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Evaluating tilted plane models for solar radiation using comprehensive testing procedures, at a southern hemisphere location

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Abstract

This work aims at assessing the performances of four models for their relative merits in estimating diffuse solar radiation on tilted planes for a southern location (Reunion Island). This comparative study benefits from a sound and consistent experimental set-up and is more detailed than the previous research works as 14 inclined planes are available for testing.

Keywords: Solar radiation; diffuse solar radiation; Tilted irradiance models; Ground albedo;

Nomenclature

\begin{align*}
I_{B,n} & \quad \text{Normal beam radiation} & \text{W.m}^{-2} \\
I_{D} & \quad \text{Diffuse horizontal radiation} & \text{W.m}^{-2} \\
I_{D,\beta} & \quad \text{Diffuse radiation on tilted plane} & \text{W.m}^{-2} \\
I_{E,n} & \quad \text{Extraterrestrial normal incidence radiation} \\
F & \quad \text{Beam fraction} \\
m & \quad \text{Air mass} \\
N_{pt} & \quad \text{Weighting factor} \\
Z & \quad \text{Solar zenith angle} & \text{rad.} \\
\alpha & \quad \text{Solar altitude} & \text{rad.} \\
\beta & \quad \text{Tilted angle} & \text{rad.} \\
\gamma & \quad \text{Azimuth of a titled plane} & \text{rad.} \\
\epsilon_{i} & \quad \text{Sky clearness} \\
\theta & \quad \text{Angle of incidence of solar beams on a titled plane} & \text{rad.} \\
\Delta & \quad \text{Sky brightness}
\end{align*}

1. Introduction

Gueymard [1] introduces why we need to address the issue of modelling the solar resource on a tilted surface.
Accurate solar radiation resource data are necessary at various steps of the design, simulation, and performance evaluation of any project involving solar energy. Solar energy systems are installed on either fixed tilted planes or tracking receivers. Similarly, glazed envelopes are installed vertically on the facades of buildings or at a variety of tilt angles in atria.

The most usual situation is that we have satellite inferred solar radiation data on a horizontal surface, and restricted to the global radiation only. If we are to infer from that the global radiation on a tilted plane, we are going to have to go through a chain of models. One can predict the diffuse on a horizontal surface using the BRL model for example [2]. Then, one can calculate the direct on the horizontal. From there we can use a model to predict the diffuse on the tilted plane. Even if the diffuse (or direct) is measured on the horizontal, we will still need a model to go to the tilted plane. It would in fact be a bit ridiculous to be measuring solar radiation on a tilted plane if we did have the resources because what tilt and orientation would be of use for a project would only be known when that project were designed.

We are investigating four models for their relative merits in estimating diffuse solar radiation on a tilted plane given measurements on a horizontal plane. No such model needs to be used for the direct solar radiation as simple trigonometry can be employed. This is not the case for diffuse as it is anisotropic over the sky dome. The models are Hay [3], Skartveit and Olseth [4], Gueymard [5] and Perez et al. [6].

2. Previous Studies

2.1. Literature review

Kambezidis et al. [7] presented a comparative assessment of tilted irradiation models, using hourly measurements of total solar irradiation on a surface tilted 50 degrees and oriented south in Athens. Twelve sky diffuse submodels were used with four albedo submodels to estimate the global irradiation on the tilted surface from data on the horizontal plane. Root mean square errors (RMSE) and mean bias errors (MBE) were used to determine the intrinsic performance of each diffuse tilt/albedo submodel combination. Gueymard [5], Hay [3], Reindl [8], and Skartveit and Olseth [4] diffuse tilt submodels were found to have the best overall performances, in conjunction with either one of three albedo submodels (constant albedo, seasonally varying albedo, and anisotropic albedo). The anisotropic and seasonally varying albedo submodels do not improve the performance of the four better diffuse tilt
models (compared to their performance using an albedo fixed at 0.2) for the moderately tilted surface investigated in this article. Kambezidis et al. [9] performed a similar analysis for daily radiation totals, finding that the Gueymard model [5] performed the best.

Nijmeh and Mamlook [10] evaluated only two models and they were early ones, so we can discount this work. Diez-Mediavilla et al. [11] compared 10 models used to estimate diffuse solar irradiance on inclined surface. Among the models are the models of Hay [3], Gueymard [5] and Perez [6]. The models were tested against data collected from a south facing surface inclined at 42°. RMSE and MBE statistics were used to rank the models. The results obtained favour the Muneer model [12], followed by the Reindl model [8], for hourly as well as for daily values.

In Notton et al [13], 15 models (including the models of Hay, Gueymard, Perez and Skarveit) were tested. The performance (given by the RMSE and MBE) of models were evaluated against data (2 inclinations 45° and 60°) measured on the French Mediterranean site of Ajaccio. Among the tested models, the Perez model exhibits the best accuracy. Regarding the reflected part of the radiation from the ground on the tilt plane, the authors chose a constant value of 0.2 for the albedo and noted that this value is the most commonly used in the literature. Further, they argued that some authors Kambezidis et al. [7] used three different albedo and showed that it did not improve the performance of their estimation compared to utilization of an albedo fixed at 0.2.

Loutzenhiser et al. [14] checked seven solar radiation models on tilted surfaces that were implemented in building energy simulation codes. Among the models tested are the models of Hay [3] and Perez [6].

Gueymard [1] compared 10 transposition (between horizontal and tilted planes) models that were appraised against global radiation measured on fixed-tilt, south-facing planes (40° and vertical) and a 2-axis tracker at NREL’s Solar Radiation Research Lab. in Golden, CO. Among the tested models are the model of Hay, Skarveit, Perez and Gueymard. It must be stressed that the models were tested against 1-minute irradiance data. In this work, the usual error metrics are used i.e. (mean bias error, MBE, and root mean square error, RMSE) to evaluate the models. The goal of the paper was not to compare the performances of the transposition models but rather to gain a better understanding of what drives the modelling errors. One may also notice that the models were also tested with suboptimal inputs (i.e.
when only global irradiance is known and specific models were used to estimate
diffuse or direct). The authors noticed that the performance of the models degrades
significantly. Under clear skies and for optimal inputs, the Gueymard and Perez
models exhibit the best estimates of global tilted irradiance. Although, the
transposition models reviewed in the paper were generally developed from hourly
data, they appear to be equally applicable to 1-minute data.
Evseev and Kudish [15] assessed 11 models that convert horizontal diffuse radiation
to diffuse radiation incident on a tilted surface utilizing data measured in Beer Sheva,
Israel. Among the 11 models are the models of Hay, Skartveit, Perez and Gueymard.
The data consist of hourly global and diffuse solar radiation on a horizontal surface,
normal incidence beam and global radiation on a south-oriented surface tilted at 40°.
The relative performance of the different models under different sky conditions has
been assessed. Indeed, the relative ability of the models to predict the global
radiation on a tilted surface is, a priori, a function of sky conditions, since all the
models make assumptions regarding the isotropy or anisotropy of the sky conditions.
Consequently, they have been analysed under all cloud conditions and, also, under
clear, partially cloudy and cloudy skies.

2.2. Summary and procedure proposed for the present work

Abundant research has been aiming at comparing several models. The anisotropic
models of Hay, Skarveit, Gueymard and Perez are often tested. All of the
comparative studies have been made for northern hemisphere sites. In most of the
works, the models are tested for a maximum of 2 tilt angles and one or two
orientations. If the ground albedo is not locally measured, it is current practice to
choose a fixed value of 0.2. Moreover, some authors stated that the use of different
albedo did not lead to improved performance of the models. Among the cited papers,
a detailed study of the performance of the models for different sky conditions is
rarely made except the work done by Evseev and Kudish [15].

The analysis of the different papers does not exhibit a particular powerful model
albeit 2 or 3 papers (Notton and Gueymard papers) mentioned the Perez model as a
good candidate.

Regarding the data, a quality check is made by some authors (Evseev and Kudish
[15] and Notton et al. [13]). Finally, one may notice that most of the authors used the
classical error metrics RMSE and MBE to assess the accuracy of the models.

In an attempt to complete the previous surveys, this work aims at assessing the
performances of the four most cited models in the literature (i.e Hay, Skartveit, Gueymard and Perez models) for a southern location (Reunion Island). This comparative study benefits from a sound and consistent experimental set-up (see section 4) and is more detailed than the previous research works as 14 inclined planes are available for testing.

2. Models Tested

2.1. Hay Model

The diffuse solar radiation on a tilted plane with tilt angle $\beta$ and azimuth $\gamma$ as given by Hay’s Model [3] is:

$$I_{D,\beta} = I_D \left[ F \frac{\cos(\theta)}{\cos(\gamma)} + (1 - F) \cos^2 \left( \frac{\beta}{2} \right) \right]$$

(1)

2.2. Skartveit and Olseth Model

Skartveit and Olseth’s model [4] was developed for high latitudes and is essentially an extension of the Hay model

$$I_{D,\beta} = I_D \left[ F \frac{\cos(\theta)}{\cos(\gamma)} + B\cos(\beta) + (1 - F - B)\cos^2 \left( \frac{\beta}{2} \right) \right]$$

$$B = \max(0.3 - 2F; 0.0)$$

(2)

2.3. Gueymard Model

From Gueymard [5], the radiance of a partly cloudy sky can be considered as a weighted sum of the radiances of a clear and an overcast sky.

$$I_{D,\beta} = I_D \left[ (1 - N_{pt})r_{d0} + N_{pt}r_{d1} \right]$$

(3)

where $N_{pt}$ is the weighting factor and the subscripts 0 and 1 refer to the opacity (0 clear and 1 overcast). $r_{d0}$ is obtained as the sum of a circumsolar component (dependent on the angle of incidence $\theta$) and a hemispheric component (dependent on the tilt of the collector $\beta$ and the solar altitude $\alpha$).

$$r_{d0} = \exp \left( a_0 + a_1 \cos(\theta) + a_2 \cos^2(\theta) + a_3 \cos^3(\theta) \right) + f(\beta)g(\alpha)$$

(4)

In this the parameters (themselves functions) were estimated and took the following forms:

$$a_0 = -0.897 - 3.364h + 3.960h^2 - 1.909h^3$$

$$a_1 = 4.448 - 12.962h + 34.601h^2 - 48.784h^3 + 27.511h^4$$

$$a_2 = -2.770 + 9.164h - 18.876h^2 + 23.776h^3 - 13.014h^4$$

$$a_3 = 0.312 - 0.217h - 0.805h^2 + 0.318h^3$$
\[ f(\beta) = \frac{[1 + b_0 \sin^2(\beta) + b_1 \sin(2\beta) + b_2 \sin(4\beta)]}{[1 + b_0]} \]

\[ g(\alpha) = 0.408 - 0.323\,h + 0.384\,h^2 - 0.170\,h^3 \]

In the last few relationships, \( h = 0.01\alpha, \quad b_0 = -0.2249, \quad b_1 = 0.1231, \quad b_2 = -0.0342. \)

### 2.4. Perez Model

The Perez et al. [6] model is based on a three components treatment of the sky diffuse irradiance. The incident diffuse energy on any tilted surface is given by

\[ I_{D,\beta} = I_D \left[ 1 - F_1 \cos^2 \left( \frac{\beta}{2} \right) + F_1 \left( \frac{a_0}{a_1} \right) + F_2 \sin(\beta) \right] \tag{5} \]

Here, \( F_1 \) and \( F_2 \) are circumsolar and horizon brightening coefficients, and \( a_0, a_1 \) account for the respective angles of incidence of circumsolar radiation on the tilted and horizontal surfaces.

\[ a_0 = \max[0, \cos(\theta)], \quad a_1 = \max[\cos(85^0), \cos(Z)] \]

With these definitions, \( \frac{a_0}{a_1} = r_B \). The brightness coefficients \( F_1, F_2 \) are functions of the zenith angle \( Z \), sky clearness \( \varepsilon_i \), and a brightness \( \Delta \). \( \varepsilon_i \) is a function of \( I_D \) and the normal beam irradiation \( I_{B,n} \).

\[ \varepsilon_i = \left\{ \left[ I_D + \frac{I_{B,n}}{I_D} \right] + 5.535 \times 10^{-6} Z^3 \right\}/\{1 + 5.535 \times 10^{-3} \} \tag{6} \]

Note that in this equation, \( Z \) is in degrees.

\[ \Delta = m I_D/I_{E,n} \]

where \( m \) is air mass and \( I_{E,n} \) is the extraterrestrial normal incidence irradiation.

\[ F_1 = \max\left[0, F_{11} + F_{12} \Delta + \left( \frac{\pi}{180} \right) Z F_{13}\right] \]

\[ F_2 = \left[ F_{21} + F_{22} \Delta + \left( \frac{\pi}{180} \right) Z F_{23} \right] \]

The \( F_{ij} \) are obtained from Table 1 for various values of \( \varepsilon_i \).

<table>
<thead>
<tr>
<th>( \varepsilon_i ) (bins)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lower Bound</td>
<td>1.000</td>
<td>1.065</td>
<td>1.230</td>
<td>1.500</td>
<td>1.950</td>
<td>2.800</td>
<td>4.500</td>
<td>6.200</td>
</tr>
<tr>
<td>Upper Bound</td>
<td>1.065</td>
<td>1.230</td>
<td>1.500</td>
<td>1.950</td>
<td>2.800</td>
<td>4.500</td>
<td>6.200</td>
<td>--</td>
</tr>
<tr>
<td>( F_{11} )</td>
<td>-0.0083</td>
<td>0.1299</td>
<td>0.3297</td>
<td>0.5682</td>
<td>0.8730</td>
<td>1.1326</td>
<td>1.0602</td>
<td>0.6777</td>
</tr>
<tr>
<td>( F_{12} )</td>
<td>0.5877</td>
<td>0.6826</td>
<td>0.4869</td>
<td>0.1875</td>
<td>-0.3920</td>
<td>-1.12367</td>
<td>-1.5999</td>
<td>-0.3273</td>
</tr>
<tr>
<td>( F_{13} )</td>
<td>-0.0621</td>
<td>-0.1514</td>
<td>-0.2211</td>
<td>-0.2951</td>
<td>-0.3616</td>
<td>-0.4118</td>
<td>-0.3589</td>
<td>-0.2504</td>
</tr>
<tr>
<td>( F_{21} )</td>
<td>-0.0596</td>
<td>-0.0189</td>
<td>0.0554</td>
<td>0.1089</td>
<td>0.2256</td>
<td>0.2878</td>
<td>0.2642</td>
<td>0.1561</td>
</tr>
<tr>
<td>( F_{22} )</td>
<td>0.0721</td>
<td>0.0660</td>
<td>-0.0640</td>
<td>-0.1519</td>
<td>-0.4620</td>
<td>-0.8230</td>
<td>-1.1272</td>
<td>-1.3765</td>
</tr>
<tr>
<td>( F_{23} )</td>
<td>-0.0220</td>
<td>-0.0289</td>
<td>-0.0261</td>
<td>-0.0140</td>
<td>0.0012</td>
<td>0.0559</td>
<td>0.1311</td>
<td>0.2506</td>
</tr>
</tbody>
</table>

Table 1: Coefficients used in the Perez et al. [6] model
3. Data collection and analysis

The instrumentation was set up in the platform of Saint-Pierre, Reunion Island (latitude 21° 20’ south, longitude 55° 29’ East, altitude 76m). This site is located on the sunniest coastal part of the island. The local climate is warm and wet with an average annual temperature close to 24°C and an average annual humidity of 70%. Trades winds blow approximately 30% of time in this part of the island.

The instrumentation is divided into two parts (see also Table 2). The first one is a complete weather station that measures the standard climate variables with a minute time step. The second part is a semi-hemispheric structure composed of fifteen different planes (Fig. 1 and Table 2). Each plane gathers two PV polycrystalline silicon cells from the two French manufacturers (Photowatt and Tenesol) and a secondary standard pyranometer from Kipp & Zonen [16]. Table 3 presents the azimuths and the inclinations of the fifteen inclined surfaces. These orientations and inclinations correspond to:

- the ideal inclination of PV fields in Reunion Island : 20° (latitude of the site),
- the common inclinations of roofs where PV modules may be installed (between 20° and 40°),
- the best orientations of PV modules for southern territories (from East to West facing North),
- the reference horizontal plane.

The weather station gives the same data as the common French weather forecast utilities’ ground stations. An albedometer is mounted at 2.5 meters high on the tower of the weather station. This tower is set up on flat place covered with grass at approximately 25 meters from the studied inclined surfaces. The semi-hemispherical structure allows concentration of the fifteen planes on a single structure without problems of shading between the different instruments and planes. For each plane, a minute time step monitoring is used for the global solar irradiance. Furthermore, a sun tracker offers additional data concerning the solar radiation (beam, diffuse and long wave irradiances). This part of the experiment is set up on a roof in order to avoid shading effects from the nearby buildings (Fig. 1).
<table>
<thead>
<tr>
<th>Sensor model</th>
<th>Variables</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Secondary standard ventilated pyranometer (Kipp&amp;Zonen CMP11)</td>
<td>Global horizontal irradiation</td>
<td>W.m(^{-2})</td>
</tr>
<tr>
<td>1 Secondary standard ventilated pyranometer (Kipp&amp;Zonen CMP11)</td>
<td>Diffuse horizontal irradiation</td>
<td>W.m(^{-2})</td>
</tr>
<tr>
<td>1 Normal incidence pyrheliometer (Kipp&amp;Zonen CH1)</td>
<td>Direct normal irradiation</td>
<td>W.m(^{-2})</td>
</tr>
<tr>
<td>1 Albedometer (2 LP02 Huksefluxpyranometers)</td>
<td>Albedo</td>
<td></td>
</tr>
<tr>
<td>14 Secondary standardpyranometers (Kipp&amp;Zonen: 10 CMP11, 3 CM11, 1 CMB6)</td>
<td>Global irradiation on titled planes</td>
<td>W.m(^{-2})</td>
</tr>
</tbody>
</table>

Table 2: Solar measurement sensors

The measurement campaign was done in two steps. The first one was a calibration procedure. The pyranometers were installed horizontally during a short period (5 weeks). This phase of calibration permitted assessment of the dispersion of the measurements and to detect material failures or incoherence in the measurements.

The pyranometer that records the global horizontal irradiance on the sun tracker was used as a reference. The measurements of the other 14 pyranometers were corrected with a linear function \(y = ax + b\) to fit exactly to the reference by minimizing the MBE (see equations 9 and 10). Table 3 provides the error metrics between the reference and the 14 other pyranometers before and after the calibration step. The raw measurements present MBE in the range ±3% according to the technical data of these sensors. We can also note that minimizing the MBE permits to reduce the RMSE (see Eq. (7) and Eq. (8)).

<table>
<thead>
<tr>
<th>Sensor model</th>
<th>Inclination</th>
<th>Azimuth</th>
<th>NRMSE (%)</th>
<th>NMBE (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Before calibration</td>
<td>After calibration</td>
<td>Before calibration</td>
<td>After calibration</td>
</tr>
<tr>
<td>CM11</td>
<td>20°</td>
<td>0° North</td>
<td>3.752</td>
<td>3.082</td>
</tr>
<tr>
<td>CM11</td>
<td>20°</td>
<td>30° North-West</td>
<td>5.307</td>
<td>2.585</td>
</tr>
<tr>
<td>CM11</td>
<td>20°</td>
<td>60° North-West</td>
<td>2.617</td>
<td>1.859</td>
</tr>
<tr>
<td>CM11</td>
<td>20°</td>
<td>0° North</td>
<td>5.320</td>
<td>4.588</td>
</tr>
<tr>
<td>CM11</td>
<td>20°</td>
<td>-30° North-East</td>
<td>3.596</td>
<td>2.470</td>
</tr>
<tr>
<td>CM11</td>
<td>20°</td>
<td>-60° North-East</td>
<td>4.696</td>
<td>3.635</td>
</tr>
<tr>
<td>CM11</td>
<td>20°</td>
<td>-90° East</td>
<td>6.683</td>
<td>5.912</td>
</tr>
<tr>
<td>CM11</td>
<td>40°</td>
<td>0° North</td>
<td>2.032</td>
<td>1.870</td>
</tr>
<tr>
<td>CM11</td>
<td>40°</td>
<td>60° North-West</td>
<td>2.992</td>
<td>2.720</td>
</tr>
<tr>
<td>CM11</td>
<td>40°</td>
<td>30° North-West</td>
<td>2.707</td>
<td>1.939</td>
</tr>
<tr>
<td>CM11</td>
<td>40°</td>
<td>-90° East</td>
<td>4.532</td>
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</tr>
<tr>
<td>CM11</td>
<td>40°</td>
<td>-30° North-East</td>
<td>7.079</td>
<td>6.443</td>
</tr>
<tr>
<td>CM11</td>
<td>40°</td>
<td>-60° North-East</td>
<td>5.233</td>
<td>4.656</td>
</tr>
<tr>
<td>CM11</td>
<td>40°</td>
<td>-90° East</td>
<td>3.455</td>
<td>2.133</td>
</tr>
</tbody>
</table>

Table 3: Error of measurement before and after the calibration step
During the second step, the pyranometers were mounted on the semi-hemispherical structure. Continuous measurements were recorded during a period of one year. They were corrected as presented in the calibration step. Unfortunately, the sensor set up on the plane with a 20° inclination and a 60° azimuth (north-west) gave erroneous data during this step. So the measurements of this plane were not used for the test of the models.

The data were all collected on a data logger with a 10 seconds time step (scan frequency) and they are averaged and recorded every minute. The two periods of record use for this study are:

- 35 days during wet season (25/02/2009 – 31/03/2009),
- 34 days during the dry season (18/07/2009 – 21/08/2009).

Finally, all the data were stored in a web database. This method of data management permits both the retrieving and the real-time access to the data. Such a method was already tested with the weather station. It offers great possibilities of control of the experimental devices and powerful data treatment.

In order to assess the models, a comparison between the calculated and measured irradiance was done. Different error metrics were used. Among others, the root mean square error (RMSE) and the normalized root mean square error (NRMSE) are found to be suitable for ranking the models. A lower RMSE or NRMSE means that
the model fits better the experimental data. In the case of solar irradiance, the mean bias error is a good indicator of the precision of the models for periods longer than a day. It is proportional to the bias on the predicted energy over the considered period. The model that presents the lowest MBE does not systematically present the lowest RMSE. The mean absolute percentage error (MAPE) is another common measure of error. It gives an intermediate measure between the MBE and the NRMSE. We also use the Bayesian information criterion (BIC). The first part of this criterion corresponds to the maximized value of the likelihood function for the estimated model. Under the assumption that the model errors are independent and identically distributed according to a normal distribution, this first term is given by the log of the maximum likelihood estimate of the variance of the residuals. The second part of the BIC introduces a penalty term for the number of fitting parameters in the model (see Table 4). This criterion permits comparison of the performance of the models by taking into account the complexity of their formulation. The model that exhibits the lowest BIC is deemed to be the best one.

\[
RMSE = \sqrt{\frac{\sum_{i=1}^{N} (G_{\text{model},i} - G_{\text{meas},i})^2}{N}} \tag{7}
\]

\[
NRMSE = \frac{RMSE}{G_{\text{meas}}} \tag{8}
\]

\[
MBE = \frac{1}{N} \sum_{i=1}^{N} \left( G_{\text{model},i} - G_{\text{meas},i} \right) \tag{9}
\]

\[
NMBE = \frac{MBE}{G_{\text{meas}}} \tag{10}
\]

\[
MAPE = \frac{1}{N} \sum_{i=1}^{N} \frac{|G_{\text{model},i} - G_{\text{meas},i}|}{G_{\text{meas}}} \tag{11}
\]

\[
BIC = \log \left( \frac{1}{N} \sum_{i=1}^{N} (G_{\text{model},i} - G_{\text{meas},i})^2 \right) + d \cdot \frac{\log(N)}{N} \tag{12}
\]

<table>
<thead>
<tr>
<th>Model</th>
<th>Number of fitting parameters (d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hay</td>
<td>1</td>
</tr>
<tr>
<td>Skartveit &amp; Olseth</td>
<td>2</td>
</tr>
<tr>
<td>Gueymard</td>
<td>30</td>
</tr>
<tr>
<td>Perez</td>
<td>61</td>
</tr>
</tbody>
</table>

Table 4: number of fitting parameters used for the BIC calculation

4. Results and discussion

5.1. Overall performance of the models
The overall performance (see Tables 5, 6, 7) is given by aggregating the hourly measured irradiance and the corresponding model predictions for all the titled planes. With a negative MBE, all the models underestimate the global irradiance on the tilted planes. The RMSE, NRMSE, MAPE and BIC show little differences between the four studied models. Nevertheless the MBE of the Hay model and the Skartveit and Olseth model are two times higher than the Perez model. For this set of data, Perez model has the best performance with a mean bias of -6.8W.m\(^{-2}\).

Fig. 2 (see also Table A.1 in the Appendix) shows the RMSE for the different tilted planes. A asymmetry of the results is observed. The models have a smaller RMSE on the east direction than on the west direction. Furthermore, the error is minimum for the facing north surfaces.

<table>
<thead>
<tr>
<th>Model</th>
<th>RMSE (W.m(^{-2}))</th>
<th>NRMSE (%)</th>
<th>MBE (W.m(^{-2}))</th>
<th>MAPE (%)</th>
<th>BIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hay</td>
<td>39.40</td>
<td>8.49</td>
<td>-15.86</td>
<td>9.32</td>
<td>7.35</td>
</tr>
<tr>
<td>Skartveit&amp;Olseth</td>
<td>39.53</td>
<td>8.52</td>
<td>-16.90</td>
<td>9.42</td>
<td>7.36</td>
</tr>
<tr>
<td>Gueymard</td>
<td>37.77</td>
<td>8.14</td>
<td>-11.99</td>
<td>9.08</td>
<td>7.26</td>
</tr>
<tr>
<td>Perez</td>
<td>37.28</td>
<td>8.04</td>
<td>-6.80</td>
<td>9.27</td>
<td>7.24</td>
</tr>
</tbody>
</table>

Table 5: Total errors for hourly data and measured albedo

![Fig. 2. Comparison of RMSE of the titled radiation models for hourly data and measured albedo](image)

5.2. Use of the measured albedo and minute time step data
The albedo was not measured exactly at the same place as the inclined planes and could be a source of error. So a constant albedo of 0.2 was also tested in the input of the models. As mentioned in previous studies (Kambezidis et al. [7], Notton el al. [13]), the results show that the use of the measured albedo do not improve the
performances of the models. In our case, a constant albedo of 0.2 gives slightly better results for all the models (see Table 6).

<table>
<thead>
<tr>
<th>Model</th>
<th>RMSE (W.m⁻²)</th>
<th>NRMSE (%)</th>
<th>MBE (W.m⁻²)</th>
<th>MAPE (%)</th>
<th>BIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hay</td>
<td>38.41</td>
<td>8.28</td>
<td>-13.79</td>
<td>9.18</td>
<td>7.30</td>
</tr>
<tr>
<td>Skartveit&amp;Olseth</td>
<td>38.50</td>
<td>8.30</td>
<td>-14.83</td>
<td>9.26</td>
<td>7.30</td>
</tr>
<tr>
<td>Gueymard</td>
<td>37.07</td>
<td>7.99</td>
<td>-9.92</td>
<td>8.96</td>
<td>7.23</td>
</tr>
<tr>
<td>Perez</td>
<td>36.93</td>
<td>7.96</td>
<td>-4.73</td>
<td>9.17</td>
<td>7.22</td>
</tr>
</tbody>
</table>

Table 6: Total errors for hourly data and albedo=0.2

The tested models were developed with hourly inputs but Gueymard [1] states they can be equally used for minute time steps. In our case, the use of minute data degrades significantly the performances of the models (Table 7). Nevertheless, the rank of the models is still the same.

<table>
<thead>
<tr>
<th>Model</th>
<th>RMSE (W.m⁻²)</th>
<th>NRMSE (%)</th>
<th>MBE (W.m⁻²)</th>
<th>MAPE (%)</th>
<th>BIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hay</td>
<td>43.60</td>
<td>9.04</td>
<td>-21.79</td>
<td>8.94</td>
<td>7.55</td>
</tr>
<tr>
<td>Skartveit&amp;Olseth</td>
<td>43.83</td>
<td>9.09</td>
<td>-23.27</td>
<td>9.02</td>
<td>7.56</td>
</tr>
<tr>
<td>Gueymard</td>
<td>41.61</td>
<td>8.63</td>
<td>-18.91</td>
<td>8.79</td>
<td>7.46</td>
</tr>
<tr>
<td>Perez</td>
<td>39.43</td>
<td>8.18</td>
<td>-14.49</td>
<td>8.47</td>
<td>7.35</td>
</tr>
</tbody>
</table>

Table 7: Total errors for minute data

5.3. Influence of the sky conditions and model’s performance vs. incidence angle of the beam radiation

The clearness index ($k_c$) is used in order to assess the influence of the sky conditions on the performances of the models. In Fig. 3, the relative error (%) is nearly equally distributed for a clearness index lower than 0.6 (cloudy skies). For the clear sky days with a high clearness index, greater than 0.6, the relative error exhibits a less important distribution. The points situated under the -50% relative error come from the planes facing west. Two phenomena can explain these underestimated values of the west facing planes. First, the albedo was not measured for each titled plan but globally for the site. So it does not take into account a possible anisotropy of the environment solar reflectivity. At the east side of the experiment is a car park and at the west side is dry vegetation. Second, the models do not integrate the diurnal asymmetry of the solar irradiance.
Fig. 3. Relative error vs clearness index (Kt) for hourly data, a) Hay model, b) Skartveit and Olseth, c) Gueymard, d) Perez model

Fig. 4 presents the correlation between the errors of the models and the angle of incidence of the beam irradiance on the tilted plane. For an angle larger than 90°, when the tilted planes are shaded, the error is relatively low. The distribution of the error shows a maximum around an angle of incidence of 40°.
Fig. 4. Error vs incidence angle of beam radiation on the tilted plane for hourly data, 
a) Hay model, b) Skartveit and Olseth, c) Gueymard, d) Perez model

5. Conclusion
In this work, a consistent experimental procedure has been setup for a comparative study of 4 models. Overall, the Perez model exhibits the best performance. Analysis of the accuracy of the models for different sky conditions shows rather high relative errors for planes 40°, Azimuth 60°, 90°. A more in-depth study is currently being undertaken in order to identify the cause of this phenomenon.

An asymmetry between east and west clearly appears for all the models and tend to confirm that a specific treatment of solar radiation in the beginning and end of the day must be undertaken.

This study also confirms that the use of a constant albedo of 0.2 for these models (instead of a measured one) led to better results. Finally, although the use of one minute data led to the same ranking of the models, in our opinion, a specific procedure to fit the models for that time scale must be conducted in order to improve the accuracy of the models.

References


