Semi-Blind Source Separation in a Multi-User Transmission System with Interference Alignment
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Abstract—In this paper we address the decoding problem in the $K$-user MIMO interference channel assuming an interference alignment (IA) design. We aim to decode robustly the desired signal without having a full Channel State Information (CSI) (i.e. precoders knowledge) at the receivers. We show the equivalence between the IA model and the Semi-Blind Source Separation model (SBSS). Then, we prove that this equivalence allows the use of techniques employed in source separation for extracting the desired signal free of interference, even though dependency exists between some components of the source signal in the SBSS model. Our simulation results illustrate a BER performance very close to the MMSE receiver with full-CSI.

1. INTRODUCTION

Interference Alignment (IA) is an interference management strategy that aims to achieve the Degrees of Freedom (DoF) of the $K$-user interference channel (IC). This technique results in a linearly scaling network sum-rate with the number of users sharing a common transmission medium. Its basic idea is a joint design of all transmitted signals such that interfering users sharing a common transmission medium. Its basic idea in a linearly scaling network sum-rate with the number of

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K
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II. SYSTEM MODEL

We consider a $K$-user quasi static IC with $K$ transmit-receive pairs with $N$ antennas at each side of the link. A given transmitter intends to have its signal decoded by a single dedicated receiver. The received signal at the $k^{th}$ receiver node and at instant $l$ is given by

$$y_k(l) = H_{kk}V_{kk}s_k(l) + \sum_{j \neq k} H_{kj}V_{j}s_j(l) + z_k(l), \quad l = 0, \ldots, L-1$$

where $L$ represents the frame length, $H_{kj} \in \mathbb{C}^{N \times N}$ is the fading channel matrix between the $j^{th}$ transmitter and the $k^{th}$ receiver, $V_{j} \in \mathbb{C}^{N \times d}$ is the precoding matrix at the $j^{th}$ transmitter, and $z_k(l) \in \mathbb{C}^{N \times 1}$ is the circular symmetric complex Gaussian noise vector at the $k^{th}$ receiver, with i.i.d. components; i.e. $z_k \sim \mathcal{CN}(0, I_N)$. $\{s_j(l) \in \mathcal{M}^{d \times 1} | l = 0, \ldots, L-1\}$ represents the $d$ streams from the $j^{th}$ transmitter during an $L$-symbol duration interval. The symbols of $s_j$ are assumed i.i.d. from a finite constellation $\mathcal{M}$. The decoding is over one frame, in which the channel is supposed unchanged.

The $K$ precoders are jointly designed to satisfy the IA conditions, which can be achieved using different solutions [1], [5]. At the receiver side, the intended signal can be recovered by projecting the received signal on the interference null space. The $l^{th}$ decoded vector signal is given by

$$\hat{y}_k(l) = U_kH_{kk}V_{kk}s_k(l) + \sum_{j \neq k} U_kH_{kj}V_{j}s_j(l) + U_kz_k(l), \quad l = 0, \ldots, L-1$$

where $U_k$ denotes the decoding matrix that we aim to estimate. To this end, we exploit the statistical independence of the transmitted streams, and we seek at each receiver for the decoding matrix that maximizes the statistical independence between the components of the mixed received signal. Then, we introduce a few training symbols in order to remove the scaling and the permutation ambiguities.

III. FROM THE FADING INTERFERENCE CHANNEL TO A BSS MODEL

This section is dedicated to the definition of a linear BSS model from the MIMO IC model, assuming an IA scheme at the transmission side. Such model is required for the application of linear BSS techniques. Before going into details, let us recall the following lemma from linear algebra

**Lemma 1:** Let $A_1 \in \mathbb{C}^{N \times n}$ and $A_2 \in \mathbb{C}^{N \times n}$ ($N > n$), where rank($A_1$) = $n$ and rank($A_2$) = $m$, ($m \leq n$) and span($A_2$) $\subset$ span($A_1$). Then, for every $s_1 \in \mathbb{C}^{n \times 1}$, $\exists$ $s_2 \in \mathbb{C}^{n \times 1}$ such that $A_1s_1 = A_2s_2$.

In the adopted transmission model, the received signal given in (1) can be expressed as

$$y_k(l) = H_{kk}^s s_k(l) + \sum_{j \neq k} H_{kj}^s s_j(l) + z_k(l), \quad l = 0, \ldots, L-1$$

where $H_{kk}^s = H_{kk}V_{k}$. Assuming the IA conditions are satisfied, and let $H_{kj}^s \in \mathbb{C}^{N \times (N-d)}$ denote the matrix spanning the $(N - d)$-dimensional interference subspaces at receiver $k$; i.e. $H_{kj}^s$ for $j \in \{1, \ldots, K\}, j \neq k$ are all spanned by $H_{kj}^s$. Thus, using the lemma above, the interference subspaces can be expressed in terms of $H_{kj}^s$ as

$$H_{kj}^s s_j = H_{kj}^s s_j(l), \quad s_j(l) = (H_{kj}^s)^{-1}H_{kj}^s H_{kj}^s s_j(l), \quad s_j(l) = (H_{kj}^s)^{-1}H_{kj}^s H_{kj}^s s_j(l)$$

Substituting (4) into (3) yields for $l = 0, \ldots, L-1$

$$y_k(l) = H_{kk}^s s_k(l) + \sum_{j \neq k} H_{kj}^s s_j(l) + z_k(l)$$

$$= [H_{kk}^s \quad H_{kj}^s] [s_k^T(l) \quad s_j^T(l)]^T + z_k(l)$$

$$= B_k s_k(l) + z_k(l), \quad l = 0, \ldots, L-1$$

where $s_k = (s_1 + \cdots + s_{(k-1)} + s_{(k+1)} + \cdots + s_K)$ is the full rank matrix that spans the union of the desired and the interference subspaces, and $\hat{s}_k(l)$ is the $N \times 1$ source vector consisting of $d$ desired streams and $(N - d)$ interference streams. (5) gives the formulation of a linear determined BSS problem where an $N$-length source vector $\hat{s}_k$ is mixed by an unknown mixing matrix $B_k$ to produce $N$ mixture signals $y_k$ observed from $N$ sensors. The BSS aims at separating the original streams from an observed sensor array without knowing the transmission parameters.

IV. EXTRACTION OF THE DESIRED SIGNAL USING BSS

A. Desired signal Extraction

The BSS standard model is defined as

$$y(l) = As(l), \quad l = 0, \ldots, L-1$$

where $s(l) \in \mathbb{C}^{N \times 1}$ is the vector of $N$ statistically independent latent variables, $y(l) \in \mathbb{C}^{N \times 1}$ is the observation vector, and $A \in \mathbb{C}^{N \times N}$ is a full rank unknown mixing matrix. The BSS technique seeks for the demixing matrix $U$ that maximizes the statistical independence between the estimated components $\hat{s}(l) = UAs(l)y(l)$. For the sake of the simplicity, the time index will be ignored in the remaining of this section.

It is shown in [7] that, under mild assumptions, the estimated variables $\hat{s}$ are similar to the original sources $s$ up to a permutation and scaling by a constant, i.e.

$$UA = PA,$$

where $P$ is a permutation matrix and $A$ is a diagonal matrix.

The previous section shows that the MIMO IC model in (5) is similar to the BSS model except that some mutual dependencies exist between some components of $\hat{s}_k = [s_k^T \; s_k^T]$. The first $d$ components of $\hat{s}_k$ are mutually independent and represent the desired streams. The $(N - d)$-components $\hat{s}_k$ are mutually dependent and represent the interference part. This situation of dependent sources has been considered in certain recent studies, e.g. [10], [11]. Next, we show how to only extract the desired signal using the method JADE.

B. Joint Approximate diagonalization of Eigenmatrices

JADE is a well known statistical technique for solving linear determined BSS problem. It is based on the fact that the fourth order cross-cumulant of independent variables are zeros. Thus, demixing a received mixed signal as in (6) involves looking
for the decoding matrix that makes the fourth order cross-
cumulant null [13]. We summarize the steps of the algorithm
JADE as follows (for further details, the reader can refer to
[9]):

1) Step 1: Compute the whitening matrix $W$ as the inverse
square root of the sample covariance matrix of the
received data. As shown in [9], $W$ transforms $A$ into a
unitary matrix $F = WA$.

2) Step 2: Form the sample 4-th order tensor $Q_2$ of the
whitened data $z = Wy$.

3) Step 3: Compute the $N$ most significant eigenpairs of
$Q_2$: $\{\lambda_n, M_n | n = 1, \ldots, N\}$

4) Step 4: Perform the approximate joint diagonalization
of matrices $\{\lambda_n M_n | n = 1, \ldots, N\}$ by a unitary matrix
$U$.

5) Step 5: An estimate of the source vector is $\hat{s} = Uz$

JADE has been proved able to separate the original streams
when all streams are statistically independent. Let us now
demonstrate that even in the presence of some mutually depen-
dent components, as in (5), JADE is able to separate the mutual
independent streams. For our considered problem, the $(N-d)$
interference sources are dependent in which case we can
easily show\footnote{The entries of $Q_2$ are given by $Q_2(i,j,k,l) = cum(z_i, z_j^*, z_k, z_l^*)$ where $cum$ refers to the fourth order cumulant and $z_k^*$ is the complex
conjugate of the $i-th$ entry of $z$.} that the set of matrices $\{\lambda_n M_n | n = 1, \ldots, N\}$
are not anymore jointly diagonalizable but are jointly block-
diagonalizable. In other words, for $n = 1, \ldots, N$, we have the
following joint matrix structure:

\[
\lambda_n M_n = F \begin{bmatrix} M_{n,1} & 0 \\ 0 & M_{n,2} \end{bmatrix} F^H
\]  

where $M_{n,1}$ are $d \times d$ diagonal matrices, $M_{n,2}$ are $(N-d) \times (N-d)$ unstructured matrices and $F = WA$. In [14], it
is shown that the joint diagonalization algorithm used in the
standard BSS method JADE can be used as well for the joint
block diagonalization of a set of matrices. Consequently, the
final transformation given by the whitening matrix and unitary
transform $U$ leads to:

\[
U W A = \begin{bmatrix} D_1 & 0 \\ 0 & D_2 \end{bmatrix}
\]

where $D_1$ is a $d \times d$ diagonal matrix and $D_2$ a $(N-d) \times (N-d)$
given matrix. Hence, the first $d$ entries of $\hat{s} = Uz$ represent the
desired source signals while its remaining $N-d$ entries represent
linear mixtures of the (non-desired) interference signals.

Another BSS technique is the iterative FastICA. This tech-
nique is characterized by its low computational complexity
and fast convergence. It also performs close to the JADE in
terms of robustness [8] (see section V).

C. Semi-Blind separation

The ambiguities on the scale and permutation of the esti-
mated streams are solved thanks to a few training symbols
inserted within each data frame. We define $x_j \in \mathbb{C}^{1 \times N_s}$ the

\[
\text{argmin}_{i,j \in \{1, \ldots, d\}} \text{NMSE}(\hat{s}_i, x_j),
\]

Next, the scale ambiguity can be solved by seeking for the
complex variable $\alpha$ that minimizes the MMSE between the
estimated variables and the training sequence

\[
\text{MMSE} = \mathbb{E} \left[ |\alpha \hat{s}_j - x_j|^2 \right].
\]

The first $N_s$ vector $\hat{s}_i$ used in (9), (10), and (11) is formed by
the first $N_s$ estimated symbols of the stream $s_k$.

Remarks: 1) The proposed technique requires the receiver to
wait for all samples within one frame. Therefore, the authors
in [15] have proposed an adaptive semi-blind high order
separation technique. This technique can be adapted to our
case since we have showed that the dependency between
interference streams does not affect the desired source extraction.
2) As shown by our simulation results, the semi-blind approach
results in a slight performance loss as compared to the stand-
ard (data-aided) MMSE. However, it is shown in [12], that in such
cases one can compensate for this performance loss using a
decision-directed MMSE detection in a two step approach, the
first step being the semi-blind approach proposed previously
(see [12] for further details).

V. SIMULATION RESULTS

In this section, we evaluate the Bit Error Rate (BER) of
the JADE and FastICA in a 3-user 4 x 4 MIMO IC with an
IA design. The IA scheme is achieved using the distributed
iterative solution proposed in [5]. Each user sends $d = 2$
data streams. The symbols are QPSK modulated. The channel
is supposed flat fading Rayleigh distributed, and remains
constant over one frame with length $L = 2000$ symbols.
The estimated channel matrices are modeled as $\tilde{H}_{kj} = H_{kj} + E_{kj}$; $\forall k, j$ (12) where $E_{kj}$ is the channel estimation error $\forall k, j$. The coefficients of $E_{kj}$ are symmetric complex Gaussian distributed with zero mean and $\sigma_{E}^2$ variance. In Fig. 3, the BER performance of both LS and JADE methods is illustrated in presence of a channel estimation error. It can be observed that in the region where $\sigma_{E}^2/\sigma_{h}^2 < 10$dB, both of the decoders results in a degraded BER. However, our proposed decoder outperforms the LS: i) beyond $\sigma_{E}^2/\sigma_{h}^2 = 10$dB when SNR=12dB, ii) beyond $\sigma_{E}^2/\sigma_{h}^2 = 14$dB when SNR=16dB, and iii) beyond $\sigma_{E}^2/\sigma_{h}^2 = 21$dB when SNR=20dB.

VI. CONCLUSION

In this paper, we have addressed the problem of detection in a MIMO IC system using IA scheme at the transmitters. The problem has been formulated as a semi-blind source separation problem, and the ability of the joint diagonalization technique (JADE) has been shown to extract the desired streams. Training sequence has been introduced to solve the permutation and scaling ambiguities. The proposed scheme performs close to full-CSI MIMO IC-IA schemes. We have also showed by simulations that it outperforms the traditional MMSE using LS for CSI estimation method with a same training sequence length.

REFERENCES


