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A Multi-Level Interface Model for Damaged Masonry

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Abstract

The aim of this paper is to propose a new micro-mechanical model in the context of the deductive approach used to derive interface models. This model, based on a previous study introduced previously by A. Rekik and F. Lebon [31, 32], is used to reproduce the damage in masonry by combining structural analysis and homogenization methods. The focal point of this method is to assume the existence of a third material, called interphase, which is a mixture of the two principal constituents of masonry, brick and mortar, and that is the interface between them. This new element presents a low thickness, a low stiffness and a given damage ratio. The mechanical problem of masonry, initially a 3D problem, is solved numerically as a 2D problem using finite element methods. The properties of the interface brick-mortar material are obtained using three essentials steps. First of all, an exact homogenization of a laminates is used to define a first homogeneous equivalent medium named HEM-1. After, the assumption of damaged material is taken into account by using the general framework given by M. Kachanov [13, 24, 35, 36] to evaluate the global behaviour of the damaged HEM-1 defining thus a second equivalent homogeneous medium noted HEM-2. The last step consists in using an asymptotic analysis technique which is performed to model HEM-2 as an interface or a joint. The properties of this joint are deduced from those of the HEM-2 material as proposed in former papers [2, 15, 17, 34]. Particularly, through the second homogenization are taken into account the variability of microcracks oriented family and simultaneously the opening-closure effects (unilateral behaviour). Numerically this interface is modelled with connector finite elements. Numerical results are compared to experimental ones available in the literature [7].

Keywords: Masonry, interfaces, damage, homogenization, microcracks, unilateral effects, asymptotic analysis.

1 Introduction

Historically, the masonry is considered one of the oldest building material although its mechanical behaviour is still the subject of many research activities. It is usually described as a composite material formed by units (bricks, natural stones, marble, granite, limestone, concrete block, etc.) and joint, with or without adhesive component (mortar, concrete, clay, etc.), and different bond arrangements. Masonry is still used nowadays to build houses because of its qualities of strength, solidity, durability and fire resistance, and its elegant appearance, etc. It is certain that the problems associated with modelling ancient and modern masonry structures are very different. Physical evidence shows us that ancient masonry is a very complex material with three-dimensional internal arrangement, usually unreinforced, but which can include some form of traditional reinforcement. Moreover, these materials are associated with complex structural systems, where the separation between architectural features and structural elements is not always clear. However, masonry, which is not generally thought to be a highly technological material, shows highly complex behaviour, due in particular to the interactions between the components and the anisotropy induced by the direction of the joints, which are a source of weakness. Even for simple geometries and far from failure loadings, masonry structures exhibit a mechanical response affected by extreme stiffness contrast between constituents, randomness of contact points between bricks where unilaterality and Coulomb friction dominate. The interplay between extreme stiffness contrast and randomness on the one hand and regularity of the fabric on the other, yields stress distributions within masonry walls that may present localized stress paths, evidencing stress concentrations and stress relieves. It is classically held that the seismic vulnerability of masonry buildings depends strongly on their resistance to shear forces. It is therefore of great interest to model and test the shear responses of building components subjected to loading of this kind, especially cyclic loading. These responses have generally been characterized by a peak load, loss of rigidity and energy dissipation. To summarize considerably, two methods of modelling masonry structures have been used so far. The first method involves macroscopic models, in which the wall is assumed to be a single structural element characterized by a non linear response when it is exposed to shear forces [5, 11, 23, 26, 27, 29, 33]. In continuum macro-models bricks, mortar and brick-mortar interfaces are smoothed out into a homogeneous continuum, the average properties of which are identified at the level of the constituents, taking their geometric arrangement into account [20, 22]. This technique is particularly indicated in global modelization of unreinforced masonry structures in which the very low tensile strength of the material renders the use of non-linear constitutive behaviour more obvious. This is particularly true in the assessment of existing structures and in seismic analysis. To describe the inelastic behaviour of structural masonry, some authors have combined homogenization techniques with a continuum damage mechanics approach [2, 38]. On the other hand, some models of micro-analysis school have been developed for predicting the evolution of damage into interface between two initially bonded deformable bodies [7, 9, 16, 18, 24, 25, 30, 35]. Two main modelling approaches used for

this purpose are phenomenological modelling and deductive modelling. In the first approach, the thickness of the interface is taken to be zero and the mechanical properties are obtained from physical considerations and experiments (see for example [3, 8–10] and references therein). The second approach consists in focusing on the thin layers materials at the micro-mechanical level, which are usually called the interphase. Several authors [1, 14, 15, 17, 19, 21, 28] have established that the interface elements reflect the main interactions occurring between bricks and mortar. For this reason, various studies have been presented for modelling the behaviour of interfaces with zero thickness and predicting their failure modes. Some studies [6], for example, expressed the constitutive law at the interface in terms of contact traction and the relative displacements of two surfaces interacting at the joint. The fracture of the joint and the subsequent sliding are associated with the interface yield condition. Method based on limit analysis combined with a homogenization technique was shown [5] to be a powerful structural analysis tool, giving accurate collapse predictions. The brittle damage model developed in [2, 28] involves an elementary cell composed of units, mortar and a finite number of fractures at the interfaces. In this paper, a multi-level model for interfaces based on homogenization and asymptotic techniques is presented. This model is based on a previous study proposed by A. Rekik and F. Lebon [31, 32]. The first part of this paper gives an accurate version of the mechanical modelling approach used. The multi-level approach used is described. This approach takes into account the mechanical characteristics of the mortar and bricks, the presence of micro-cracks and the thickness of the interface into. In the second part, the numerical procedure used and implemented using a finite element software program is presented and some numerical examples are given and compared with experimental data [7, 11].

2 An accurate version of the Rekik-Lebon model [31, 32]

Masonry units have generally been discretized using continuum elements, whereas joints have been modelled in the form of weakness planes, using interface elements. The main limitation of this approach is the fact that the interactions between joints and brick units cannot be satisfactorily described. The most original feature of this kind of model is that it includes a third material inserted between the units and the mortar, which accounts for the noticeable differences generally existing between the mechanical properties of bricks and mortar. In order to model interface damage to masonry structures, the present method based on homogenization theories, asymptotic techniques and finite element methods was developed. The main steps involved in this method will be described below. Most studies on masonry structures have dealt with only two materials: brick and mortar. In the present work, it is assumed the existence of a third material: the brick/mortar interface, which is considered as a mixture of brick and mortar with a crack density ρ. To obtain the effective properties of the damaged intermediate material, three steps are performed. First it is calculated the exact effective properties of the crack-free material using homogenization techniques for laminate composites, for the sake of simplicity it is considered to have the same

volume fraction for both constituents, and thus define a first homogeneous equivalent medium, which will be referred below as HEM-1. In the second step, it is assigned a proper crack density ρ to the previous material. To model the macroscopic behaviour of the cracked material HEM-1, it is used the Kachanov model and then it is defined a new homogeneous equivalent medium HEM-2. Finally, in order to be sandwiched between the brick and mortar, this material is given a small thickness, and its mechanical behaviour is derived using asymptotic techniques to shift from the micro to the macro level. With this interface law, the masonry structure problem can be solved using finite element methods.
The following overall scheme (Fig.1) describes the principles underlying the proposed model:

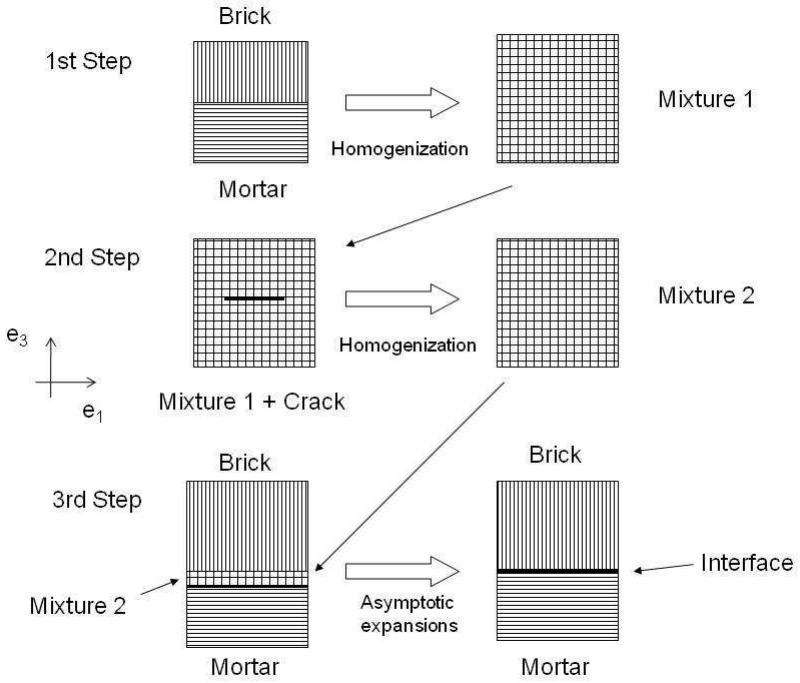


Figure 1: *Principle of the proposed model*

2.1 *Undamaged stratified composite homogenization*

It is proposed first to obtain the mechanical properties of the 3D interphase material by homogenizing those of brick and mortar. Both constituents are assumed to be isotropic and linear elastic materials. In the compliance form, their constitutive law reads:

$$\varepsilon_{ij}^{\zeta} = S_{ijkl}^{\zeta}\sigma_{ik}^{\zeta} = \frac{1+\nu^{\zeta}}{E^{\zeta}}\sigma_{ij}^{\zeta} - \frac{\nu^{\zeta}}{E^{\zeta}}\sigma_{kk}^{\zeta}\delta_{ij} \tag{1}$$

where S^{ζ}, E^{ζ} and ν^{ζ} are respectively the compliance tensor, the Young's modulus and the Poisson ratio of phase ζ ($\zeta = b$ for the brick, $\zeta = m$ for the mortar). The

macroscopic behaviour law of the laminate brick/mortar reads:

$$\bar{\varepsilon} = \bar{S}^0 : \bar{\sigma} \quad \text{where} \quad \bar{\varepsilon} = \sum_{\zeta=b,m} f^\zeta \varepsilon^\zeta = \sum_{\zeta=b,m} f^\zeta S^\zeta : \sigma^\zeta \tag{2}$$

where f^ζ denotes the volume fractions of phase ζ and $\bar{S}^0$ is the effective fourth-order compliance tensor of the homogeneous equivalent crack-free material supposed to be transversely isotropic . According to the modified Voigt notation, the macroscopic law (2) reads:

$$\begin{pmatrix} \bar{\varepsilon}_{11} \\ \bar{\varepsilon}_{22} \\ \bar{\varepsilon}_{33} \\ \sqrt{2}\bar{\varepsilon}_{23} \\ \sqrt{2}\bar{\varepsilon}_{13} \\ \sqrt{2}\bar{\varepsilon}_{12} \end{pmatrix} = \begin{pmatrix} \bar{S}^0_{1111} & \bar{S}^0_{1122} & \bar{S}^0_{1133} & 0 & 0 & 0 \\ \bar{S}^0_{1122} & \bar{S}^0_{1111} & \bar{S}^0_{1133} & 0 & 0 & 0 \\ \bar{S}^0_{1133} & \bar{S}^0_{1133} & \bar{S}^0_{3333} & 0 & 0 & 0 \\ 0 & 0 & 0 & 2\bar{S}^0_{1313} & 0 & 0 \\ 0 & 0 & 0 & 0 & 2\bar{S}^0_{1313} & 0 \\ 0 & 0 & 0 & 0 & 0 & \bar{S}^0_{1111} - \bar{S}^0_{1122} \end{pmatrix} \begin{pmatrix} \bar{\sigma}_{11} \\ \bar{\sigma}_{22} \\ \bar{\sigma}_{33} \\ \sqrt{2}\bar{\sigma}_{23} \\ \sqrt{2}\bar{\sigma}_{13} \\ \sqrt{2}\bar{\sigma}_{12} \end{pmatrix} \tag{3}$$

The five independent coefficients of the compliance tensor $\bar{S}^0$ are determined by applying three independent loads. The same results may be found in [4], where the macroscopic law of the laminate is given in the stiffness form $\bar{C}^0 = \left(\bar{S}^0\right)^{-1}$.
For the sake of simplicity the model is developed assuming the plane stress hypothesis in direction e_2 on the effective material, so that the 3D problem is reduced to a 2D problem in the (e_1, e_3) plane.

2.2 *Homogenization of the micro-cracked composite*

In this section, the material HEM-1 is assumed to contain an arbitrary distribution of rectilinear cracks located on the plane (e_1, e_3) in a representative area $A = L_0\varepsilon$, where L_0 is the bed mortar length and ε is the thickness of the micro-cracked HEM-2 material. Kachanov et al. [24, 35] provided an accurate approximation of the effective behaviour of such a material for open cracks; the average strain $\bar{\varepsilon}$ in a solid with N families of microcracks can be written in the form:

$$\bar{\varepsilon} = \bar{S} : \bar{\sigma} = (\bar{S}^0 + \Delta\bar{S}) : \bar{\sigma} \tag{4}$$

where $\bar{S}$ (resp. $\bar{S}^0$) is the effective compliance of the cracked (resp. crack-free) material and

$$\Delta\bar{S}_{ijhl} = \frac{1}{2A}\sum_{k=1}^{N}[n_i^{(k)}n_h^{(k)}B_{jl} + n_i^{(k)}n_l^{(k)}B_{jh} + B_{ih}n_j^{(k)}n_l^{(k)} + B_{il}n_j^{(k)}n_h^{(k)}](l^{(k)})^2 \tag{5}$$

in which $2l^{(k)}$ and $n^{(k)}$ are mean length and normal of the k^{th} family of cracks and A is the area of the representative 2D-domain.
The second rank symmetric tensor $B^{(k)}$ can be called the crack compliance tensor of the k^{th} family of cracks, which depends on the anisotropy of the virgin material [36]:

$$B^{(k)} = (C(1-D)e_1 \otimes e_1 + C(1+D)e_3 \otimes e_3) \tag{6}$$

where

$$\begin{cases} C = \dfrac{\pi}{4}\dfrac{\sqrt{E_1^0}+\sqrt{E_3^0}}{\sqrt{E_1^0 E_3^0}}\left(\dfrac{1}{G_{13}^0}-2\dfrac{\nu_{13}^0}{E_1^0}+\dfrac{2}{\sqrt{E_1^0 E_3^0}}\right)^{\frac{1}{2}} \\ D = \dfrac{\sqrt{E_1^0}-\sqrt{E_3^0}}{\sqrt{E_1^0}+\sqrt{E_3^0}} \end{cases} \tag{7}$$

E_1^0, E_3^0, ν_{13}^0 and G_{13}^0 are the effective elastic engineering constants of the crack-free material HEM-1. These constants are easily derived from the effective elastic compliances $\bar{S}^0$ as follows:

$$\begin{cases} E_1^0 = \dfrac{1}{S_{1111}^0} \\ E_3^0 = \dfrac{1}{S_{3333}^0} \\ \nu_{13}^0 = -\dfrac{S_{1133}^0}{S_{1111}^0} \\ G_{13}^0 = \dfrac{1}{4S_{1313}^0} \end{cases} \tag{8}$$

We note in (7) the dependence of the sign of D on the anisotropy ratio E_1^0/E_3^0.
We consider here a single family of microcracks normal to direction e_3 with mean length $2l$. Substituting (7) and (8) into (6), we obtain the change of elastic compliance:

$$\Delta\bar{S}_{ijhl} = \frac{1}{2}\rho[(e_3\otimes e_3)_{ih}B_{jl}+(e_3\otimes e_3)_{il}B_{jh}+B_{ih}(e_3\otimes e_3)_{jl}+B_{il}(e_3\otimes e_3)_{jh}] \tag{9}$$

where $\rho = 1/A\sum(l)^2$ is the proper scalar crack density parameter in 2D case The compliance of the cracked material HEM-2 is thus given by:

$$\begin{cases} S_{1111} = S_{1111}^0 \\ S_{3333} = S_{3333}^0 + 2\rho C(1+D) \\ S_{1133} = S_{1133}^0 \\ S_{1313} = S_{1313}^0 + \dfrac{1}{2}\rho C(1-D) \end{cases} \tag{10}$$

The elastic constants C and D are given by (7). The engineering constants E_1, E_3, ν_{13} and G_{13} of the cracked material HEM-2 are given by relations similar to (10) thus:

$$\begin{cases} E_1 = E_1^0 \\ E_3 = \dfrac{E_3^0}{1+2\rho C(1+D)E_3^0} \\ \nu_{13} = \nu_{13}^0 \\ G_{13} = \dfrac{G_{13}^0}{1+2\rho C(1-D)G_{13}^0} \end{cases} \tag{11}$$

2.3 *Third step: asymptotic analysis and interface law*

In this section, it is considered a thin joint composed of the material defined above, which is sandwiched between brick and mortar. Since the joint is thin and soft, it is natural to use asymptotic techniques, to study the limit problem (by tending the thickness to zero) and to replace the joint by an interface law defined along the limit surface. It is taken ε to denote the thickness of the joint, which is assumed to be constant and S to denote the limit surface of the joint (a line in 2D), corresponding to a thickness equal to zero (see fig. 2). We take $\mathbf{C}$ to denote the elasticity tensor of the joint and $\lim c_{ijkl}/\varepsilon = \bar{c}_{ijkl}$. The limits are assumed to exist. We take $[\,]$ to denote the

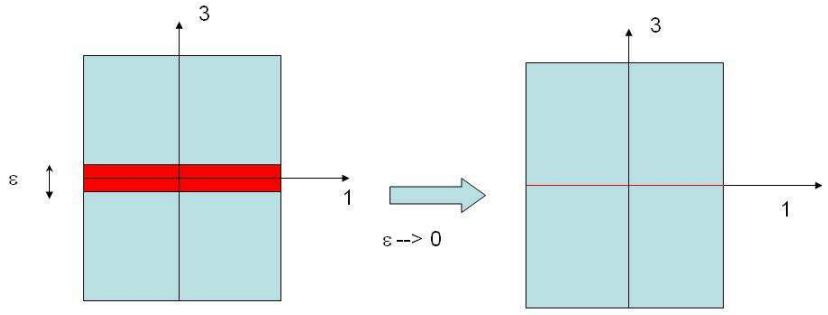


Figure 2: *the principle of the asymptotic techniques in a* 2*-D case*

jump along S. We obtain

$$\sigma^0_{i3} = \bar{C}_{i3i3}\left[u^0_i\right]$$

It is found an interface law which links the stress vector to the jump in the displacement via a diagonal matrix. It is important to remark that this is a simplified choice to reduce the continuous model of interface material to a simple mechanistic model obtained considering springs in the normal and tangential direction. In this case, the terms C_N and C_T in this matrix (corresponding to the normal and the tangential springs stiffnesses) are given by

$$\begin{cases} C_N = \bar{C}_{3333}(\varepsilon \to 0) \quad \text{where} \quad \bar{C}_{3333} = \dfrac{\bar{C}_{3333}}{\varepsilon} \\ C_T = \bar{C}_{1313}(\varepsilon \to 0) \quad \text{where} \quad \bar{C}_{1313} = \dfrac{\bar{C}_{1313}}{\varepsilon} \end{cases} \tag{12}$$

Using expressions (12) and writing the crack density scalar in the form: $\rho = l^2/\varepsilon L_0$, where L_0 denotes the joint length, it can be established that the normal and tangential joint stiffnesses read:

$$\begin{cases} C_N = \dfrac{L_0}{2C(1+D)\,l^2} \\ C_T = \dfrac{L_0}{2C(1-D)\,l^2} \end{cases} \tag{13}$$

Note that the choice of HEM-1 is quite irrelevant because the same functions C_N and C_T can be obtained with various combinations of HEM-1 and material parameters. The present model takes the evolution of the micro-crack into account by taking a

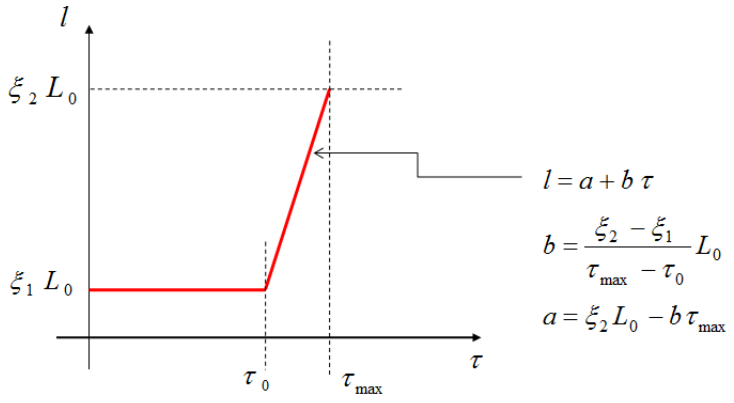


Figure 3: *Function describing the evolution of the crack half length with respect to the applied shear stress*

variable crack half length l depending on the load. For the sake of simplicity, it is first assumed that the half length l depends only on the predominant tangential stress τ by neglecting its dependence on the normal stress. For its evolution, it is assumed that l remains constant $l = l_c$ until a certain value τ_c of the shear stress has been reached. From this value, the crack half length l evolves linearly with respect to the shear stress τ up to a second value of the crack length l_u reached at the maximum shear stress value τ_u. It is experimentally proved that this evolution law can accurately model the response of a quasi-brittle non-confined masonry structure subjected to a shear load. Fig. 3 describes the evolution of the half crack length with respect to the shear stress τ. The first step (where l is constant) corresponds to a stable state of the interface material in which crack propagation occurs. The second step (where l evolves linearly

as a function of the shear stress τ) includes the crack propagation, which leads to the failure of the interface.
The values of the shear stresses τ_c and τ_u are fixed in advance, based on experimental 'stress-displacement' diagrams obtained on various masonries subjected to shear conditions. The values of the crack evolution law parameters (l_c and l_u) result from the minimization of the difference between the numerical and experimental 'stress-displacement' diagrams.

3 Numerical modelling

In this section, some numerical results are illustrated and a brief review of the experiments which validate this damage model is done. The main goal is to evaluate the strain and damage distribution in the unreinforced and strengthened masonry panels which are submitted to a predominant shear load, taking into account the non-linear behaviour of the material. It is known from the laboratory experiences that this strongly non-linear shear behaviour of the masonry is mainly governed by phenomena that occur at the brick/mortar interface.

3.1 *An outline on experimental validations*

The local behaviour of the interfaces at mode I and mode II fracture of brick masonry bed joints, which are typical quasi-brittle interfaces has been studied by various laboratory experiments [11].
In the following, the capability of the proposed interface constitutive model in representing the behaviour of the brick/mortar joints under different load conditions is validate by experimental results obtained by F. Fouchal et al. [7]. The experimental device (Fig. 4) was designed to study on the local scale the shear behaviour of a simple assembly consisting of two and three full or hollow bricks ($210\,mm \times 100\,mm \times 50\,mm$) connected by a mortar joint $10\,mm$ thick. The samples were subjected to a monotonous increasing load up to failure.

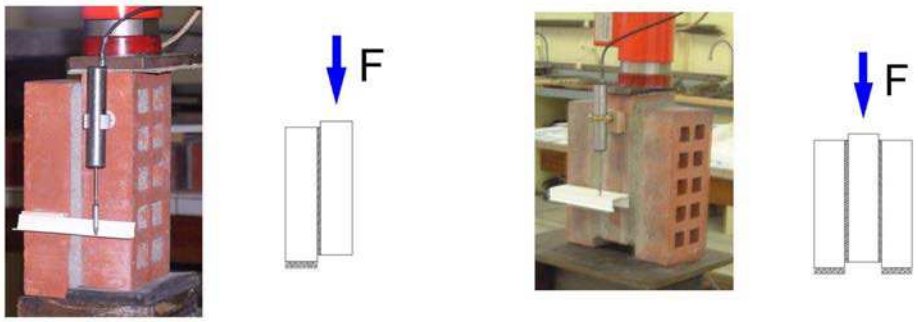


Figure 4: *a) Experimental device involving two bricks, b) Experimental device involving three bricks*

The following findings were obtained :

- occurrence of two kinds of fracture processes, the fracture can occur along the interface or it can begin along the interface and then it propagates into the mortar joint;
- the variations in the shear stresses show that the stress concentration develops in the regions containing the discontinuities
- rigid elastic behaviour up to the failure, followed by friction sliding behaviour;
- the behaviour of full bricks was fragile beyond the limit strength;
- the behaviour of hollow bricks was quasi-fragile beyond the limit strength;
- hollow brick samples showed great dispersion, mainly due to the non uniform distribution of the mortar spikes and local defects in the components of the bricks;
- samples consisting of two and three bricks showed similar behaviour, so the choice of basic cell therefore has no effect on the local scale.

3.2 *Numerical processing*

For the first numerical processes and in according to experiments is chosen to study a simple model: a triplet of three full bricks bounded by two mortar joints. The geometrical and mechanical properties of the sample are totally in agreement with those used in Fouchal's experiments (Tab.1). The boundary conditions are given in Fig. 5. The finite element method is used to perform the spatial discretization. In the subsequent modelling study, only the interface's behaviour will therefore be of importance and not the basic components used. The principal goal of the model is to analyze interface's stiffness degradation caused by microcracks. To begin with a model as simple as possible brick and mortar are assumed to be linear elastic isotropic materials. A plane stress modelling is pursued using a regular mesh of four node quadrangular elements having two degree of freedom per nodes, four Gauss integration points and lagrangian polynomials as shape functions in displacement formulation. This classical mesh choice is used both for brick and mortar elements. For brick/mortar interface elements quadrangular finite elements are chosen also. Their modelling is made explicit below.

Let us briefly recall the weak formulation of a standard elastic problem, having the following form :

$$\int_{\Omega} A\,\varepsilon(u)\,.\,\varepsilon(v)\,d\Omega \;-\; \int_{\Gamma_s} C\,[u].[v]\,d\Gamma_s \;=\; \int_{\Omega} f\,v\,d\Omega \;+\; \int_{\Gamma_1} s\,.\,v\,d\Gamma_1 \qquad (14)$$

Young's moduli (MPa) of full brick	9438
Poisson ratio of full brick	0.13
Young's moduli (MPa) of mortar	4000
Poisson ratio of mortar	0.3

Table 1: Mechanical properties of the three-fold masonry constituents

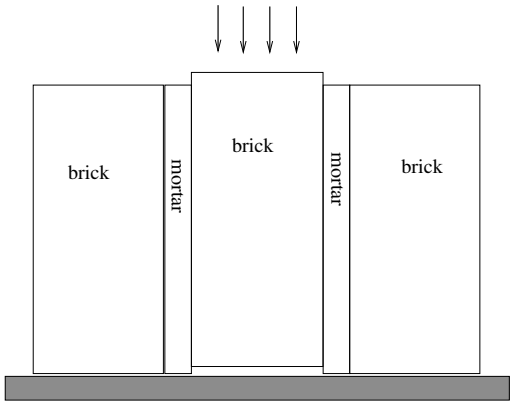


Figure 5: *Initial geometrical configuration and loading conditions imposed*

where f and s denote the volume and the surface forces, respectively, Γ_s is part of the boundary $\partial\Omega$. A is the fourth order elasticity tensor, and C is the symmetric stiffness matrix depending on the damage:

$$C = \begin{pmatrix} C_T & 0 \\ 0 & C_N \end{pmatrix} \tag{15}$$

Writing the displacement jump $[u] = N\,\delta[u]$, the discretization of the surface term is obtained as follows:

$$\int_{\Gamma_s} C\,[u].[v]\,d\Gamma_s = \sum_{seg} \int_{seg} \delta[v]\,N^t\,C\,N\,\delta[u]\,dx = \sum_{seg} \int_{seg} \delta[v]\,\tilde{V}\,\delta[u]\,dx \tag{16}$$

A linear interpolation is performed (Fig. 6.a), taking x to denote the abscissa associated with the segment of length h. V^{el} is the order (8×8) elementary matrix associated with the length of the segment h, .

$$V^{el} = \int_0^h \tilde{V}\,dx = \int_0^h N^t\,C\,N\,dx \tag{17}$$

After proceeding to the assembly of the matrix, we then obtain the linear system where K and V are the rigidity matrix and the matrix associated with the damage interface, respectively.

$$K\,\delta u - V\,\delta[u] = F \tag{18}$$

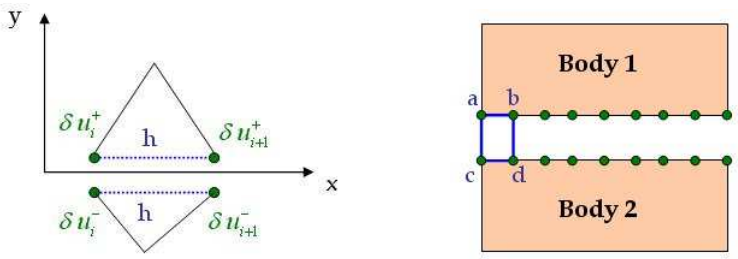


Figure 6: *a) Displacement jump, b) Quadrangular element at the interface.*

An incremental explicit algorithms is used to solve the local problem. Due to the contact conditions, a fairly small increment is chosen. The mesh consists of 665 Q4 finite elements for the whole domain and of 76 Q4 finite elements for brick/mortar interfaces (Fig. 7). The loading on the middle brick ranges from 0 to 53 KN.

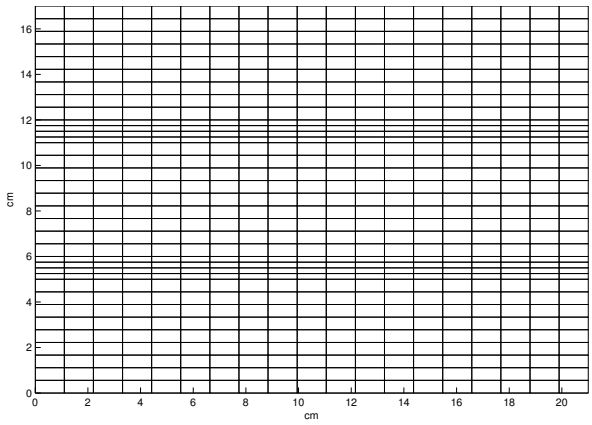


Figure 7: *Meshing detail*

The main goal of computation consists in determining the stiffness values of the interfaces, C_N and C_T. The values of the load parameters F_c and F_u (or stresses τ_c and τ_u) are determined from the experimental "load-displacement" curves [7]. The values of the lengths l_c and l_u are chosen so that the numerical global response matches the experimental "stress-strain" (or "stress-displacement") diagram satisfactorily.
The rigidity of the assembly depends mainly on the interface stiffness as expected. Since the problem is highly non linear, even small perturbations in the stiffness coefficients can greatly affect the numerical responses. In Figs. 8 e 9 stiffnesses C_N and C_T degradation with respect to load increments and with respect to the micro-crack length l evolution are plotted.

The analysis of the shear stresses σ_{xy} distribution map (Fig. 11) reveals that the stress concentration develops in the regions containing the discontinuities, or more specifically, at the interface level. Moreover, Figs. 10 show the evolution of the jump

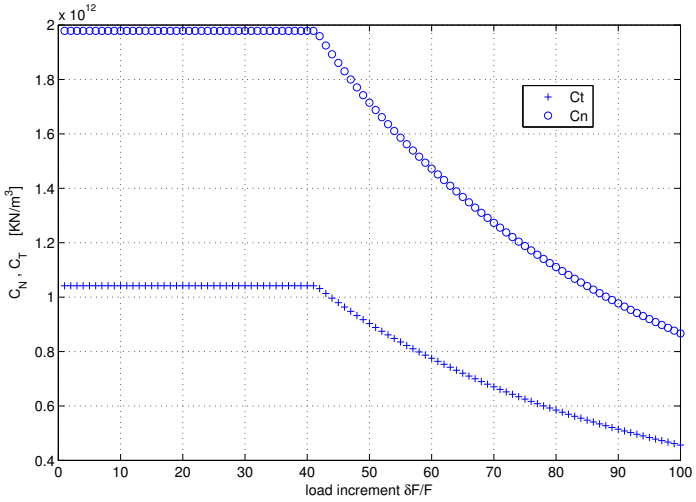


Figure 8: *Variation of the overall elastic coefficients C_N and C_T with respect to load incrementation*

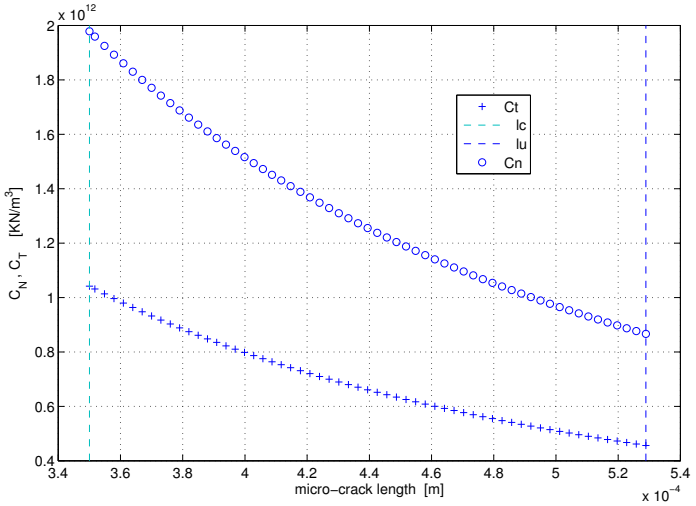


Figure 9: *Variation of the overall elastic coefficients C_N and C_T as function of the crack length*

in the tangential displacements depending on the shear stresses increasing values at interface. When the yield is reached, it is observed a sudden change of the global stiffness which predicts the degradation of the mechanical properties and the failure.

It is possible to conclude that the future failure of the triplet model occurs in brick/mortar interface zone by the excess of the strain capability of this interphase, which agrees with the experimentally observed failure mode.

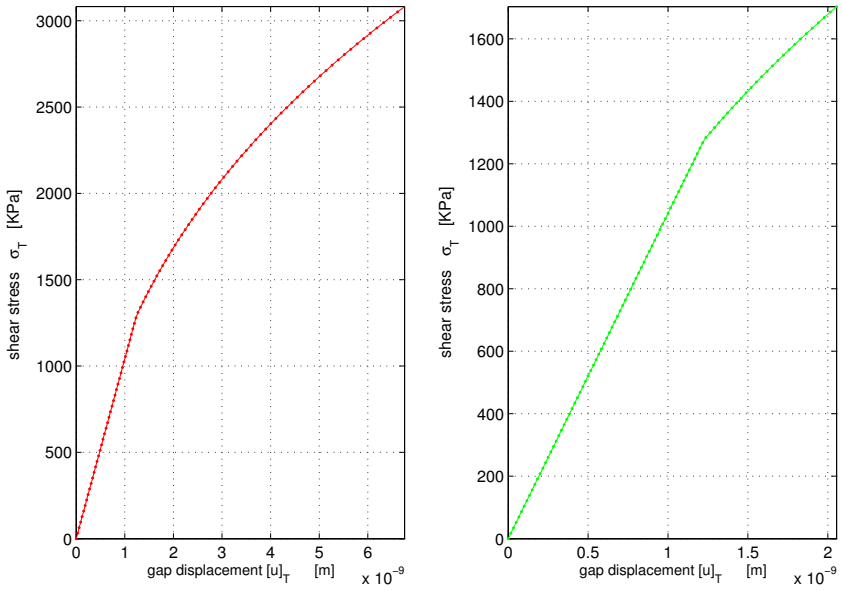


Figure 10: *Shear stresses at the brick/mortar interface*

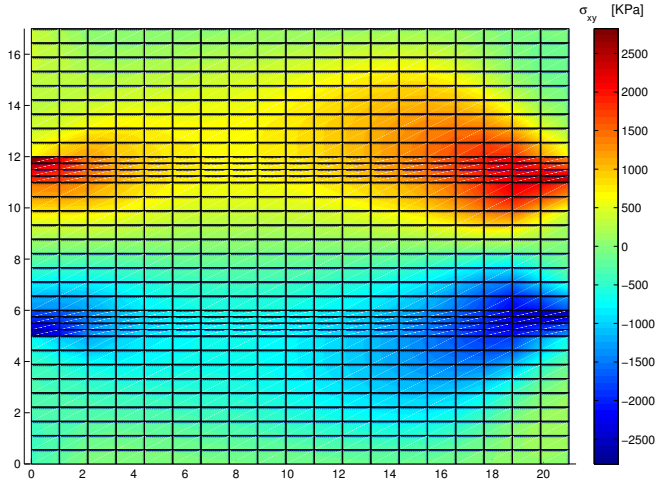


Figure 11: *Shear stresses at final loading step*

4 Conclusions

An accurate version of the multi-scale model proposed by A. Rekik and F. Lebon [31, 32] it is presented here. It is successfully used to simulate the experimental tests in which failure occurred at the brick/mortar interface presented in [7], which provided the coefficients required to model the interface, namely the stiffness parameters and

the length of the micro-cracks. The model is sensitive to these characteristics but the results obtained are in line with the experimental data. From the practical point of view, an optimization routine is needed to systematically determine the values of the parameters describing the evolution of the crack length. This idea will be applied to more complex masonry structures in a future study.

It is proposed in the future to enrich this model with a more complete homogenization technique, in particular the one performed by C. Goidescu and H. Welemane [12, 37]. Their work is devoted to a continuum micromechanics-based investigation of the anisotropic multi-linear response of orthotropic materials containing microcracks. This response is often a very complex combination of two specific features of such deteriorating phenomenon. First, the oriented nature of microcracks induces an evolution of the material symmetry. Secondly, a change in the elastic response of the material is expected, based on opening-closure microcracks state with respect to loading situations. Their procedure leads to the proposal of a closed-form expression of the macroscopic free energy corresponding to two dimensional initially orthotropic materials weakened by arbitrarily oriented microcracks systems.

In a future study, it is planned to implement and validate this enriched model with a software suite for finite element analysis and computer-aided engineering like ABAQUS.

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