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Macroscopic Fundamental Diagram for Urban Streets and Mixed Traffic: Cross-comparison of Estimation Methods

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Abstract

In the last decade, many papers focused on describing vehicular traffic stream of an arterial on an aggregate level. Unfortunately, among this considerable body of research, only few papers account for bus systems. Our paper tries to fill this gap by investigating two potential methods to estimate macroscopic fundamental diagrams of multimodal transport systems of a signalized arterial. The first approach models endogenously motion of bus by extending the work by the moving bottleneck theory, whereas the second approach proposes to incorporate exogenously effects of buses. The estimated macroscopic fundamental diagrams are then cross-compared to results provided by micro-simulation software that finely reproduce traffic stream. It turns out that mean speeds of vehicles and buses produced by the different methods are similar and consistent. Finally, results of the three methods are expressed in terms of levels of service and confronted with the levels of service of the HCM2010.
INTRODUCTION

Study of mixed traffic streams in cities and arterials is a topic of increasing concerns. A large body of works is dedicated to analyze explanatory variables that characterize dynamics of such networks (1-3). These papers aim to evaluate arterials and city networks performance by using traffic measurements. The largest part of this literature has been directed toward improving techniques to monitor traffic streams (4-5), to measure travel times (6) or to derived relationships between traffic flow and signal control settings (7). However, evaluation remains a tedious task because of the complicated traffic behavior especially caused by (i) traffic signals and (ii) bus transit systems traversing arterials. Another approach for a better understanding of arterial traffic dynamics is to resort to accurate models. Indeed, aggregated and parsimonious models often provide valuable insights when analyzing and comparing different traffic situations and transit strategies.

Various theories have been proposed to reproduce traffic stream on an aggregate level. Among this existing body of works, models that are adapted to characterize traffic in arterials have to account for (i) traffic signal and (ii) transit systems. Many of these papers are based on the key idea that it exists a macroscopic fundamental diagram (MFD) able to reproduce both free-flow and congested traffic conditions. Earlier studies were devoted to look for such relationship in data of real-world network or arterials. However, evidences of existence of MFD have been exhibited only very recently (3,7). On their seminal works, the authors pointed out a major insight: the MFD is an intrinsic property of the network itself and remain invariant when demand changes. MFD is thus a reliable tool for traffic agency to manage and evaluate solutions for improving mobility. (8) furnished a very good example of how MFD can be used to dynamically control signals to prevent congestion. It is thus appealing to estimate an accurate MFD for various urban sites and traffic. In this sense, many papers have been recently directed toward highlighting links between shapes of the MFD and different simulated or measured parameters (9-10), exploring impact of distributions of vehicles and space on MFD (11) or investigating bifurcation and instability issues of and MFD (12-13). Unfortunately, this large body of works does not account for multiple modes.

(14-15) have presented the only instance trying to overcome this drawback of representation to the author’s knowledge. They introduced a methodology to estimate a MFD for an arterial with mixed-traffic bus-car lanes or with dedicated bus lanes. This work extends for multi-modal networks the Variational theory (VT) proposed in (7). Impacts of bus stops are incorporated in the estimation method by considering these stops as point bottlenecks that locally reduce capacity. However, a bus has also a major influence on traffic when bus travels at a lower speed than the flow speed, i.e. when bus acts as an active moving bottleneck (MB). (16) also propose a MFD estimation method for heterogeneous hyperlinks, i.e. a series of successive links with different traffic signal. Based on the VT, this method directly provides the upper envelop of the MFD.

However, the existing methods do not account for buses. Our paper tries to fill this gap by investigating two potential methods to model multimodal transport systems on a signalized arterial. The final goal is to provide an aggregated, parsimonious and effective model to reproduce and then evaluate traffic stream of a multimodal urban corridor. To this end, we resort to the MFD that needs to be accurately estimated by appropriate methods. These methods are build on past research in arterial modeling based on MFD representation as well as past research in accounting for buses in traffic flow as moving bottlenecks. Two different approaches are proposed: (i) to improve the method of (16) and (ii) to propose a new estimation process.

So, the first method (M1) extends the work of (16) by introducing the effect of buses with a temporal and local reduction of the capacity of the arterial when a bus is present. This reduction will influence the shape of the estimated MFD. Consequently, buses are endogenously incorporated in the MFD estimation process.

On the contrary, the second method (M2) will exogenously account for impacts of buses. Time-space diagrams of an urban arterial are analytically calculated based on the associated MFD estimated without buses (7). Bus is considered as a MB (17-18) that will modify the time-space diagrams. Then, traffic flow dynamics of arterials can be easily assessed through these diagrams.

Finally, results of these two methods are cross-compared with results obtained with micro-simulation software (method M3). This is a convenient way to verify that the level of details of previous methods does not introduce bias in results. We use the software SymuVia that is a Lagrangian discretization of the kinematic wave model. Consequently, M3 is fully consistent with both previous methods. Their results will also be confronted with the multimodal Level of Services (LOS) provided in the HCM2010 (19) because this corresponds to the current reference for practitioners.

Section 2 briefly specifies the study case. Section 3 presents details of the endogenous approach whereas Section 4 is focused on the exogenous method. Then, Section 5 is devoted to the micro-simulation of the urban corridor. Finally, Section 6 proposes to cross-compare the results of the different estimation methods with the multimodal LOS provided in the HCM2010.
CASE STUDY

Consider here a hypothetic urban arterial (see Figure 1) composed of $n$ successive links with traffic signal and $p$ lanes. The length of link $i$ is $l_i$ and its signal settings are: green $g_i$, cycle $c_i$ and offset $\delta_i$ from a common reference. The total length of the arterial is $L$. Traffic on each link is supposed to obey a triangular fundamental diagram (FD) that only depends on three observable parameters: free-flow speed $u$ [m/s], wave speed $w$ [m/s] and jam density $k$ [veh/m] (20-21). Capacity $q_c$ [veh/s] and optimum density $k_c$ [veh/m] can be easily derived: $q_c = u w k / (u + w)$ and $k_c = q_c / u$. Notice that all the links share the same FD for sake of simplicity.

![Figure 1: study case](image)

In the remaining of the paper, we assumed that the arterial is composed of 11 successive 3 lanes links ($l=200$m). Concerning the traffic signal settings, $g_i$ is equal to 60s and $c_i$ to 90s. For sake of simplicity, we supposed that there is no offset. Finally, $u$ is equal to 15 m/s, $w$ is fixed to 5m/s and $k$ to 0.185veh/m ($q_c = 2.08$ veh/s $=7500$ veh/h). We also assumed that buses travel along the arterial with a lower average speed to mimic existence of bus stops: $u_w=8$ m/s. Consequently, bus stops are not explicitly model in this paper.

ENDOGENOUS APPROACH

The aim of this section is to propose a first method that endogenously estimates MFDs and accounts for traffic signal settings and buses represented as a moving obstruction. This method extends previous works from (16) that defines an accurate estimation method founded on VT for arterials with heterogeneous traffic signal parameters. The extension deals with introducing the impacts of moving bottlenecks into the proposed framework. The detailed presentation of the variational method proposed by (16) is out of the scope of this paper, only the key elements will be recap below. Note that VT was described in Daganzo’s seminal papers (22-23).

The foundation of the variational method comes from (7). This paper shows that a MFD can be defined by a set of cuts $\{C_j\}$. A cut corresponds to a line in the $(k,q)$ plane parameterized by its $y$-intercept $r_j$ and its slope $v_j$, i.e. $q=r_j + kv_j$. Cuts are associated to moving observers that moves into the considered arterial with a constant speed $v_j$. Such observers have no dimension and are not influenced by traffic signals. $r_j$ corresponds to the maximal passing rate that such an observer can encounter depending on the FD and the signal settings. In practice, to piecewise-linearly estimate a MFD one only have to define a set of discrete values $\{v_j\}$ and determine the associate $r_j$ values. (7) propose to only focus on practical cuts to estimate $r_j$. Practical cuts correspond to specific paths of the moving observer that can only experiment speeds $u$ and 0 when mean speed $v_j$ is positive or $w$ and 0 when mean speed $v_j$ is negative. These paths are constructed in practice by assigning a speed $u$ (or $w$) to observers when moving into a link and then delaying them at green signals with different constant values $\xi$. The mean speed $v_j$ is then calculated afterwards and only depends on $\xi$. This method has been applied in (14-15) but suffers from a limitation that is not compatible with further introducing buses. Indeed, in unregular cases (heterogeneous signal timings or presence of moving obstructions), this method only provides an upper bound for $r_j$. Thus, cuts do not necessarily tightly define the MFD.
(16) blows up with this limitation by generalizing the concept of practical cuts. Instead of focusing on particular paths into the arterials, they define a sufficient but minimal variational graph that encompasses the practical cut paths but also all the other optimal paths associated to a mean speed $v_j$ in order to properly estimate $r_j$ even in heterogeneous cases. To be precise, two variational graphs have to be constructed to deal with free-flow ($v_j<0$) and congested cuts ($v_j>0$). We will only present how to define the free-flow graph (the congested graph can be obtained by replacing $u$ by $w$ and considering the arterials in the reserve direction). This graph is composed of three kinds of edges, see Figure 2a:

a. the red phases of all traffic signals;

b. the green phases of all traffic signals;

c. the paths with speed $u$ that start from the ends of all red times of each signal and propagate until another edge (a). Vertices should be added anytime such a path crosses edges (b) and (a).

The proof of sufficiency can be found in (16). Cuts $j$ are defined by all the paths into the variational graph that have the same initial and final points, i.e. the same speed $v_j$. The associated $r_j$ corresponds to the least-cost between these initial and final points. Note that cost rate associated to edges (a) (b) and (c) are respectively 0, $q_c$, and 0 (costs associated to edges (c) in the congested graph are $w_K$). Note also that in regular cases (identical traffic signal settings, constant offset and no buses) only one initial point has to be considered because the network is periodic. In unregular cases, $r_j$ values have first to be calculated for different origin points and then averaged for all origin points to properly define cut $C_j$.

The above variational method can be easily extended to account for buses. Indeed, (17) and further other authors see (18) for a review, shows that the impacts of such vehicles on the global traffic stream can be represented as a moving bottleneck that locally reduces the available capacity. Note that such a bottleneck is said to be active when it really has an influence on the surrounding traffic. (24-25) have shown that such a moving bottleneck can also be approximated by a succession of fixed bottlenecks that follows the bus trajectory with a bounded error. This key result provides a simple way to extend a variational method. Indeed, we can consider that a bus inside a link reduces the available capacity from one lane until it leaves the links. This defines into the arterial time-link regions where the capacity is reduced, see gray shape around the bus trajectory in Figure 2b. This capacity reduction can be taken into account by modifying the costs of the edges that cross such regions. In practice, only cost rates on edge (b) have to be modified and switched to $(p-1)/p.q_c$, where $p$ is the number of lanes. Note that introducing buses makes the studied case unregular (bus are introduced according to a given frequency) and considering the arterials in the reserve direction). Thus, several initial points have to be considered. To be sure that the mean value of $r_j$ is properly estimated, i.e. that we consider enough initial points, we check that the standard deviation of $r_j$ is lower than 10% of its mean value.

Figure 3a presents the resulting free-flow and congested cuts calculated for an arterial and a bus system headway of 3 min. Note that none of the two variational graphs provide the stationary cut corresponding to the minimum capacity observed at the most constraining traffic signals. This cut can easily be added. Red lines in Figure 3a show the only relevant cuts that fully define the MFD. Figure 3b depicts the estimated MFDs for various values of bus time-headway $h$. It clearly shows that the presence of bus transit systems on the arterials reduces the maximal capacity. Moreover, it is not surprising that the maximal capacity decreases with the increase of $h$. Figure 3b also reveal that bus has a major effect on the MFD shape in the vicinity of the top of the MFD. Hence, bus is an active MB in this domain of traffic conditions.
**EXOGENOUS APPROACH**

On the contrary of the previous model, method M2 endogenously encompasses effects of buses. To this end, time-space diagrams are calculated based on the MFD estimated without the presence of buses. This MFD can easily be calculated according to findings of (7) or through the process introduced by method M1. This MFD is displayed in Figure 4. Notice that we use a MFD that is bi-linear in both free-flow and congested situations, for sake of simplicity. However, method M2 can be applied to any piece-wise linear MFD. Figure 4 displays the equilibrium states of traffic flow on arterial that turn out to be of interest throughout the paper. For any equilibrium traffic state $A$ (point of the MFD), the flow and the density are respectively denoted $q_A$ and $k_A$. State $F$ corresponds to the limit between maximal free-flow speed $u$ and free-flow speed $u_f$. $C_1$ is the full arterial capacity; $C_2$ corresponds to the change of congested wave speed; $J$ is the full arterial jam density.

The associated equilibrium states are depicted in Figure 4. State $D$ corresponds to the downstream traffic conditions, which is in free-flow situation. The associated flow can be completely determined based on the state $D$ and the speed of the MB $u_b$ (see Figure 4). According to KWT, interface between states $U$ and $D$ travels at speed $u_b$ along the arterial. It is worth noticing that the MB is active for a given traffic state $A$ when passing rate associated to state $A$ is higher the passing rate of state $D$, i.e. $r_A = q_A u_b k_A > r_D = q_D u_b k_D$. 

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**Figure 3: Estimated MFD with method M1**

(a) whole sets of cuts for a given headway ($h=3\text{min}$)  
(b) estimated MFDs for different headways

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**Figure 4: Equilibrium states of traffic flow on arterial**

- State $F$ corresponds to the limit between maximal free-flow speed $u$ and free-flow speed $u_f$.
- $C_1$ is the full arterial capacity; $C_2$ corresponds to the change of congested wave speed.
- $J$ is the full arterial jam density.
- The associated equilibrium states are depicted in Figure 4.
- State $D$ corresponds to the downstream traffic conditions, which is in free-flow situation.
- The associated flow can be completely determined based on the state $D$ and the speed of the MB $u_b$ (see Figure 4).
- According to KWT, interface between states $U$ and $D$ travels at speed $u_b$ along the arterial. It is worth noticing that the MB is active for a given traffic state $A$ when passing rate associated to state $A$ is higher the passing rate of state $D$, i.e. $r_A = q_A u_b k_A > r_D = q_D u_b k_D$. 

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**Case Analysis**

- **Case 1**: $D$ to $U$ transition.
- **Case 2**: $U$ to $C_2$ transition.
- **Case 3**: $C_1$ to $D$ transition.
- **Case 4**: $D$ to $F$ transition.
- **Case 5**: $F$ to $J$ transition.
Figure 4: MFD of the arterial without bus

Time-space diagrams can now be calculated. The associated patterns depend on the input flow and the exiting rate of the arterial. Indeed, various levels of the entry demand and the exiting rate of the arterial are tested to mimic all the traffic conditions described by the MFD. Situations can be classified based on three criterions: traffic state conditions, active/inactive MB and propagation of the upstream shockwave generated by the entry of the bus in the arterial. Based on these levels, one can identified five cases that are sum-up in Figure 4:

a. case 1: free-flow (FF) state, bus is an inactive MB \((r_A<r_D)\), no upstream shockwave;

b. case 2: FF state, bus is an active MB \((r_A>r_D)\), upstream shockwave moves forward \((q_A<q_U)\);

c. case 3: FF state, bus is an active MB \((r_A>r_D)\), upstream shockwave moves backward \((q_A>q_U)\);

d. case 4: congested (C) state, bus is an active MB \((r_A>r_D)\), upstream shockwave moves backward;

e. case 5: C state, bus is an inactive MB \((r_A<r_D)\), upstream shockwave moves backward.

For each of these cases, many subcases can also be identified based on the bus headway value \(h\). We will only detail the obtained patterns for case 2 and let the reader verify for the remaining cases. Indeed, case 2 is the more generic and easiest situation to explain the framework of our method.

In case 2, the demand inflow \(q_A\) is comprised between \(q_D\) and \(q_U\) and the exiting rate is equal to the maximum capacity \(C_1\). Because \(q_A<q_D\), when the bus enters in the arterial, it becomes an active MB and generates state \(D\) downstream and state \(U\) upstream. Both states are separated by a wave travelling downstream at speed \(u_b\). Once the bus has left the arterial, state \(C_1\) is created and propagates upstream at speed \(w\). Figure 5a displays the resulting pattern.

This pattern remains valid if the next bus enters in the arterial a time \(h\) later when the congestion generated by the first bus has totally been vanished. In that case, the same pattern is observed for all the buses (see Figure 5a, case 2i). Otherwise, state \(D\) generated by the entry of the next bus reaches existing state \(U\). It modifies the patterns as depicted by Figure 5b (case 2ii). It is worth noticing that the patterns are periodic after the passage of the second bus (hatched areas in Figure 5). Consequently, it is sufficient to focus on only one pair of buses once the stationary situation has been reached.

Figure 5: Time-space diagram for case 2i (large bus headways) and case 2ii (small bus headways)

We let the reader verify that this work can be easily extended for the other cases. Figure 6 shows the results of the remaining cases. It is important to notice that these time-space diagrams are analytically calculated, i.e. coordinates of every states area are known. Moreover, the patterns are always periodic, which also make the evaluation of the method easier. Consequently, we can calculate the average (in the sense of Edie’s definition) flow \(Q\) and density \(K\) for each pattern and then estimate the MFD. Details of the calculation of \(Q\) and \(K\) are presented in the final section.
Figure 6: Time-space diagrams for the remaining cases

Figure 7 displays MFDs estimated with method M2 for different values of bus headway. The results are not surprising because high headways decrease the capacity of the arterial. For example, the maximal capacity is reduced of more than 8%.

Figure 7: MFD estimated with method M2 for different bus headways

MICRO-SIMULATION

The third method (method M3) proposed in this paper is based on microscopic simulation software. This allows controlling every aspect of the simulated environment and gives also access to vehicles trajectories. The
The simulation setup is close as possible to characteristics of the arterial used for both previous methods. The used software, called SymuVia, is based on a Lagrangian discretization of the KWT. Details of the resulting car-following law can be found in \((30-31)\). This car-following law relies on the existence of a triangular FD. We used the same traffic parameters, \(u_0\), \(w\), and \(k\) as in previous methods. Moreover, the model has been refined to take into account bounded acceleration, lane-changing phenomenon and relaxation after lane-changing phenomenon \((31-32)\). Thus, the simulation software is able to accurately reproduce the impacts of the MB on the remaining of the traffic in the arterial.

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MFD can now be estimated with the simulated results. We used the trajectories-based approach proposed by \((33)\). The authors resort to Edie’s definitions \((34)\) to calculate flow and density. \((34)\) computes density \(k\) and flow \(q\) based on the observation of vehicles across a space-time window \(A\):

\[
k = \sum_j \frac{l_j}{|A|} \quad (2)
\]

\[
q = \sum_j \frac{l_j}{|A|} \quad (3)
\]

where \(l_j\) and \(t_j\) are respectively the length traveled and the time spent in the area by vehicle \(j\) and \(|A|\) the area of \(A\). It turns out that such measurements perfectly match MFD definition \((34)\). In the case of a single pipe, the estimated measurements perfectly match the theoretical MFD. Thus, simulations can be performed for any value of demand. We also add an exit bottleneck to generate congested states. Thereby, the congested part of the MFD will also be appraised.

Time-space diagrams similar to the diagrams of method M2 are obtained. Impacts of the MB are clearly highlighted. It generates lane-changing phenomenon upstream of the MB and reduces the capacity downstream. Figure 8 shows MFDs obtained for various values of bus headway \(h\).

![Figure 8: MFD estimated from simulated trajectories (Method M3)](image)

**CROSS-COMPARISON**

The three methods we proposed aim to reproduce the effects of buses on arterial traffic dynamics. A bus may create a local and temporal capacity reduction, which leads to an increase of travel-time. This increase clearly depends on the demand level \(q_a\), but also on the bus headway \(h\). It is thus appealing to study the evolution of vehicles and bus mean speed with respect to \(q_a\) but also to \(h\). Results of the methods are cross-compared based on this indicator but also with LOS of the HCM 2010, which provided a frame of reference.

**Calculation of mean speed values**

Consistent mean speeds of vehicles and buses have to be calculated to cross-compare the three methods. To this end, we will use instantaneous travel-times. Indeed, formulations that are fully consistent between the three approaches can be found. The centerpiece of these formulations is the definitions of density and flow proposed by \((35)\).
**Method M1:** This is the simplest case. It does not require any extra calculation. Mean speeds of the vehicles $V_{veh}^{1}$ are directly derived from estimated MFD and from the fundamental relationship $q = k \cdot v$. Thus, it turns out that $V_{veh}^{1}$ is equal to $k/q$. Mean speeds of the buses $V_{bus}^{1}$ can be calculated in the same way except that $V_{bus}^{1}$ cannot exceed the maximal speed of the buses $u_{bus}$. Figure 9a displays the mean speed values for the range of possible traffic conditions. It is interesting to notice that, in congestion, speed of the buses is not reduced until the speed of the equilibrium state is lower than $u_{bus}$. It means that even if $q$ is lower than $q_{U}$, i.e. the MB is not active, the buses are not impacted by the traffic congestion.

**Method M2:** Calculating mean speeds is a more tedious in that case. Edie’s definitions have to be adapted to this particular case. It is worth noticing that periodic patterns can be identified for each case (hashed areas in Figure 5 and Figure 6). These patterns are composed of different areas $A_i$ where traffic is in equilibrium state. Coordinates of these areas are analytically known such as values of flow and density. It is thus appealing to calculate averaged density $K$ and flow $Q$ based on these patterns. Then, $K$ and $Q$ are calculated as

$$K = \frac{1}{h_i} \sum_{i}^{k_i} \frac{k_i}{A_i} \quad (4)$$

$$Q = \frac{1}{h_i} \sum_{i}^{q_i} \frac{q_i}{A_i} \quad (5)$$

Where $A_i$ are the different equilibrium space-time area, $k_i$ and $q_i$ the respective density and flow. This leads to the averaged speed $V_{veh}^{2} = Q/K$. As previously explained, mean speeds of the buses $V_{bus}^{2}$ can be calculated in the same way except that $V_{bus}^{2}$ cannot exceed the maximal speed of the buses $u_{bus}$. Of course, these results depend on the bus headway, the entry demand and the exit rate. Figure 9b depicts the mean speeds of vehicles and buses for the range of possible traffic conditions.

**Method M3:** Results provided by method M3 make possible to calculate directly $k$ and $q$ as in Edie’s original definitions. As previously explained, trajectories of vehicles are available. Consequently, $l_j$ and $t_j$ can be easily
computed. The main difficulty for method M3 is to use a space-time window consistent with the previous approaches. However, this window cannot exactly match the analytical areas calculated by method M2 otherwise methods M2 and M3 will be redundant. Consequently, we decided to use, in addition to an arterial warm-up time, a very long space-time window (more than ten times the bus headway \( h \)). Figure 9c highlights the mean speeds \( V_{\text{veh}}^{3} \) and \( V_{\text{bus}}^{3} \) for the same range of traffic conditions than methods M1 and M2.

Results of the three approaches can now be cross-compared. Figure 10a-d sums up the mean speed values for each method. At a first glance, it is worth noticing that results are close from each other. However, it turns out that differences exist. Indeed, the effects of the MB are observed for different ranges of flow values depending on the used method.

Maximal observed capacities are consistent within the different methods with slight differences. Each method includes different modeling assumption. Notably, impacts of traffic signals and moving obstructions are non-correlated with M2, while being correlated with M1 and M3. This surely explains why maximal capacity is bigger when applying M2 because worth cases with high correlations are not reproduced. Higher capacities of method M2 are also due to the fact that the trajectory of the MB is discretized in the framework of method M1. This discretization induces an extra delay in the vehicle travel time calculation according to (24-25). Furthermore, MB can be inactive in method M2 whereas the MB can be active in some part of the arterial in method M1. It explains why the ranges of MB activity differ from one method to another. This also leads to the fact that method M2 underestimates the effects of the bus and provides higher capacities.

One can also see that method M3 furnishes mean speed values lower than the two analytical methods. Lane-changing maneuvers caused by the MB increase the vehicle travel time. Consequently, it also reduces the capacity and explains results of Figure 10a-d.

Figure 10e depicts the relative results provided by the three methods from the reference scenario corresponding to the same arterial with no bus. We have calculated the total difference in flow for the whole range of traffic conditions. This aggregated indicator reveals that methods can always be classified in the same order. Method M1 is the most sensitive to the presence of bus whereas method M2 seems to under-estimate impacts of bus. This discrepancy confirms the observations mentioned above. Results of method M3 stands between estimations of both previous methods. Finally, it is worth noticing that estimations tend to converge to the MFD estimated without bus when the headway increases.
Figure 10: Comparison of mean speed obtained with the different methods
(a) \( h = 3 \text{min} \)  (b) \( h = 6 \text{min} \)  (c) \( h = 12 \text{min} \)  (d) \( h = 30 \text{min} \)
(e) Difference in flow density with a no bus situation

Level Of Services (LOS)
The results are now confronted to the LOS proposed by the HCM 2010 \((/9)\). We only focused on the automobile LOS because transit LOS are not consistent with our approach. Indeed, this methodology is applicable for bus system that stops along the arterials and boards passengers. This is not the case in our paper. Then, the
automobile LOS measures are computed based on the methodology proposed for urban street segments. Details can be found in (19).

Speed-flow relationship is calculated for our study case (see Figure 11a). The point is that LOS of the HCM underestimated effects of the traffic signal on the running speed of the vehicles. It is not surprising because correlation between successive traffic signal is not accounted for in this methodology. The second drawback is that the HCM does not provided running speed values for congested situations and especially when spillbacks occur. Our methods clearly fill this gap that is of great importance for city managers.

Then, the HCM running speed thresholds for urban street segments are used to convert mean speed estimated for methods M1 and M2 into LOS letter grades. Figure 11b clearly shows that the grade A is never obtained. Grade D and E corresponds to transient situations. Ranges of mean flow are very small compare to ranges of grade B, C and F. Notice that Figure 11b displays only results of method M1. Similar results are obtained for method M2.

![Figure 11: (a) Comparison of the three methods with HCM 2010 (b) LOS obtained with method M1 for the different headway values](https://via.placeholder.com/150)

**CONCLUSION**

By extending past research on MFD estimation methods, this paper shows the potential for using MFD to analytically evaluate performance of an arterial. To this end, we first refine the work of (16) to account for buses in traffic. This method is based on the VT and consists in defining a suitable graph. It provides all the necessary cuts that defined the upper envelop of the MFD. The second approach relies on the analytical study of time-space diagrams. According to Edie’s definitions, it has been shown that it is possible to estimate MFDs that account for buses based on the KW theory and the MFD calculated without presence of buses. Finally, we directly estimate MFDs from vehicle trajectories provided by micro-simulation software that is fully consistent with both previous methods.

Then, these three methods have been cross-compared based on estimated mean speed values. The methods furnish average running speed of vehicles and buses for range of all possible traffic conditions (both free-flow and congested situations). It turns out that discrepancy exist in results. Method M1 is the most sensitive to the presence of buses, method M2 always under-estimates impacts of buses whereas results of method M3 falls between estimations of both previous method. These observed differences are easily understandable by the fact that discordances exist between the modeling processes: MB discretization, aggregation of impacts of traffic signal, lane-changing phenomenon, etc. Readers and practitioners can then choose between a detailed and complex method (M1), an analytical method (M2) or micro simulation (M3) to evaluate performance of an arterial. However, M1 presents the best trade-off and turns to be the more appropriate method to estimate accurate MFD.

Finally, methods are compared with the LOS of the HCM 2010. These results are promising, especially in the light of the fact that methods fill the drawbacks of the LOS: congested situations are also appraised. Furthermore, running speed of vehicles estimated by method M1 and M2 are more realistic because effects of successive traffic signal are accounted for. These methods significantly improved the understanding and the
evaluation of traffic dynamics of an arterial. One of the next extensions of the methods presented in this paper will be a more realistic modeling of bus systems. Although impacts of buses on traffic are already accounted for, ongoing work is investigating how to incorporate dynamics of motion laws of the bus systems (boarding and alighting of passengers at bus stops). These results can be of great importance to practitioner to manage arterials and bus systems.

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**REFERENCES**

2. Mahmassani, H. S., Williams, J. C., Herman, R., 1984, Investigation of Network-level traffic flow relationships: some simulation results, Transportation Research Record: Journal of the Transportation Research Board 971, 121 - 130.