Turbulence and columnar vortex formation through inertial-wave focusing
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In rotating turbulence, the Coriolis acceleration is responsible for the presence of inertial waves (also known as gyroscopic or Kelvin waves) and the formation of large columnar vortices [1]. The formation of columnar structures is an ever-present feature in rotating turbulence that has been observed in flows forced by a continuously oscillating grid [1,2], electromagnetic forcing [3,4], and a translating grid (for decaying flows) [5].

Previously, the formation of these columnar structures was explained using the Taylor-Proudman theorem, but this theorem is strictly valid only for infinite rotation rates and not for turbulent flows. More recent studies have pointed out the importance of inertial waves for this process [2,5,6]. Depending on the value of the Rossby number Ro, which represents the ratio between convective and Coriolis acceleration, different regimes have been observed. For strong rotation rates, the columnar structures are seen as zero-frequency inertial waves [2]. For intermediate Ro values, weakly nonlinear triad interactions are responsible for the formation of the columnar vortices [6]. Because of their relevance to momentum transport, inertial waves are at the base of the inverse energy cascade observed in rotating flows, where energy transfers from large to small wave number, which is opposite to the direct energy cascade resulting in angular momentum mixing and the generation of a columnar cyclonic vortex. The results suggest that nonlinear dynamics is essential for the wave induced momentum transport towards columnar vortices in rotating turbulence.

In this experimental and numerical study, we consider the role of inertial waves in the inverse energy cascade and the transfer of momentum in a rotating fluid. An oscillating torus generates two inertial-wave cones with their energy focusing at their apex. For high wave amplitudes, turbulence is generated locally around the focal point, resulting in angular momentum mixing and the generation of a columnar cyclonic vortex. The results suggest that nonlinear dynamics is essential for the wave induced momentum transport towards columnar vortices in rotating turbulence.

In the current Rapid Communication, we attribute the vortex formation to angular momentum mixing due to the turbulence generated by the inertial-wave breaking. Some applications of this process that have been considered previously are the generations of tornadoses in the atmosphere [8], vortex formation in accretion disks (see Ref. [9] and references therein), and the generation of mean flows in ocean basins by focusing of inertial waves into attractors [10]. The principle of this process is that the horizontal exchange of fluid rings in a rotating fluid gives rise to a cyclonic flow, and at larger radii, anticyclonic flow (see Refs. [8,10,11]).

Experiments were conducted in a cubic tank with sides 100 cm long. The tank was filled to a depth $H = 80$ cm with salt water ($\rho = 1.02$ kg/m$^3$) to match the density of particles used for PIV and placed on top of a rotating table, which rotated at a constant rate $\Omega$. In the center at 40 cm height above the bottom, a solid Plexiglas torus with a small radius $R = 1.5$ cm and a large radius $R_L = 13.5$ cm oscillated vertically. The oscillation was driven by an electric motor, and the forcing frequency $\sigma$ was controlled through an electric feedback system. We define the coordinate system $(\hat{\mathbf{e}}_1, \hat{\mathbf{e}}_2, \hat{\mathbf{e}}_3)$ with $\hat{\mathbf{e}}_3$ parallel to the axis of rotation. The velocity and the vorticity vectors are given by $\mathbf{u} = (u_1, u_2, u_3)$ and $\mathbf{\omega} = (\omega_1, \omega_2, \omega_3)$, respectively.

The torus oscillated as $K \cos(\sigma t)$ with $K$ the amplitude and $0 < \sigma < 2\Omega$, generating two inertial-wave cones. According to linear theory, the waves are characterized by the dispersion relation

$$\cos \alpha = \frac{\sigma}{2\Omega},$$

i.e., the angle of propagation of the waves, with respect to the horizontal $\alpha$, is determined by the ratio of the forcing frequency to the rotation frequency. This is similar to the radiation of internal waves in a uniformly stratified fluid. In a vertical section, the energy propagates along rays away from the oscillating torus and forms a pattern similar to the diagonal cross (often called the St. Andrew’s Cross) for planar internal
 gravity waves [12]. The phase velocity is perpendicular to these rays.

The three velocity components were measured on a vertical plane crossing the center of the torus using S-PIV. For this, the water was seeded with 60 μm polymer particles. A vertical laser sheet, of 5 mm thickness and passing through the center of the ring, illuminated the particles. On one side of the tank, two cameras were positioned, observing the center of the ring, illuminated the particles. On one vertical laser sheet, of 5 mm thickness and passing through the water was seeded with 60

plane crossing the center of the torus using S-PIV. For this, the UVMAT/CIV software packages developed at the Coriolis Rotating Platform at the LEGI (Grenoble, France)1 were used.

For fixed ratio $R/R_L$, there are three nondimensional parameters governing the problem: the Reynolds number,

$$\text{Re} = \frac{\sigma K (2R)}{\nu},$$

which represents a ratio of convective acceleration to viscous momentum diffusion; the Rossby number

$$\text{Ro} = \frac{\sigma K}{2R \Omega};$$

and the angle of propagation $\alpha$ obtained from the dispersion relation (1). Sometimes, the ratio $\sigma/2\Omega$ is referred to as the Rossby number, but since the relative magnitude of convective acceleration with respect to that of the Coriolis acceleration is better represented by (3) for the flow studied, we prefer this definition.

To explore the parameter space, nine different experiments were performed for various values of the frequency $\sigma$, the amplitude $K$ of the oscillation, and the rotation rate of the table $\Omega$. In the current paper, we focus on the results of two of these experiments (see Table I) and their corresponding numerical simulations. These experiments are extreme cases within the parameter space explored: with Expt. 1 having a low Re value and a low Ro value, and hence, being characterized by weak wave-breaking effects due to the wave focusing, and Expt. 2 having the highest Re value for Ro < 0.5 and a clearly turbulent focal region. The other experiments represent intermediate situations with only quantitative differences.

Numerical simulations were performed using a pseudospectral code based on Fourier polynomials that solves the full Navier-Stokes equations with a resolution of 5763 and volume penalization for the moving torus and the upper and lower boundaries (see Ref. [13] for more details). Side boundaries were periodic. In view of the flow symmetry and the weak wave energy near these boundaries, this choice does not affect the results.

Figure 2 shows isosurfaces of the vertical component of the velocity $u_3$ for the simulation equivalent to Expt. 1. The inertial-wave cone excited by the oscillating torus can be clearly observed. Figure 3(a) shows the phase averaged velocity taken at phase $t = 2n\pi$ with $n = 0,1,2,\ldots$ and Fig. 3(b) the velocity magnitude averaged over 60 periods, with both figures clearly revealing the wave beams. The wave energy grows from the torus towards the apex of the cones, and it decreases afterwards due to the radial spreading. However, the energy does not increase monotonically towards the apex but it is concentrated within a close to spherical region. The diameter of this region is given by the thickness of the wave beams

$$\delta = 2R + 6.84 \left( \frac{\nu r}{\sigma \tan \alpha} \right)^{1/3},$$

at $r = R_L \sin \alpha$, where $r$ is the distance from the center of the tube along a wave beam (see Ref. [14] for details). For this experiment, the average velocity at the apex of the cones

FIG. 2. (Color online) Inertial-wave cones excited by the oscillating torus as depicted by isosurfaces of $u_3$ from a numerical simulation equivalent to Expt. 1. The figure has been rotated 90° for presentation purposes. The torus is shown at the center of the pictures (in blue).

FIG. 1. (Color online) Schematic representation of the experimental setup.

TABLE I. Parameter values for the experiments and numerical simulations presented.

<table>
<thead>
<tr>
<th>Expt.</th>
<th>$\Omega$ (rad/s)</th>
<th>$\sigma$ (rad/s)</th>
<th>K (cm)</th>
<th>Re</th>
<th>$\alpha$</th>
<th>Ro</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.889</td>
<td>1.26</td>
<td>0.25</td>
<td>94</td>
<td>$\pi/4$</td>
<td>0.12</td>
</tr>
<tr>
<td>2</td>
<td>1.110</td>
<td>1.56</td>
<td>1.00</td>
<td>470</td>
<td>$\pi/4$</td>
<td>0.47</td>
</tr>
</tbody>
</table>

1Information and code are available at http://coriolis.legi.grenoble-inp.fr
is about eight times larger than the velocity along the wave beam closer to the torus.

Figure 4 shows the experimentally (a) and numerically (b) obtained nondimensional time-averaged velocity magnitude for the parameters of Expt. 2. The two (nondimensionalized) results reveal good agreement in velocity magnitude and also in flow structure: The kinetic energy concentrates at the apex of the inertial-wave cones, from which a long vertical cyclonic vortical structure emanates. However, the simulations seem to be less dissipative than the experiments as more energy is found outside the wave cones.

The vortical structure can be clearly observed by averaging either the azimuthal velocity, or the vertical vorticity over an integer number of periods [see Figs. 5(a) and 5(b), respectively]. Turbulence is generated by the breaking of the inertial waves around the apex of the cone and mixes angular momentum irreversibly [8]. This explanation is supported by the temporal spectra of the flow. Figure 6 shows power spectral density (PSD) of $u_3$ at three different points along the inertial wave beam [their locations are shown in Fig. 4(b)]. The spectra were calculated from the numerical simulations since the experimental measurements have a temporal resolution which is too coarse to resolve the frequencies of the turbulence. For frequencies higher than $2\Omega$, the spectra quickly decay outside the high-energy region, whereas inside much more energy content is found for these frequencies. This is a clear indication of a more efficient cascade towards higher frequencies inside the blob, and that this nonlinear behavior is confined within the focal region of wave breaking.

The wave beams are much weaker after the apex of the cone since part of their energy is transferred to the vortex (see Fig. 4). The transfer of energy from the waves towards the columnar vortex is determined from the energy of the mean flow by integration of the velocity field over an integer number of forcing periods. The integration filters out the wave motion but retains the mean flow. The experimental and numerical results for Expt. 2, both show that the mean azimuthal flow contains 45% of the total energy within the experimental measurement area, thus confirming the quantitative agreement between both approaches. The numerics allow therefore to extrapolate and to calculate the energy content of the mean flow in the entire area, thus confirming the quantitative agreement between both approaches.
FIG. 6. (Color online) Power spectral density (PSD) of the vertical velocity component \(u_3\) at points A, B, and C of Fig. 4 along the wave beam for the simulation corresponding to Expt. 2.

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\[7\] L. Messio, C. Morize, M. Rabaud, and F. Moisy, Exp. Fluids 44, 519 (2007).