State-parameter identification for accurate building energy audits
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ABSTRACT

Building performance simulation often fails to predict accurately the actual energy performance, mostly due to great uncertainties in the input data. Errors in computed performance are particularly significant in the case of existing buildings, for which the amount of information about intrinsic characteristics is low. However, efficient energy retrofit operations make necessary an accurate understanding of the initial state of a building using a calibrated prediction model. Several works have investigated the use of identification techniques for model calibration. The present paper investigates the use of such techniques to derive an energy audit procedure suitable to be an efficient aid for retrofit. In particular, we study here the possibility to determine the unknown thermal conductivity of the envelope and the internal gains based on temperature measurements only. We show how the adjoint method can be used to solve efficiently the inverse problem, while providing a fast method for computing model’s sensitivities.

INTRODUCTION

Building energy performance simulation often fails to predict accurately the actual energy performance, (Norford et al., 1994; Bordass et al., 2004; Demanuele et al., 2010). Errors in computed performance are particularly significant in the case of existing buildings, for which the uncertainties on intrinsic characteristics are high. Most of today’s audit methods are unable to provide reliable data under cost and implementation constraints. However, efficient energy retrofit operations make necessary an accurate understanding of the models used for studying and choosing the rehabilitation procedures. This need has fostered the research for the development of model calibration techniques. The present paper investigates the use of inverse modelling to identify some among the most significant parameters of a model.

State-parameter identification aims at determining heat sources and intrinsic properties of a mathematical model based on partial observations of the thermal state of the building. Using optimal control theory, it can be formulated as a minimization problem, where the unknowns are sought as minimizers of a cost function evaluating the gap between the measures and the response of the model. Such inverse problems are ill-posed in the sense of Hadamard, which means that their solution, if it exists and is unique, does not depend continuously on the data, (Tikhonov and Arsenin, 1977). In order to obtain a stable numerical scheme, some regularization has to be introduced. We use here Tikhonov regularization, which consists in adding a regularization term to the cost function thus providing local convexity and transforming the initial problem into a well-posed one, (Chavent, 2009).

The minimization is done using a descent algorithm, and the gradient is computed with the adjoint state method, (Lions, 1968). This method gives a way to derive the gradient by solving a problem which has the same structure than the direct one, meaning that the same tools can be used for both computations. An additional advantage is that the minimization framework gives a complete local sensitivity analysis of the model (Griesse, 2007), as it will be illustrated in the last section of this document.

The choice of the regularization term greatly influences the shape of the reconstructed functions. The internal gains modelling the use of heating devices are discontinuous functions of time. However, the usual $L^2$ framework in least squares minimization with Tikhonov regularization fails to render discontinuities in time. Several works, in the field of image restoration, showed that the total variation (TV) is an efficient regularization term for the reconstruction of piecewise constant functions, (Rudin et al., 1992; Vogel and Oman, 1996; Osher et al., 2005).

We focus here on the identification of thermal conductivities and internal gains. These two parameters are reconstructed using simple ambient and wall surface temperatures. Tikhonov regularization is used in a TV framework to catch time discontinuities in internal gains. The adjoint method is employed to construct a non linear minimization algorithm. We show in the last section how the adjoint model can be used to efficiently compute model’s sensitivities.

The general methodology framework is presented here on a very simple test case composed of a two-zone building for illustration purpose, see figure 1. Solar radiation is neglected.
Inverse identification aims at determining the input parameters of a mathematical model that minimize the discrepancy between the model’s response and the data. We describe in this section the model used for this study.

The mathematical model describing the thermal state of the building builds upon standard multizone assumptions for the air temperatures, and partial differential equations for the heat flow in the envelope, (Boland, 1997). In this framework, the entire zone temperature and pressure are assumed to be homogeneous, and the heat transfers through the walls are assumed one-directional. The equations system is composed of eleven partial differential equations for the temperature distribution into walls, and two ordinary differential equations for the zone temperatures. All equations are coupled by radiative and convective exchange coefficients.

The evolution of the temperature $T_i$ of the room $i \in \{1;2\}$ is governed by:

$$
\begin{align*}
C_i \frac{dT_i}{dt} &= \sum_{p \in P_i} S_p h^0_p (\theta^0_p(t) - T_i) + W_i \\
T_i(t = 0) &= T^0_i
\end{align*}
$$

where $C_i$ is the room heat capacity in $J.K^{-1}$, $S_p$ is the area of the wall $p$ in $m$, $h^0_p$ is the convective exchange coefficient with the surface of the wall $p$ whose temperature is denoted $\theta^0_p(t)$, and $W_i(t)$ the internal gains in $J.s^{-1}$. $P_i$ is the index set of walls having a surface adjacent to the zone $i$.

The evolution of the temperature field $\theta_p(x; t)$ inside a wall $p$ of thickness $L_p$ adjacent to the zone $i$ is given by an equation of the form:

$$
\begin{align*}
S_p \frac{\partial \theta_p}{\partial t} - S_p \frac{\partial}{\partial x} \left( k_p \frac{\partial \theta_p}{\partial x} \right) &= 0 \\
-k_p S_p \frac{\partial \theta_p}{\partial x}(0; t) &= S_p h^0_p (T_i - \theta_p(0; t)) \\
&+ \sum_{m \in P_i} S_p \alpha_{p,m} (\theta_m(0; t) - \theta_p(0; t)) \\
k_p S_p \frac{\partial^2 \theta_p}{\partial x^2}(L_p; t) &= S_p h^L_p (T_a - \theta_p(L_p; t)) \\
&+ S_p \beta_p (T^\infty - \theta_p(L_p; t)) \\
\theta_p(x, t = 0) &= \theta^0_p(x)
\end{align*}
$$

where $c_p$ is the equivalent heat capacity in $J.m^{-3}.K^{-1}$, $k_p$ is the equivalent thermal conductivity in $J.s^{-1}.m^{-1}.K^{-1}$, $\alpha_{p,m}$ is the radiative exchange coefficient with the wall $m$, $h^0_p$ is the convective exchange coefficient with the room temperature and $h^L_p$ is the convective exchange coefficient with the outside temperature $T_a(t)$, and $\beta_p$ is the radiative exchange coefficient with the sky temperature $T^\infty(t)$. Equation (2) is written for a wall with surface $(x = 0)$ adjacent to the zone $i$ and surface $(x = L_p)$ facing outside. Of course, the various boundary conditions have to be adapted to each wall situation.

For the sake of simplicity, we will not write the heat equation inside every wall. They are all the same, only boundary conditions differ.

**INVERSE PROBLEM**

We present in this section the identification method based on the model described in the previous section. We suppose that we have air and surface temperature measurements obtained from sensors deployed on site for a given period of time. Let $t_a$ be the actual time, we suppose having data starting from the initial time to the actual time : $t \in [0; t_a]$. We set $T^0_i(t)$ the inside air temperature measurements of the room $i \in \{1;2\}$, and $\theta^0_p(0; t)$ and $\theta^L_p(L_p; t)$ the temperature measurements of the surface $(x = 0)$ and $(x = L_p)$ of the wall $p$, respectively. More precisely, the two surface temperatures of walls 1 and 2, and two zone temperature are measured. We suppose that all the walls in contact with the outside have the same internal composition. That is, their thermal conductivity are the same, denoted $k$ hereafter.

Based on the measurements described above, we aim at reconstructing this equivalent thermal conductivity $k$ of the walls having a surface in contact with the outside, the equivalent thermal conductivity $k_2$ of the wall 2, and the internal gains in each room. The internal gains $W_i$ heating the rooms are due to heating devices. They are typically discontinuous functions of time, and traditional identification techniques are unable to catch discontinuities in their time evolution. To deal with the discontinuous nature of the unknowns, we use here a regularization technique first developed in the field of image restoration, (Rudin et al., 1992; Vogel and Oman, 1996). It consists in looking for the unknowns in the space of bounded variation functions, thus separating, in the set of unknowns, the smoothly varying components from the rapidly varying ones. We make use of this technique here for the reconstruction of the internal gains, (Brouns et al., 2013).

Let $u = \{k; k_2; W_1, W_2\} \in \mathcal{P}$ be the vector of unknowns. It belongs to the parameters space $\mathcal{P} = (\mathbb{R}^+)^2 \times BV(0; t_a)^2$, where $BV(0; t_a)$ is the space of bounded variation functions. We set the problem as an optimization problem, consisting in the minimization of a cost function evaluating the gap between the data and the model’s response. The problem reads: find
\[ u \in \mathcal{P} \text{ that minimizes} \]
\[ J(u) = \frac{1}{2} \sum_{p=1}^{2} \| \theta_p(0; t) - \theta_p^d(0; t) \|^2_{\mathcal{M}} + \frac{1}{2} \sum_{p=1}^{2} \| \theta_p(L_p; t) - \theta_p^d(L_p; t) \|^2_{\mathcal{M}} + \frac{1}{2} \sum_{i=1}^{2} \| T_i(t) - T_i^d(t) \|^2_{\mathcal{M}} + \frac{\epsilon}{2} \| u \|^2 \]  

(3)

with \( \mathcal{M} = L^2(0; t_0) \) the measurements space, and \( \epsilon \) the regularization parameter.

**Levenberg-Marquardt algorithm**

Since the model response is not linear with respect to the thermal conductivities, we use an iterative method based on the Levenberg-Marquardt approach (Moré, 1977), coupled with the conjugate gradient to solve the minimization problem. This technique relies on the following approximation: let \( \delta u = \{ \delta k; \delta k_2; \delta W_1, \delta W_2 \} \in \mathcal{P} \) be a small perturbation of the unknowns ; the approximation writes:

\[ \theta_p(u + \delta u) \approx \theta_p(u) + \delta \theta_p(\delta u), \quad p \in [1; 11] \]

\[ T_i(u + \delta u) \approx T_i(u) + \delta T_i(\delta u), \quad i \in [1; 2] \]

(4)

where \( \delta T_i \) and \( \delta \theta_p \) are the solutions to the sensitivity model around \( u \), written in (5)-(7):

\[
\begin{aligned}
C_i \frac{d\delta T_i}{dt} &= \sum_{p \in \mathcal{P}} S_p h_p^0(\delta \theta_p(t) - \delta T_i) + \delta W_i \\
\delta T_i(t = 0) &= 0
\end{aligned}
\]

(5)

\[
\begin{aligned}
S_p c_p \frac{\partial \delta \theta_p}{\partial t} - S_p \frac{\partial}{\partial x} \left( k \frac{\partial \delta \theta_p}{\partial x} \right) &= S_p \frac{\partial}{\partial x} \left( \frac{\partial \theta_p}{\partial x} \right) \\
-k S_p \frac{\partial \delta \theta_p}{\partial x}(0; t) &= S_p h_p^0(\delta T_i - \delta \theta_p(0; t)) \\
&+ \sum_{m \in \mathcal{P}} S_p \alpha_{p,m}(\delta \theta_m(0; t) - \delta \theta_p(0; t)) \\
&+ \delta k S_p \frac{\partial \theta_p}{\partial x}(0; t) \\
k S_p \frac{\partial \delta \theta_p}{\partial x}(L_p; t) &= - S_p \left( h_p^L + \beta_p \right) \delta \theta_p(L_p; t) \\
&- \delta k S_p \frac{\partial \theta_p}{\partial x}(L_p; t) \\
\delta \theta_p(x, t = 0) &= 0
\end{aligned}
\]

(6)

Therefore, we introduce a new cost function:

\[
\tilde{J}_u(\delta u) = \frac{1}{2} \sum_{p=1}^{2} \| \theta_p(0; t) - \delta \theta_p(0; t) \|^2_{\mathcal{M}} + \frac{1}{2} \sum_{i=1}^{2} \| T_i(t) - \delta T_i(t) \|^2_{\mathcal{M}} + \frac{\epsilon}{2} \| u \|^2
\]

(7)

The new unknown is the optimal local increase \( \delta u \) which will be used for the next linearization step around \( u_{n+1} = u_n + \delta u \). The cost function (8) is quadratic, which ensures convexity. Its minimum corresponds to the solution of Euler’s equation:

\[
\delta u = \arg \min_{\delta \pi \in \mathcal{P}} \tilde{J}_u(\delta \pi) \iff \nabla \tilde{J}_u(\delta u) = 0
\]

(9)

The problem (9) is solved using the conjugate gradient method. In order to do this, the gradient of (8) is computed using the adjoint method. With this method, an exact expression of the gradient can be obtained by solving the so-called adjoint problem, which has the same structure as the initial problem (1)-(2).

**Adjoint state method**

Let \( V_i, i \in [1; 2] \), and \( A_p, p \in [1; 11] \), be the solutions to the adjoint state equations (10)-(12):

\[
\begin{aligned}
-C_i \frac{dV_i}{dt} &= \sum_{p \in \mathcal{P}} S_p h_p^0(\delta A_p(t) - V_i) \\
&+ T_i + \delta T_i - T_i^d \\
V_i(t = t_0) &= 0
\end{aligned}
\]

(10)
\[
\begin{aligned}
-S_p \frac{\partial A_p}{\partial t} &= S_p \frac{\partial A_p}{\partial x} \left( k \frac{\partial A_p}{\partial x} \right) \\
&+ (\theta_p + \delta \theta_p - \theta_p^0) (\delta_0(x) + \delta L_2(x)) \\
-kS_p \frac{\partial A_p}{\partial x} (0; t) &= S_p h^0_p (V_1 - A_p (0; t)) \\
&+ \sum_{m \in \mathcal{P}_1} S_p \alpha_{p,m} (A_m (0; t) - A_p (0; t)) \\
kS_p \frac{\partial A_p}{\partial x} (L_p; t) &= -S_p (\delta^2 h^0_p + \beta_p) A_p (L_p; t) \\
A_p (x, t = t_a) &\equiv 0
\end{aligned}
\]  

where \( \delta_0 \) is the Dirac measure at \( a \). The optimal control theory tells us that the components of \( \nabla J_u (\delta u) \) are the following:

\[
\begin{aligned}
\langle \nabla J_u; \delta \hat{k} \rangle &= \sum_{p \in \{1; 2\}} \int_0^{t_a} \int_0^{L_p} \delta \hat{k} S_p \frac{\partial \theta_p}{\partial x} \frac{\partial A_p}{\partial x} \\
&+ \epsilon \langle \delta \hat{k}; \delta \hat{k} \rangle \\
\langle \nabla J_u; \delta \hat{\theta} \rangle &= -\int_0^{t_a} \int_0^{L_p} \delta \hat{\theta} S_p \frac{\partial A_p}{\partial x} \\
&+ \epsilon \langle \delta \hat{\theta}; \delta \hat{\theta} \rangle \\
\langle X_i; \delta \hat{W} \rangle_{L^2(0; t_a)} &= \int_0^{t_a} \delta \hat{W}, V_i \\
&+ \epsilon \langle \delta \hat{W}; \delta \hat{W} \rangle_{L^2(0; t_a)}
\end{aligned}
\]

with \( i \in \{1; 2\} \).

The third and fourth components of the gradient are obtained by the projection in the space \( BV(0; t_a) \) using the following computation (see (Brouns et al., 2013) for more details):

\[
\| X_i - \nabla J_u \|_{L^2(0; t_a)} = \min_{v \in BV(0; t_a)} \| X_i - v \|_{L^2(0; t_a)}
\]

with \( i \in \{1; 2\} \).

Once we have the gradient, we use a descent method for solving (9). We choose the conjugate gradient method (see (Nassiopoulos and Bourquin, 2013) for more details).

**IDENTIFICATION RESULTS**

In order to give numerical evidence of the performance of the method, we generate simulated temperature measurements with our model. The virtual sensors are distributed on the two surfaces of the walls 1 and 2 (meaning that one of the surface temperature sensors of the wall 1 is outside of the building), and in each room. The data are sampled at \( 10^{-3} \mathrm{Hz} \) (every 15 min), during 24h. We add a gaussian white noise with standard deviation \( 10^{-2} \) to the data. These data are then used as inputs for the inverse algorithm.

We reconstruct simultaneously the thermal conductivities \( k \) and \( k_2 \), and the internal gains \( W_i, i \in \{1; 2\} \). The algorithm is initialized by 0.7 for the conductivities, and by the zero function for the internal gains. All other parameters are assumed to be known.

The figures 2 and 3 represent the reconstruction of the thermal conductivities \( k \) and \( k_2 \), respectively. The reconstruction of \( W_1 \) and \( W_2 \) are presented in the figures 4 and 5, respectively.
A retrofit operation aims at reducing the overall energy demand of a building. This demand is computed as the overall heat needed to maintain a given setpoint temperature. The optimal solution $w^*$ also depends on $k$ and $T^\infty$. The sensitivity of $w^*$ with respect to $k$ and $T^\infty$ is defined as the variation of $w^*$ under perturbations $\delta k$ and $\delta T^\infty$ respectively. These sensitivities, denoted $\tilde{\omega}_{\delta k}$ and $\tilde{\omega}_{\delta T^\infty}$ respectively, are given by the Gâteaux derivatives:

$$\tilde{\omega}_{\delta k} = \lim_{\epsilon \to 0} \frac{w^*(k + \epsilon \delta k; T^\infty) - w^*(k; T^\infty)}{\epsilon}$$  \hspace{1cm} (21)$$

$$\tilde{\omega}_{\delta T^\infty} = \lim_{\epsilon \to 0} \frac{w^*(k; T^\infty + \epsilon \delta T^\infty) - w^*(k; T^\infty)}{\epsilon}$$  \hspace{1cm} (22)$$

One can show (Griesse, 2007) that the solutions of the problems (21)-(22) are in fact the solutions of the following problems:

$$\min_{w \in L^2(0,t_u)} \left\{ \sum_{i=1}^{M} \frac{\mu}{2} ||T_i - T_i^*||^2_M + \frac{1}{2} ||w||^2_{L^2(0,t_u)} \right\}$$  \hspace{1cm} (23)$$

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with \( \tilde{w} \in L^2(0; \tau_c) \), and \( \tilde{T}_i, i \in \{1; 2\} \) the solutions of equations (24)-(25):

\[
\begin{align*}
C_i \frac{d\tilde{T}_i}{dt} &= \sum_{p \in \mathcal{P}_i} S_p h_p^0 \left( \tilde{\theta}_p^s(t) - \tilde{T}_i \right) + \tilde{w} & (24) \\
\tilde{T}_i(t=0) &= 0
\end{align*}
\]

\[
\begin{align*}
S_p c_p \frac{\partial \tilde{\theta}_p}{\partial t} &= S_p \frac{\partial}{\partial x} \left( k \frac{\partial \tilde{\theta}_p}{\partial x} \right) = S_p \frac{\partial}{\partial x} \left( \delta k \frac{\partial \tilde{\theta}_p}{\partial x} \right) \\
-kS_p \frac{\partial \tilde{\theta}_p}{\partial x}(0; t) &= S_p h_p^0 \left( \tilde{T}_i - \tilde{\theta}_p(0; t) \right) \\
&+ \sum_{m \in \mathcal{P}_i} S_p \alpha_{p,m} \left( \tilde{\theta}_m(t; 0) - \tilde{\theta}_p(t; 0) \right) \\
&+ \delta k S_p \frac{\partial \tilde{\theta}_p}{\partial x}(0; t) \\
kS_p \frac{\partial \tilde{\theta}_p}{\partial x}(L_p; t) &= -S_p \left( h_p^L + \beta_p \right) \tilde{\theta}_p(L_p; t) \\
&+ S_p \beta_p \delta T^\infty - \delta k S_p \frac{\partial \tilde{\theta}_p}{\partial x}(L_p; t)
\end{align*}
\]

where \( \delta k \) and \( \delta T^\infty \) are small perturbations of \( k \) and \( T^\infty \), respectively. The term \( \tilde{\theta}_p(x; t) \) appearing in (25) is the solution of the initial problem (1)-(2) for the given \( k \) and \( T^\infty \).

Let \( \tilde{w}_{\delta k}^* \) and \( \tilde{w}_{T^\infty}^* \) be the solution of the minimization problem (23) for the problem (21) and (22), respectively. They satisfy locally:

\[
\begin{align*}
\tilde{w}_k^*|_{k+\delta k} &\simeq \tilde{w}_k^* + \tilde{w}_{\delta k}^* & (26) \\
\tilde{w}_{T^\infty+\delta T^\infty}^* &\simeq \tilde{w}_{T^\infty}^* + \tilde{w}_{T^\infty}^* & (27)
\end{align*}
\]

Since the model’s response is linear with respect to the sky temperature, the equation (27) is an equality, and is globally satisfied. The equation (26) is an approximation only satisfied for a small perturbation \( \delta k \).

This sensitivity analysis could be a precious aid for retrofit. Firstly, it helps the professionals to evaluate the uncertainty of the energy performance computation with the knowledge of the uncertainties on the other parameters. Secondly, it can help the professionals to determine the best rehabilitation scenario by highlighting the most sensitive parameters.

**CONCLUSION**

We showed the possibility to reconstruct thermal conductivities and internal gains of a two-zone building based on six temperature sensors. We use a projection method in the space of bounded variation functions to increase the accuracy of the reconstruction of the internal gains. The procedure has been validated using synthetic data obtained from simulation, where numerical noises has been added. We showed that the adjoint model can be used efficiently to perform a local sensitivity analysis: the problem is equivalent to a minimization problem similar to the initial inverse problem. We outlined the methodology in the case of the computation of sensitivities of the theoretical consumption of the building with respect to its thermal conductivities and the sky temperature.

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