Yield Improvement by the Redundancy Method for Component Calibration

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We explore the benefits of a redundant channels methodology for the component calibration. We propose a normal approximation of the yield in order to estimate the number of redundant components needed to provide a minimal area occupied by the components.

Introduction: The demand for high data rate in communication puts stringent requirements on components' dynamic range and especially on high speed ADC. However, at the same time, the extreme size reduction in advanced technology has resulted inadvertently in increased process variability, which inherently limits the performances. For several years, the redundancy approach has been identified as complementary to digital calibration [1] to improve the performances. The approach is particularly efficient for interleaved ADCs. Recently, it has been shown in [2] that the redundancy resulted from dividing an elementary component (capacitor, resistor, transistor) into several subsets can be even more efficient in improving the matching precision. Paradoxically, increasing the number of elementary components by adding the redundant ones can lead to a decrease of the total area, since the needed precision for an elementary component becomes lower and the variance is inversely proportional to the area. However, the reduction in the total area depends on the target yield as well as on the number of needed elementary components. Therefore, it is hard for a designer to select an optimal number of redundant components and the usual way to solve this problem is to resort to statistical simulations which are time consuming and sometimes misleading. In this paper, we explore the benefits of the redundancy method and derive some expressions which can be useful to fully exploit the approach.

The approach is as follows. To produce a given number N of capacitors with a high yield we produce a larger number of capacitors in such a way that the yield of a subset of N capacitors was a priori high. The problem is to estimate the minimal number of redundant capacitors N_{red} required to provide the given target yield.

Methodology: The capacitance X of a single capacitor has a Gaussian distribution with mean μ and variance σ^2. Let ε be the accuracy level for the mean capacitance μ such that we have to produce capacitors with capacitance within the interval $\mu \pm \varepsilon$ with high probability. This value is fixed and $\varepsilon/\mu \approx 10^{-2}$–$10^{-5}$. The initial yield is the probability for a capacitor to satisfy the specifications, $Y_0 = \mathbf{P}\{|X-\mu| < \varepsilon\}$.

Assume that we need to produce N capacitors such that each capacitor out of those N satisfies the specifications with high probability. Obviously, if we produce exactly N capacitors, then their total yield is equal to Y_0^N, which is not high enough even if N is not large. For example, if $Y_0 = 0.95$ and $N = 4$, the total yield of four capacitors is only equal to 0.8145. To increase the total yield we produce N_{red} additional capacitors and then choose N capacitors out of those $N + N_{\mathrm{red}}$. The number of redundant capacitors N_{red} has to be chosen in such a way that a total yield of N capacitors is greater than some given value Y_T. We call Y_T a target yield.

Optimization problem: The number of capacitors Z satisfying the specifications follows the binomial distribution with $N + N_{\mathrm{red}}$ trials and the probability of success Y_0. The total yield is the probability that at least N capacitors satisfy the specifications,

$$Y_N(N_{\mathrm{red}}, Y_0) = \sum_{i=N}^{N+N_{\mathrm{red}}} \binom{N+N_{\mathrm{red}}}{i} Y_0^i (1-Y_0)^{N+N_{\mathrm{red}}-i}. \qquad (1)$$

The goal is to find a minimal number of redundant channels N_{red} such that the total yield is at least Y_T, $Y_N(N_{\mathrm{red}}, Y_0) \ge Y_T$, and the total area occupied by all capacitors

$$S_N\left(N_{\mathrm{red}}, \frac{\varepsilon}{\sigma}\right) = A^2 (N + N_{\mathrm{red}}) \frac{\varepsilon}{\sigma},$$

is minimal. Here A^2 is the area proportionality constant [3]. Denote by $\Phi(t)$ the standard normal distribution function. Then the initial yield is given by $Y_0 = \mathbf{P}\{|X-\mu| < \varepsilon\} = 2\Phi(\varepsilon/\sigma) - 1$, and we have $\varepsilon/\sigma = \Phi^{-1}((Y_0+1)/2)$. Now we can rewrite the total area in terms of the initial yield,

$$S_N(N_{\mathrm{red}}, Y_0) = (N + N_{\mathrm{red}}) \Phi^{-1}\left(\frac{Y_0+1}{2}\right). \qquad (2)$$

Thus, we have the following optimization problem,

$$S_N(N_{\mathrm{red}}, Y_0) \to \min_{N_{\mathrm{red}}, Y_0} \qquad (3)$$

$$\sum_{i=N}^{N+N_{\mathrm{red}}} \binom{N+N_{\mathrm{red}}}{i} Y_0^i (1-Y_0)^{N+N_{\mathrm{red}}-i} \ge Y_T. \qquad (4)$$

A natural way to solve this problem is to find the minimal number of redundant capacitors $N_{\mathrm{red}}(Y_0)$ satisfying (4) as a function of the initial yield Y_0. Then substituting it into (2) and minimizing the objective with respect to Y_0 gives the desired optimal numbers of N_{red} and Y_0.

The exact numerical solution to problem (3)–(4) can be obtained easily, but it is time-consuming due to the cost of computation of binomial coefficients. We propose to use the following normal approximation of binomial distribution with the continuity correction,

$$Y_N(N_{\mathrm{red}}, Y_0) \approx 1 - \Phi\left(\frac{N + 1/2 - (N+N_{\mathrm{red}})Y_0}{\left[(N+N_{\mathrm{red}})Y_0(1-Y_0)\right]^{1/2}}\right). \qquad (5)$$

Denote $q = \Phi^{-1}(1 - Y_T)$. Using (5) we can estimate from below the minimal number of redundant capacitors satisfying (4) by a function of Y_0, such that $N_{\mathrm{red}} \ge N_{\mathrm{red}}^{\mathrm{app}}(Y_0)$, where

$$N_{\mathrm{red}}^{\mathrm{app}}(Y_0) = \frac{(2N+q^2)(1-Y_0)+1}{2Y_0} + \frac{q(1-Y_0)}{2Y_0}\left(q^2 + \frac{4N+2}{1-Y_0}\right)^{1/2}. \qquad (6)$$

Figure 1 shows that the target function $S_N(N_{\mathrm{red}}^{\mathrm{app}}, Y_0)$ is convex, therefore, the minimum does not fall on on the boundary of $[0,1]$ interval. Unfortunately, it is quite hard to prove the convexity analytically. We can see also that for a large number N of capacitors we have quite a "flat" minimum and, consequently, many possibilities to choose the number of redundant capacitors within some reasonable accuracy level for the area.

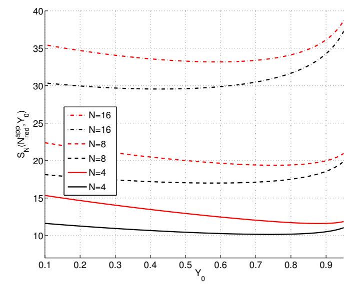

Fig. 1 *The area $S_N(N_{\mathrm{red}}^{\mathrm{app}}, Y_0)$ for the target yield $Y_T = 0.95$ (black) and $Y_T = 0.99$ (red).*

Taking into account (6) we obtain that an approximate solution to (3) minimizes the approximate area

$$S_N(N_{\mathrm{red}}^{\mathrm{app}}, Y_0) = (N + N_{\mathrm{red}}^{\mathrm{app}}(Y_0)) \Phi^{-1}\left(\frac{Y_0+1}{2}\right) \to \min_{Y_0}$$

so that the approximate solution is given by

$$Y_0^{\mathrm{app}} = \arg \min_{0 \le Y_0 \le 1} \left\{ (N + N_{\mathrm{red}}^{\mathrm{app}}(Y_0)) \Phi^{-1}\left(\frac{Y_0+1}{2}\right) \right\}.$$

To compare the results with the non-redundancy case we introduce the non-redundancy area $S_0(Y_T)$ that is used for production of N capacitors with the target yield Y_T. It is easy to show that $Y_0 = A^2 N \Phi^{-1}((Y_T^{1/N} + 1)/2)$. In Figure 2 the ratio of $S_0(Y_T)$ to the approximate optimal area is shown depending on $(1 - Y_T)$ for different values of desired number of capacitors N. In Figure 3, the ratio $R_N = (N + N_{\mathrm{red}})^{-1} S_N / (N^{-1} S_0)$

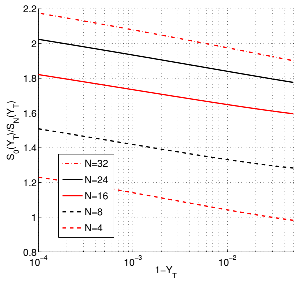

Fig. 2 *The ratio of the non-redundancy area to the approximate optimal area in log-scale depending on the target yield.*

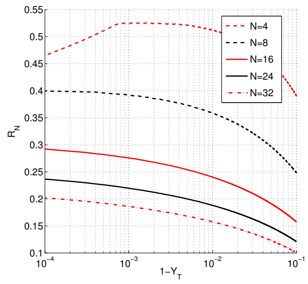

Fig. 3 *The ratio of size of the new elementary capacitor to the size of the original capacitor.*

of the size of the new elementary capacitor to the original one (without redundancy) is plotted. It illustrates that in all configurations both the total area and the elementary capacitor's size are reduced. For a small number of capacitors N, the method is more efficient in reducing the component size whereas for large arrays, the total area can be reduced a factor of two.

Figure 4 shows the approximate optimal number of redundant capacitors depending on the target yield Y_T. We can see that the higher is the target yield, the smaller number of redundant capacitors we need in order to obtain an optimal area. Of course, for this gain in the number of elements we have to pay by increasing the initial yield Y_0. Note also that the function $N_{\text{red}}^{\text{app}}(Y_0)$ is a decreasing function of Y_0. Thus we can choose a larger value of Y_0 and still obtain the desired target yield.

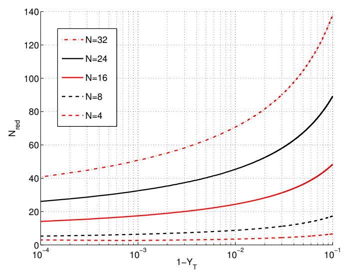

Fig. 4 *Approximate optimal number of redundant capacitors in log-scale depending on the target yield.*

The results of optimization for $Y_T = 0.99$ are presented in the table. Comparing the values of the yield Y_N and Y_N^{app} we can see that the normal approximation works quite well if N is large. As to the values of N_{red}, the normal approximation has a tendency to choose a smaller number comparing to the exact solution, but the yield remains quite close to the desired. We can see that using normal approximation gives a good idea about the initial yield of a single capacitor Y_0 that is required to provide the target yield. The difference in the approximate optimal area S_N^{app} and the area obtained directly, S_N, is less than 1%. At the same time, the total yield (1) remains very close to the target yield when using approximate number of redundant capacitors $N_{\text{red}}^{\text{app}}$.

Table 1: The exact and approximate values of the initial yield, Y_0 and Y_0^{app}, the minimal area S_N and S_N^{app}, the number of redundant capacitors N_{red} and $N_{\text{red}}^{\text{app}}$ corresponding to the target yield $Y_T = 0.99$. The corresponding values of the yield Y_N and Y_N^{app} are also presented.

N	Y_0	Y_0^{app}	S_N	S_N^{app}	N_{red}	$N_{\text{red}}^{\text{app}}$	Y_N	Y_N^{app}
4	0.782	0.878	12.311	11.598	6	3	0.990	0.957
8	0.653	0.753	19.741	19.379	13	9	0.990	0.989
16	0.485	0.589	33.181	33.181	35	24	0.990	0.988
24	0.406	0.493	45.830	46.009	62	45	0.990	0.989
32	0.366	0.430	58.068	58.355	90	71	0.990	0.991

Conclusion: We applied the redundancy method to the problem of yield improvement in capacitance calibration. By using of the normal approximation of the yield, we proposed a faster way of solving the total area optimization problem. This approach can help to select an optimal number of redundant components. We have also shown that the redundancy method can be applied to simultaneous reduction of the cost (total area) and the power consumption (component size) of advanced systems.

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