

Equations (4.1) constitute a system of governing equations of the linear thermoelasticity with microlocal parameters expressed in terms of macrodisplacements $w_i(\cdot,\cdot)$, macrotemperature $\vartheta(\cdot,\cdot)$, kinematical and thermal microlocal parameters $q^a_i(\cdot,\cdot)$ and $\pi^a(\cdot,\cdot)$, where $a = 1, \ldots, n$. Eliminating from Eqs. (4.1) all microlocal parameters q^a_i, π^a, we arrive at the system of four linear partial differential equations with constant coefficients for $w_i(\cdot,\cdot)$ and $\vartheta(\cdot,\cdot)$. The mixed boundary conditions for this system can be formulated in terms of $w_i(\cdot,t)$, $\vartheta(\cdot,t)$ (similarly to Eqs. (1.5) in the linearized case) and in terms of stresses and heat fluxes

$$\begin{aligned}(4.2)\quad &T_{ij}(x,t)=\lambda(x_1)\delta_{ij}E_{kk}(x,t)+2\mu(x_1)E_{ij}(x,t)+\lambda(x_1)\delta_{ij}l_{a,1}(x_1)\delta_k^1 q^a_k(x,t)+\\ &\qquad+2\mu(x_1)l_{a,1}(x_1)\delta_j^1 q^a_i(x,t)+\beta(x_1)\delta_{ij}\vartheta(x,t)+O(\delta;x,t),\\ &h_i(x,t)=k(x_1)\vartheta_{,i}(x,t)+k(x_1)l_{a,1}(x_1)\delta_i^1\pi^a(x,t)+O(\delta;x,t),\end{aligned}$$

where $E_{ij}(\cdot,\cdot)$ is defined by Eqs. (3.5).

Now we assume the shape functions $l_a(\cdot)$ in the form proposed in [13] and given by the picewise linear functions

$$(4.3)\qquad l_a(x_1)=\begin{cases}x_1-\dfrac{h_a}{2} & \text{for}\quad 0\leqslant x_1\leqslant h_a,\\[2mm] \dfrac{h_a}{h_a-\delta}x_1-\dfrac{h_a}{2}-\dfrac{h_a\delta}{h_a-\delta} & \text{for}\quad h_a\leqslant x_1\leqslant\delta,\end{cases}$$

where

$$(4.4)\qquad h_a\equiv\delta_1+\ldots+\delta_a,\quad \delta=h_{n+1},\quad a=1,\ldots,n.$$

Using formulae (2.2) and (4.3) for an arbitrary δ-periodic function $\varphi(\cdot)$, we obtain from Eq. (2.2)

$$\begin{aligned}(4.5)\quad &\langle\varphi\rangle=\sum_{i=1}^{n+1}\varphi_i\eta_i,\\ &\langle\varphi l_{a,1}\rangle=\sum_{i=1}^{a}\varphi_i\eta_i-\alpha_a\sum_{i=a+1}^{n+1}\varphi_i\eta_i,\\ &\langle\varphi l_{a,1}l_{b,1}\rangle=\sum_{i=1}^{b}\varphi_i\eta_i-\alpha_b\sum_{i=b+1}^{a}\varphi_i\eta_i+\alpha_a\alpha_b\sum_{i=a+1}^{n+1}\varphi_i\eta_i,\quad a,b=1,\ldots,n,\end{aligned}$$

where

$$(4.6)\qquad \eta_i \equiv \frac{\delta_i}{\delta}, \quad \alpha_i \equiv \frac{h_i}{\delta} = \frac{\eta_1 + \ldots + \eta_i}{1-(\eta_1 + \ldots + \eta_1)}, \quad i = 1, \ldots, n.$$

Using Eqs. (5.5) we can calculate all material modulae in Eqs. (4.1), substituting for $\varphi(\cdot)$ the δ-periodic functions $\varrho(\cdot)$, $\lambda(\cdot)$, $\mu(\cdot)$, $k(\cdot)$, $\beta(\cdot)$, $s(\cdot)$. Eqs. (4.1) with the coefficients defined by Eqs. (4.5) constitute a system of equations describing the periodic multilayered linear thermoelastic body under assumption of the sectionally linear shape functions.

5. Examples of Special Problems

As examples of application of the linear thermoelasticity with microlocal parameters we shall consider two two-dimensional static problems for periodic layered elastic composites, occupying a half-space loaded on the boundary by normal concentrated forces, and a space containing a Griffith crack at the interface of two layers. We shall restrict ourselves to the layered composites in which every repeating basic unit (of thickness δ) is composed of two homogeneous isotropic linear-elastic layers (of thicknesses δ_1 and δ_2). Using Eqs. (4.5) and (4.6) we obtain

$$(5.1)\qquad \begin{aligned} &\tilde{\lambda} \equiv \langle\lambda\rangle = \lambda_1\eta_1 + \lambda_2\eta_2, \qquad \tilde{\mu} \equiv \langle\mu\rangle = \mu_1\eta_1 + \mu_2\eta_2,\\ &[\lambda] \equiv \langle\lambda l_{1,1}\rangle = \eta_1(\lambda_1 - \lambda_2), \qquad [\mu] \equiv \langle\mu l_{1,1}\rangle = \eta_1(\mu_1 - \mu_2),\\ &\hat{\lambda} \equiv \langle\lambda l_{1,1} l_{1,1}\rangle = \eta_1\left(\lambda_1 + \frac{\eta_1}{1-\eta_1}\lambda_2\right), \qquad \hat{\mu} \equiv \langle l_{1,1} l_{1,1}\rangle = \eta_1\left(\mu_1 + \frac{\eta_1}{1-\eta_1}\mu_2\right). \end{aligned}$$

Assuming that the total displacement vector $u(x, t) = w(x, t) + l_a(x_1)q^a(x, t)$ is independent on the variable x_3 and $u_3 \equiv 0$, $b \equiv 0$, Eqs. (4.1) and (5.1) yield

$$(5.2)\qquad \begin{aligned} &(\tilde{\lambda}+\tilde{\mu})(w_{1,11}+w_{2,21})+\tilde{\mu}(w_{1,11}+w_{1,22})+([\lambda]+2[\mu])q^1_{,1}+[\mu]q^2_{,2} = 0,\\ &(\tilde{\lambda}+\tilde{\mu})(w_{1,12}+w_{2,22})+\tilde{\mu}(w_{2,11}+w_{2,22})+[\lambda]q^1_{,2}+[\mu]q^2_{,1} = 0,\\ &(\hat{\lambda}+2\hat{\mu})q^1+[\lambda](w_{1,1}+w_{2,2})+2[\mu]w_{1,1} = 0,\\ &\hat{\mu}q^2+[\mu](w_{2,1}+w_{1,2}) = 0. \end{aligned}$$

By using Eqs. $(5.2)_{3-4}$ we can express Eqs. $(5.2)_{1-2}$ in terms of macrodisplacements as follows [6 - 8]

$$(5.3)\qquad \begin{aligned} A_1 w_{1,11}+(B+C)w_{2,12}+Cw_{1,22} &= 0,\\ A_2 w_{2,22}+(B+C)w_{1,12}+Cw_{2,11} &= 0, \end{aligned}$$

where

$$(5.4)\qquad \begin{aligned} A_1 &= \tilde{\lambda}+2\tilde{\mu}-([\lambda]+2[\mu])^2/(\hat{\lambda}+2\hat{\mu}),\\ C &= \tilde{\mu}-[\mu]^2/\hat{\mu},\\ A_2 &= \tilde{\lambda}+2\mu-[\lambda]^2/(\hat{\lambda}+2\hat{\mu}),\\ B &= C+\tilde{\lambda}-[\lambda]([\lambda]+2[\mu])/(\hat{\lambda}+2\hat{\mu}). \end{aligned}$$

From Eqs. (4.2), (3.5), (4.3), (5.1), (5.4) for the two-dimensional problem under consideration it follows that the stresses in the subsequent layers take the form

(5.5)
$$T_{11}^{(j)} = A_1 w_{1,1} + B w_{2,2},$$
$$T_{12}^{(j)} = C(w_{1,2} + w_{2,1}),$$
$$T_{22}^{(j)} = D_{(j)} w_{1,1} + E_{(j)} w_{2,2}, \quad j = 1, 2,$$

where

(5.6)
$$D^{(j)} = \frac{\lambda_j}{\lambda_j + 2\mu_j} A_1, \qquad E^{(j)} = \frac{4\mu_j(\lambda_j+\mu_j)}{\lambda_j+2\mu_j} + \frac{\lambda_j}{\lambda_j+2\mu_j} B$$

and index $j = 1$ is related to the layers of the first kind (with material constants λ_1, μ_1) and $j = 2$ is related to the layers of the second kind (with material constants λ_2, μ_2).

To solve the problem of periodic two-layered half-space loaded by concentrated forces as well as the problem of interface crack in the periodic two-layered space, the complex potential method can be applied. According the results obtained in [6] for the case of $\mu_1 \neq \mu_2$, we have the following complex potential representation

(5.7)
$$T_{11}^{(j)}(x_1, x_2) = 2\mathrm{Re}\,(\varphi_1'(z_1) + \varphi_2'(z_2)),$$
$$T_{12}^{(j)}(x_1, x_2) = 2\mathrm{Im}\,(k_1\varphi_1'(z_1) + k_2\varphi_2'(z_2)),$$
$$T_{22}^{(j)}(x_1, x_2) = 2\mathrm{Re}\,(c_{1j}\varphi_1'(z_1) + c_{2j}\varphi_2'(z_2)),$$
$$w_1(x_1, x_2) = -2\mathrm{Re}\,(q_1\varphi_1(z_1) + q_2\varphi_2(z_2)),$$
$$w_2(x_1, x_2) = -2\mathrm{Re}\,(p_1\varphi_1(z_1) + p_2\varphi_2(z_2)),$$

where

(5.8)
$$k_1 = \left\{(A_1A_2 - 2BC - B^2 - \sqrt{\delta_0})/2A_1C\right\}^{1/2},$$
$$k_2 = \left\{(A_1A_2 - 2BC - B^2 + \sqrt{\delta_0})/2A_1C\right\}^{1/2},$$
$$\delta_0 = (A_1A_2 - 2BC - B^2)^2 - 4A_1A_2C^2 > 0,$$
$$c_{\alpha j} = \frac{(A_2 + k_\alpha^2 B)D^{(j)} - (B + k_\alpha^2 A_1)E^{(j)}}{A_1A_2 - B_2},$$
$$p_\alpha = \frac{A_1k_\alpha^2 + B}{A_1A_2 - B^2}, \qquad q_\alpha = i\,\frac{A_2 + Bk_\alpha^2}{k_\alpha(A_1A_2 - B^2)},$$
$$i^2 = -1, \quad z_\alpha = x_2 + ik_\alpha x_1, \quad \alpha = 1, 2,$$

and $\varphi_1(\cdot)$, $\varphi_2(\cdot)$ are complex holomorphic potentials.

We consider now the following boundary value problems for periodic two-layered composites:

EXAMPLE 1

Let the periodic two-layered elastic half-space $x_1 \geqslant 0$ be loaded on the boundary $x_1 = 0$ by the normal concentrated force of magnitude—P [5, 6]

(5.9)
$$T_{11}^{(1)}(0, x_2) = -P\delta(x_2), \qquad T_{12}^{(1)}(0, x_2) = 0, \qquad x_2 \in R.$$

Solution of this problem is described by Eqs. (5.7), where

(5.10)
$$\varphi_1(z_1) = C_1 z_1^{-1}, \qquad \varphi_2(z_2) = C_2 z_2^{-1},$$
$$C_1 = Pk_2/\{2\pi i(k_2 - k_1)\}, \qquad C_2 = -Pk_1/\{2\pi i(k_2 - k_1)\}.$$

The detailed discussion of the singularity of stresses due to concentrated loads can be found in [5].

EXAMPLE 2

Consider a single Griffith crack situated at the interface of two layers in an infinite periodic two-layered body. The solution of this problem in the case of Mode I is given in the terms of complex potentials as follows [8]

$$\varphi_1'(z_1) = k_2 H_1(z_1)/\{2(k_2-k_1)(z_1^2-a^2)^{1/2}\},$$
$$\varphi_2'(z_2) = k_1 H_1(z_2)/\{2(k_2-k_1)(z_2^2-a^2)^{1/2}\}, \tag{5.11}$$

where

$$H_1(z_\alpha) = \frac{1}{\pi}\int_{-a}^{a}\frac{f_1(\tau)(a^2-\tau^2)^{1/2}}{z_\alpha-\tau}\,d\tau, \qquad \alpha = 1, 2$$

is Cauchy integral and $f_1(x_2)$ is a normal load acting on the crack surfaces, $2a$ is the length of the crack.

The stress intensity factor k_1 at the crack tip $(0, a)$ is given by

$$k_1 = \frac{1}{\pi\sqrt{a}}\int_{-a}^{a} f_1(\tau)\sqrt{\frac{a+\tau}{a-\tau}}\,d\tau.$$

The detailed analysis of the interface crack problems in periodic laminated bodies is given in [8].

6. Final Remarks

In this paper a certain class of homogenized models of thermoelastic periodic composites is presented. The main feature of these models is that in the modelling of strains, stresses and heat fluxes they describe the microlocal effects, i.e. the effects due to the periodic structure of the body by means of microlocal parameters. By using the theory of linear elasticity with microlocal parameters, certain problems of propagation of plane harmonic waves in periodic multilayered elastic composites are examined in [3]. The papers [11] and [18] are devoted to the solutions of certain axially symmetric problems of laminated bodies for heat conduction and linear elasticity with microlocal parameters. Certain theorems connected with the existence and uniqueness of solutions of boundary value problems within the framework of the elasticity with microlocal parameters are proved in [15]. Paper [10] contains a detailed discussion of the interrelations between microlocal parameter and homogenization approach to the periodic elastic bodies. Certain problems of micromorphic plates are examined in [9]. In the paper [12] the nonlinear as well as linear heat conduction problems of periodic rigid bodies are considered by using the microlocal parameter approach. Paper [22] presents a discussion of application of the nonstandard analysis as a tool in the modelling of mechanics problems for periodic bodies. Recapitulation of the nonstandard analysis approach to the homogenization as well as the solutions of singular wave problems in the periodic elastic composites are given in [14]. Paper [16] contains problems of the mathematical modelling of delamination of linear-elastic composites.

The microlocal parameter approach can also be applied to the modelling of periodic thermo-inelastic composites, [29], elastic-plastic periodic bodies, [19, 20, 21], and elastic bodies with internal constraints [27].

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Streszczenie

O MIKROLOKALNYM MODELOWANIU TERMOSPRĘŻYSTYCH KOMPOZYTÓW PERIODYCZNYCH

Praca ma dwojaki cel. Po pierwsze, przedstawiono w niej pewną klasę homogenizowanych modeli termosprężystych periodycznych kompozytów. Po drugie, przeprowadzono dyskusję zastosowań proponowanych modeli do różnych szczególnych zagadnień termomechaniki.

Резюме

О МИКРОЛОКАЛЬНОМ МОДЕЛИРОВАНИИ ТЕРМОУПРУГИХ ПЕРИОДИЧЕСКИХ КОМПОЗИТОВ

Работа имеет двойную цель. Во-первых, представлен в ней некоторый класс гомогенизованных моделей термоупругих периодических композитов. Во-вторых, проведено обсуждение применений предложенных моделей к разным частным задачам термомеханики.

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