Stochastic Uncertainty Quantification of the Eddy Current in Human Body by using Polynomial Chaos Decomposition

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Problem

How to estimate the effect of the uncertainty on ELF induced currents?

Maxwell’s equations

Quasi-static Maxwell’s equation: \( \text{curl} E + \partial_t B = 0 \); \( \text{curl} H = J \); \( \text{div} E = 0 \)

\[ j = \sigma E + \mu B \]

Faraday’s law: \( \phi \rightarrow e. a \rightarrow b \rightarrow 0 \)

Ampère’s law: \( \sigma \mu \frac{\partial}{\partial t} \rightarrow \mu \frac{\partial}{\partial t} \rightarrow 0 \)

The quantity of interest are expanded in a similar way:

\[ J_{\text{avg}}(\omega) = \sum_{i=0}^{\infty} I_{\text{avg}}(\omega) \psi_i(\xi(\omega)) \]

The unknown coefficients are computed by a Hermite-Gauss numerical quadrature:

\[ J_{\text{avg}}(\omega) = \frac{\int J_{\text{avg}}(\omega) \psi_i(\xi(\omega))}{\int \psi_i(\xi(\omega))^2} \]

A deterministic Finite Element code (GetDP) is used as a “black box” for computing the values of \( J_{\text{avg}} \) in the quadrature points.

Quantification of the uncertainty

The probabilistic density of the average, maximum and 99% percentile are computed for the induced current density \( j \), and electric field \( e \).

Exposure to the field generated by a wire (I=1kA, 50Hz)

Exposure conditions: the wire is placed close to the left side of the head (ELLA)

Two basic restrictions are defined in the 2010 edition of ICNIRP recommendations which are never exceeded. Conversely, the basic restriction defined in the 1998 edition would be certainly exceeded.

Conclusion

— The proposed method allows to quantify the dosimetric uncertainty with respect of the conductivities of tissues.
— A characterization of the randomness of the conductivities is required.
— Not all the sources of uncertainty are taken into account.
— It is possible to increase the speed of computations by using sparse grid schemes.