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Quench-induced breathing mode of one-dimensional Bose gases

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We measure the position- and momentum-space breathing dynamics of trapped one-dimensional Bose gases at finite temperature. The profile in real space reveals sinusoidal width oscillations whose frequency varies continuously through the quasicondensate to ideal Bose gas crossover. A comparison with theoretical models taking temperature into account is provided. In momentum space, we report the first observation of a frequency doubling in the quasicondensate regime, corresponding to a self-reflection mechanism due to the repulsive interactions. Its disappearance through the crossover is mapped out experimentally, giving insights to the dynamics of the breathing evolution.

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The field of ultracold atomic and molecular gases has gained increasing significance in the study of nonequilibrium phenomena in quantum many-body systems. The accurate time-dependent control of microscopic parameters enables the realization of prototypical nonequilibrium processes, complementary to those studied in condensed matter physics. In particular, isolated quantum many-body systems can be driven out of equilibrium and accurately monitored. The realization of such experimental simulators [1] therefore constitutes a unique tool to investigate fundamental questions. Remarkable examples are the observation of a light-cone effect in the spreading of correlations [2, 3], as well as the studies of quantum ergodicity [1, 4–6], where prethermalization was observed in (nearly) integrable systems.

One-dimensional (1D) systems are ideal test benches for the study of out-of-equilibrium phenomena because of their intrinsic strong correlations and the possibility to realize integrable models. The experimental investigation of their quantum dynamics is of particular relevance in clarifying questions for which a theoretical treatment is challenging. The understanding of the time-dependent behavior of interacting quantum systems is indeed restricted to a few limiting cases [7], and plagued by the lack of systematic ab initio approaches. This stands in contrast to the well consolidated understanding of thermodynamic properties of such systems, for which exact analytical and numerical results are available [8].

Among the simplest out-of-equilibrium situation is the dynamics of a gas after a sudden change (quench) of the external harmonic potential in which the atoms are confined. The resulting coherent, breathing-like oscillations of the atomic cloud are governed by the collective excitations of the quantum gas, and serve to identify the collective mode cannot be accessed in terms of the standard sum-rule approach. Moreover, breathing oscillations of the momentum distribution provide complementary insight on the dynamics of interacting quantum gases, information which is not yet investigated experimentally nor accessible theoretically to date.

In this Letter, we study experimentally the breathing mode of harmonically confined 1D Bose gases at finite temperature, comparing the evolution in real and momentum space. We report the first observation of a self-reflection mechanism due to repulsive interaction, expected of a strongly interacting system [13]. We measure how the dynamics evolves through the quasicondensate (qBEC) regime to ideal Bose gas (IBG) crossover, and develop two complementary theoretical treatments for the breathing modes, taking finite-temperature effects into account. The combination of an exact short-time expansion of the dynamics and a long-wavelength hydrodynamic analysis reveals that the finite-temperature mass transport happens in an isentropic way, and provides a qualitatively good agreement with the experimental observations.

System and quench protocol. The Lieb-Liniger (LL) model of 1D bosons in continuum, with a Hamiltonian $H_{\text{LL}} = \sum_j \frac{p_j^2}{2m} + \sum_{j<k} g_{1D} \delta(z_j-z_k)$, where $m$ is the particle mass, and $g_{1D}(>0)$ is the coupling constant, is realized experimentally for gases confined in elongated traps, provided the transverse degrees of freedom are frozen out [14]. Various experimental probes have been developed for such gases, permitting not only thermometry methods [15–20], but also the study of nonequilibrium behavior. Of special importance is the measurement of the momentum distribution, now feasible via Bragg spectroscopy [21] or focusing technique [20].

Our experiment prepares a single tube of $^{87}$Rb gas using an atom-chip setup [17]. The final samples typically constitute 800 to 8000 atoms for this experiment. The in situ density profile [15] indicates a temperature around
FIG. 1. (Color online) Quench sequence and subsequent density evolution in position and momentum space for a qBEC (data A). (a) Longitudinal trapping frequency evolution in position and momentum space for a qBEC regime: Figures 1(b) and (c) show 

at the origin, we use the hydrodynamic equations (HDE) [36] for a data set that lies in this regime (data A). The breathing mode is 

100 mK, corresponding to a central chemical potential $\mu_0 \in [0.04, 0.5] \times k_BT$ [35]. Since the transverse confinement $\omega_\perp = 2\pi \times 2$ kHz, we have $\mu_0 \ll k_BT \simeq \hbar \omega_\perp$, achieving a nearly 1D scenario. The breathing mode is excited by quenching the axial confinement $\omega_z$ from $\omega_0$ to $\omega_D$, see Fig. 1(a) for a sketch. We keep the quench strength $\alpha \equiv \omega_0/\omega_D \simeq 1.3$ constant. The resulting cloud evolves for a duration $\tau$ before an absorption image is taken, either in situ to yield the density profile $\rho(z, \tau)$, or at focus to yield the momentum distribution $n(p, \tau)$. By varying $\tau$, we map out the evolution in real and momentum space.

Quasicondensate regime. We start by considering the qBEC regime: Figures 1(b) and (c) show $\rho(z, \tau)$ and $n(p, \tau)$ for a data set that lies in this regime (data A). The time evolution of these two quantities reveals a remarkable frequency mismatch. In order to elucidate its origin, we use the hydrodynamic equations (HDE) [36]

$$\partial_\tau \rho + \partial_\tau (\rho v) = 0,$$
$$\partial_\tau v + v \partial_x v = -\partial_z (\frac{1}{2} \omega_D^2 z^2) - \frac{1}{m_p} \partial_z P,$$

where $\rho = \rho(z, \tau)$ and $v = v(z, \tau)$ are the density and velocity fields, and $P$ is the pressure. For a qBEC, $P = \mu_0 \rho^2/2$, so that the scaling solution $\rho(z, \tau) = \frac{b}{\tilde{b}(\tau)} \rho_0 \left( \frac{z}{\tilde{b}(\tau)} \right)$ [23] is valid provided $\rho_0$ is the steady state inverted parabola, and the scaling factor $b$ obeys $\tilde{b} + \omega_z(\tau)^2 b = \frac{\omega_p^2}{\tilde{b}}$. Such $\rho(z, \tau)$ is periodic in time with a frequency $\omega_{Bz} \simeq \sqrt{3} \omega_\perp$. Neglecting thermal fluctuations, it follows that the momentum distribution satisfies [23] $n(p, \tau) \propto \rho_0 \left( \frac{\pi}{2m_p} \right)^{3/2}$, which is periodic in time with a frequency $\omega_{Bp} = 2\omega_{Bz}$. Minimal momentum width occurs when the real-space distribution is both the largest and the thinnest. The latter scenario, which does not occur for a non-interacting gas, corresponds to a self-reflection of the cloud due to repulsive interactions. The frequency doubling ($\omega_{Bp} = 2\omega_{Bz}$) is thus a direct consequence of atomic interactions. In higher dimensions, the scaling solutions also predicts a frequency doubling of the breathing modes [37]. What may have prevented its observation is perhaps the fact that early experiments using time-of-flight (TOF) techniques do not measure the true momentum distribution due to the contribution of interaction energy.

It is worth mentioning that such a momentum-space frequency doubling in the oscillatory behavior of $w_p$, the half width at half maximum (HWHM) in momentum space, is expected to occur for a Tonks gas ($g_{1D} \rightarrow +\infty$) [25]: the momentum distribution for large $\alpha$ oscillates between a Fermi-like and a Bose-like distribution, a thin Bose-like distribution occurring both when the cloud is the thinnest and the largest [13], while in the expansion and compression phases the momentum distribution is dominated by the large Fermi-like hydrodynamic component. This marks the difference between the breathing behaviour of a strongly interacting 1D Bose gas and that of a noninteracting gas, even though the breathing frequency in real space is $\omega_{Bz} = 2\omega_D$ for both cases. Similar collective oscillations of strongly interacting 1D Bose gases have been studied in two experiments [4, 10] to our knowledge, neither reporting any frequency doubling. In the former [4], a different excitation scheme is employed, potentially rendering the frequency doubling inaccessible. In the latter [10], the finite TOF may have prevented the direct measurement of the momentum distribution.

In our experiment the momentum-space frequency doubling is clearly seen in Fig. 1(b-c). Moreover, the data are in good agreement with the zero-temperature theory above, whose predicted real- and momentum-space density evolutions for a system initially at equilibrium in the trap of frequency $\omega_0$ are shown in Fig. 1(d-e), with $n(p, \tau)$ broadened to account for finite resolution [38]. For a more quantitative comparison, we show two frames of the instantaneous densities. At $\tau = 1$ ms (minimal in situ width, Fig. 1(f)), the scaling solution predicts a vanishing momentum width, but thermal fluctuations dominate such that the finite-temperature momentum distribution computed using quantum Monte Carlo (QMC) methods [20] (dashed lines) accounts for the data better than the sole effect of resolution [39]. However, thermal fluctuations are small corrections at $\tau = 21$ ms (maximal momentum width, Fig. 1(g)), and both $\rho(z)$ and $n(p)$ are
in excellent agreement with the inverted parabolas of the scaling solution. Self-similarity of the shapes is due to the fact the momentum distribution is dominated by the hydrodynamic velocity field, which is linear in position. The same phenomena is at the origine of the dynamic fermionization for a strongly interacting system [13].

Ideal Bose gas regime. For an IBG, a single-particle description suffices and breathing amounts to a rotation of the phase-space density, so that the widths in real and momentum space oscillate out of phase at the same frequency \( \omega_B = 2\omega_D \). Fig. 2 shows a data set close to this regime (data C), where the \textit{in situ} mean-square width \( \langle z^2 \rangle \) (momentum HWHM \( w_p \)) is obtained from Gaussian (Lorentzian) fit. The antiphase is apparent from the plots. Fitting both time evolution with damped sinusoids, we measure identical breathing frequency, \( \omega_B/\omega_D = 1.84 \pm 0.04 \), where \( \omega_D \) is determined by monitoring the center-of-mass (dipole) oscillations.

Crossover in real space. A central point to be understood is how the breathing frequency varies through the qBEC to IBG crossover. To address this question, we vary the total atom number in order to explore different regimes. The inset of Fig. 3 shows the region spanned by the data in the \((\gamma, t)\) phase diagram of the LL model [28], where \( \gamma = \frac{mg_\alpha}{\hbar \omega_B} \) is the (local) interaction parameter, and \( t = \frac{2\pi^2 \hbar^2 \omega_B}{mg_\alpha} \) is the reduced temperature. Each sample is characterized by \( \gamma_0 \), evaluated at the peak density. We extract \( \langle z^2 \rangle \) from fitting \( \rho(z) \) with either an inverted parabola for \( \gamma_0 < 0.004 \) [40] or a Gaussian otherwise. We obtain the real-space breathing frequency \( \omega_{Bz} \) by fitting \( \langle z^2 \rangle(\tau) \) with a damped sinusoid. The measured \( \omega_{Bz}/\omega_D \) as a function of \( \gamma_0 \), shown in Fig. 3(a), displays a smooth crossover between the asymptotic theoretical limits \( \sqrt{3} \) and 2 [11]. Such a frequency shift was first observed in [9], where the authors probed the breathing behaviour of an ensemble of 1D gases in real space. In addition to the issue of thermalization mentioned in [9], the intrinsic ensemble averaging renders it problematic to characterize the system with a single value of temperature \( T \) or interaction parameter \( \gamma \). In contrast, our current experiment with a single 1D system allows for a more quantitative study of the problem at hand.

In order to provide a theoretical treatment of the crossover, we devise two complementary approaches. On one hand, we model the crossover using the HDE Eq. (1). Since long-wavelength density waves in a fluid are adiabatic [41], we use the isentropic pressure curves derived numerically from the YY EoS [31]. The breathing mode frequency measured experimentally does not depend on the oscillation amplitude for the explored parameter range. We thus linearize the HDE for small displacement and \( \omega_{Bz} \) is obtained by solving an eigenvalue problem. Results evaluated at \( t = 1100 \) are shown as a dashed line in Fig. 3(a). On the other hand, we provide a microscopic treatment of the breathing frequency that accounts for the effect of temperature. Assuming the system at \( \tau = 0 \) is at thermal equilibrium with a Hamiltonian \( H = H_{LL} + H_{pot} \), \( H_{pot} = m\omega_0^2 \sum_j \delta^2 j^2 / 2 \), the expansion of the Heisenberg equation of motion after a quench \( \Delta H = m(\omega_B^2 - \omega_0^2) \sum_j \delta^2 j^2 / 2 \) gives \( \langle \Delta H \rangle(\tau) = \langle \Delta H \rangle_T - \frac{i}{\hbar} \frac{\delta}{\delta \tau} \langle C_2 \rangle_T + \frac{1}{4} \langle C_4 \rangle_T + \ldots \), where \( C_2 \) and \( C_4 \) are the second and fourth order nested commutators with \( H_f = H + \Delta H \), and the thermal average \( \langle \ldots \rangle_T \) is taken over the thermal state of \( H \). Suppose the time evolution \( \langle \Delta H \rangle(\tau) \propto \langle z^2 \rangle(\tau) \) is purely sinusoidal at the frequency \( \omega_{Bz} \), we have

\[
\frac{\omega_{Bz}^2}{\omega_D^2} = \frac{\langle C_4 \rangle_T}{\langle C_2 \rangle_T} = 4 - \frac{1}{2} \frac{\langle H_{int} \rangle_T}{\langle H_{pot} \rangle_T},
\]

where \( H_{int} = g_{1D} \sum_{j<k} \delta (z_j - z_k) \) is the interaction part of the Hamiltonian. The final equality comes from explicitly working out \( \langle C_2 \rangle_T \) and \( \langle C_4 \rangle_T \), and taking the limit of an infinitesimal quench amplitude \( \alpha \to 1 \), where the sinusoidal approximation is valid [42].
It is remarkable that both approaches (HDE and the exact short-time expansion) give the same results for the breathing frequencies through the crossover. This indicates that the finite-temperature mass transport is indeed isotropic. Moreover, Eq. (2) extends the regime of validity of the existing sum-rule approaches, which erroneously predicts \( \omega_{Dz} = \sqrt{2} \omega_D \) in IBG, whereas the true asymptotic limit is \( \omega_{Dz} = 2 \omega_D \).

Our theories provide a qualitatively good agreement with the experimental data, despite an overall overestimation of the breathing frequency. This quantitative departure most likely indicates that the atomic ensemble is not at thermal equilibrium at the end of the preparation (evaporation). Although the \textit{in situ} profiles calibrated with the YY EoS generally yields about 100 nK, the atom-number fluctuation indicates approximately 35 to 45 nK at the center of the cloud. We thus include in Fig. 3(a) our prediction at \( t = 400 \) (dash-dotted line). We believe that such a lack of equilibrium is a direct consequence of evaporating into the integrable (1D) regime. Further investigation in this direction is underway.

**Disappearance of the self-reflection mechanism.** To address the behavior of the breathing mode in momentum space, we investigate the time evolution of \( w_p \), extracted from a Lorentzian fit of \( n(p) \). Since \( w_p \) shows a periodic behavior at the frequency \( \omega_{Bz} \), it can be expanded in a discrete Fourier spectrum. The relative weight of the Fourier components varies through the crossover, and we obtain quantitative information by fitting \( w_p \) with a function

\[
y = A e^{-\frac{\tau}{\tau_z}} + B e^{-\frac{\tau}{\tau_2}} \left[ \sqrt{K \cos(\omega_{Bz} \tau)} - \sqrt{1 - K \cos(2\omega_{Bz} \tau)} \right],
\]

shown in Fig. 2 (bottom, solid line), with \( A \) fixed at the average width during the initial cycle and the phase corresponds to a minimal \textit{in situ} width at the start of the oscillations. The relative power of the first harmonic \( K \) (squares) and second harmonic \( 1 - K \) (diamonds) are shown in Fig. 3(b) as a function of \( \gamma_0 \). In qBEC, the second harmonic dominates, as predicted by the scaling solution, and signals the self-reflection mechanism. In IBG, the first harmonic dominates, as expected for a non-interacting gas where the self-reflection is absent. Both weights vary gradually through the crossover, indicating a smooth disappearance of the self-reflection mechanism.

This can be seen as the effect the breathing mode has on the thermally excited Bogoliubov modes of high energy, e.g., their frequency and wavefunction would be modulated in time, such that \( w_p \) is larger at minimal \( \langle z^2 \rangle \) than that at maximal \( \langle z^2 \rangle \), see Fig. 2 (data B). The periodicity at \( 2\omega_{Bz} \) is then broken and the first harmonic at \( \omega_{Bz} \) emerges. Figure 3 shows that the first harmonic starts to gain weight at a value of \( \gamma_0 \) significantly smaller than that where the frequency shift takes place in real space.

The breathing mode observed has lifetime estimated to be on the order of seconds. This is in stark contrast with situations in 3D where damping of the collective modes occurs, mainly via Landau damping mechanism [33, 34]. The long lifetime of the breathing mode in 1D may be related to the integrability of the underlying LL model.

**Conclusions.** In this Letter we have probed the breathing mode in real and momentum space through the qBEC to IBG crossover. The shift of the real-space frequency between the asymptotic values \( 3 \omega_D \) and \( 2 \omega_D \) is demonstrated. Our theory models that assume thermal equilibrium before the quench do not agree with the measurement quantitatively, indicating the possibility of a non-Gibbs initial state produced by evaporation. We report the first observation of a momentum-space frequency doubling in qBEC, corresponding to an interaction-induced self-reflection mechanism that is expected of a Tonks gas. We experimentally map out the disappearance of the self-reflection through the crossover, for which no theoretical predictions exist to date. This illustrates the richness of out-of-equilibrium dynamics and the importance of the momentum-space observation to unveil the underlying physics.

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