An Observer with Measurement-triggered Jumps for Linear Systems with Known Input
Francesco Ferrante, Frédéric Gouaisbaut, Ricardo Sanfelice, Sophie Tarbouriech

To cite this version:
Francesco Ferrante, Frédéric Gouaisbaut, Ricardo Sanfelice, Sophie Tarbouriech. An Observer with Measurement-triggered Jumps for Linear Systems with Known Input. IFAC World Congress, Aug 2014, Le Cap, South Africa. Paper MoA04.6, 2014. <hal-00911851v3>

HAL Id: hal-00911851
https://hal.archives-ouvertes.fr/hal-00911851v3
Submitted on 3 Oct 2014

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L’archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d’enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.
An Observer with Measurement-triggered Jumps for Linear Systems with Known Input *

F. Ferrante * F. Gouaisbaut * R. G. Sanfelice **
S. Tarbouriech *

* CNRS, LAAS 7, Avenue du Colonel Roche F-31400 Toulouse, France and Univ de Toulouse, UPS, ISAE, F-31400 Toulouse, France.
Email:{ferrante, fgouaisb, tarbour} @laas.fr

** Department of Aerospace and Mechanical Engineering, Department of Electrical and Computer Engineering, University of Arizona, Tucson, Email: sricardo@u.arizona.edu

Abstract: This paper deals with the estimation of the state of linear time invariant systems for which measurements of the output are available sporadically. An observer with jumps triggered by the arrival of such measurements is proposed and studied in a hybrid systems framework. The resulting system is written in estimation error coordinates and augmented with a timer variable that triggers the event of new measurements arriving. The design of the observer is performed to achieve uniform global asymptotic stability (UGAS) of a closed set including the points for which the state of the plant and its estimate coincide. Furthermore, a computationally tractable design procedure for the proposed observer is presented and illustrated in an example.

Keywords: Observers, hybrid systems, linear systems, linear matrix inequalities, networked control systems

1. INTRODUCTION

State observer design is undoubtedly a difficult problem, with high relevance in applications. Indeed, observers can be employed to obtain an estimation of certain state variables, which are not directly accessible or also to reduce the number of the sensors used in control systems. Many of the most interesting recent applications pertain to controlled systems linked together through data networks. The nature of such networks may often introduce time delays, asynchronism, packages drop-out, and communication channel limitations; see, for example, Lopez Hurtado et al. (2009). Moreover, in modern distributed systems, the communication mechanisms across the network are governed by logic statements, which aim at reducing the required bandwidth over the communication channel; see, for example, Wong and Brockett (1997). Such mechanisms lead to an intermittent availability of the measured variables. In this setting, the classical paradigm of continuously measured variables needs to be reconsidered to face the new challenges induced by data network constraints. Indeed, an observer can employ the measured output only at discrete-time instants, which are a priori unknown, that is the estimation algorithm is actually governed by an event-triggered mechanism (see Aström and Bernhardsson (2002) for further details). It is worthwhile to notice that for the periodic sampling case, several solutions are shown in the literature, (see for example Maroni et al. (2000)).

In this paper, we focus on the estimation problem for linear systems where the input injected into the plant is known and the measured output is gathered in an intermittent fashion. Building from the idea in Raff and Allgöwer (2007), we propose an open-loop observer along with a suitable event-triggered updating of the estimated state. Since the evolution of the considered observer exhibits both continuous-time behavior and instantaneous updating, we provide a hybrid model of the observer including the triggering logic. Then, using a Lyapunov function, we propose a condition that guarantees global uniform asymptotic stability (UGAS) of a closed set including the points for which the state of the plant and its estimate coincide. Furthermore, a computationally tractable design procedure for the proposed observer is presented and illustrated in an example.

The proposed hybrid model allows us to effectively exploit the properties of the time domain of the solutions to the resulting hybrid system, in particular, the persistence of jumps. This feature not only provides a tighter understanding of the system behavior but also enables us to construct a more general Lyapunov function, so as to
overcome the convexity issues induced by non-uniformity in sampling time, which are also pointed out in Raff and Allgöwer (2007), and, moreover, to characterize the effect of measurement noise via input-to-state stability.

The paper is organized as follows. Section II presents the system under consideration, the problem we intend to solve, and the hybrid modeling of the proposed observer. Section III is dedicated to the main results, which provide a solution to the stated estimation problem. Section IV is devoted to numerical issues and provides a convex design algorithm for the proposed observer. In Section V, the effectiveness of the approach is illustrated through a numerical example. Due to space limitations, proofs of the results will be published elsewhere.

Notation: The set $\mathbb{N}_0$ is the set of the positive integers including zero and $\mathbb{R}_{>0}$ represents the set of the nonnegative real scalars. For every complex number $\omega$, $\Re(\omega)$ and $\Im(\omega)$ stand respectively for the real and the imaginary part of $\omega$. $I$ denotes the identity matrix whereas $0$ denotes the null matrix (equivalently the null vector) of appropriate dimensions. For a matrix $A \in \mathbb{R}^{n \times n}$, $A'$ denotes the transpose of $A$ and $\|A\|$ denotes the Euclidean induced norm. $\text{He}(A) = A + A'$. For two symmetric matrices, $A$ and $B$, $A > B$ means that $A - B$ is positive definite. In partitioned symmetric matrices, the symbol $\|$ stands for symmetric blocks. The matrix diag$(A_1; \ldots; A_n)$ is the block-diagonal matrix having $A_1, \ldots, A_n$ as diagonal blocks. For a vector $x \in \mathbb{R}^n$, $x'$ denotes the transpose of $x$, whereas $\|x\|$ denotes the Euclidean norm. For a function $s \in [0, +\infty) \to \mathbb{R}^n$, $\|s\| = \sup_{t \in [0, \|t\|]} \|s(t)\|$. Let $X$ be a given set, $\text{Co}(X)$ represents the convex hull of $X$. $\delta B$ is the closed ball with radius $\delta$ of appropriate dimension in the Euclidean norm. A function $\alpha: \mathbb{R}_{>0} \to \mathbb{R}_{>0}$ is said to belong to the class $\mathcal{K}$ if it is continuous, zero at zero, and strictly increasing. A function $\beta: \mathbb{R}_{>0} \to \mathbb{R}_{>0}$ is said to belong to class $\mathcal{R}$ if, for each $r$, $\beta_{\cdot, \cdot, \cdot, r}$ and $\beta_{\cdot, r, \cdot}$ belong to class $\mathcal{K}$.

2. PROBLEM STATEMENT

2.1 System description

Consider the following continuous-time linear system:

$$
\dot{z} = Az + Bu \quad y = Mz
$$

(1)

where $z \in \mathbb{R}^n$, $y \in \mathbb{R}^r$ and $u \in \mathbb{R}^p$ are, respectively, the state, the measured output, and the input of the system, while $A, B$ and $M$ are constant matrices of appropriate dimensions. Assume also that the input $u$ belongs to the class of the measurable and locally bounded functions $u: [0, \infty) \to \mathbb{R}^p$. We want to design an observer providing an estimate $\hat{z}$ of the state $z$ when the output $y$ is available only at some times $t_k$, for $k \in \mathbb{N}_0$, not known a priori. Figure 1 illustrates such a setting in the context of network control. Suppose that $\{t_k\}_{0}^{\infty}$ is a strictly increasing unbounded real sequence of times. Furthermore, assume that there exist two positive real scalars $T_1, T_2$ with $T_1 < T_2$ such that

\footnote{Concerning this assumption, see Postoyan et al. (2011); Briat and Seuret (2012) and the references therein. Notice that, as pointed out in Raff and Allgöwer (2007), condition (2) prevents the existence of accumulation points in the sequence $\{t_k\}_{0}^{\infty}$, and, hence, it avoids the existence of Zeno behaviors, which is typically undesired in practice.}

\begin{equation}
\begin{array}{c}
T_1 \leq t_{k+1} - t_k \leq T_2.
\end{array}
\end{equation}

(2)

Since the information on the output $y$ is available in an impulsive fashion, motivated by the work of Raff and Allgöwer (2007), to solve the considered estimation problem, we design an observer with jumps in its state following the law:

$$
\dot{\hat{z}} = A\hat{z} + Bu \quad \text{when } t \notin \{t_k\}_{0}^{+\infty}
$$

(3a)

$$
\hat{z}(t_k^+) = \hat{z}(t_k) + L(y(t_k) - M\hat{z}(t_k)) \quad \text{when } t \in \{t_k\}_{0}^{+\infty}
$$

(3b)

where $L$ is a real matrix of appropriate dimensions to be designed. It is worthwhile to point out that in Sur and Paden (1997) the same observer is adopted to state estimation in presence of quantized measurement. Following the lines of Sanfelice and Praly (2012), the state estimation problem can be formulated as a set stabilization problem. Namely, define

$$
A_k = \{(z, \hat{z}) \in \mathbb{R}^{2n}: z = \hat{z} \}
$$

(4)

our goal is to design the matrix $L$ such that $A_k$ is globally asymptotically stable for the plant (1) interconnected with the observer in (3a). At this stage, as usual in estimation problems, one considers the estimation error defined as $\varepsilon := z - \hat{z}$, so the error dynamics are given by the following dynamical system with jumps:

$$
\dot{\varepsilon} = A\varepsilon \quad \text{when } t \notin \{t_k\}_{0}^{+\infty}
$$

(5a)

$$
\varepsilon(t_k^+) = (I - LM)\varepsilon(t_k) \quad \text{when } t \in \{t_k\}_{0}^{+\infty}.
$$

(5b)

Due to the linearity of the system (1), the estimation error dynamics and the dynamics of $z$ are decoupled. Then, for the purpose of stabilizing the set $A_k$, one can effectively just consider system (5).

Remark 1. Notice that assuming the knowledge of the input is not overly restrictive. Indeed, in many practical settings, all of the devices employed to control and supervise the plant may be embedded into the same system. This situation is depicted in Figure 1, where the dotted arrows denote impulsive data streams, while the solid arrows denote continuous data streams. Notice also that, often, the estimated state is part of a feedback controller (e.g. in linear observer-based controller architectures), in which case the input $u$ is a static function of the estimated state that is perfectly known.

2.2 Hybrid modeling

The fact that the observer experiences jumps when a new measurement is available suggests that the updating also in Hetel et al. (2012), condition (2) prevents the existence of accumulation points in the sequence $\{t_k\}_{0}^{\infty}$, and, hence, it avoids the existence of Zeno behaviors, which is typically undesired in practice.
process of the error dynamics can be described via a
hybrid system. Due to this, we represent the whole system
composed by the plant (1), the observer (3), and the logic
triggering jumps as a hybrid system (see Li and Sanfelice
(2013) where similar techniques are adopted to model a
finite time convergent observer).

Such a hybrid systems approach requires to model the
hidden time-driven mechanism triggering the observer
jumps. To this end, we augment the system state with an auxiliary timer variable \( \tau \), which keeps track of the
duration of flows and triggers a jump whenever a
certain condition is verified. This additional state allows
to describe the time-driven jump triggering mechanism as
a state-driven jump triggering mechanism, which leads
to a model that can be efficiently represented by relying
on the framework for hybrid systems proposed in Goebel et al. (2012). More precisely, we make \( \tau \) to decrease
as ordinary time \( t \) increases and, whenever it reaches zero,
triggers a jump that makes a self reset of \( \tau \). In fact, after
a jump occurs, \( \tau \) is re-initialized to some value belonging
to the interval \([0, T_2]\), and, after the reset, it flows again.

Therefore, the whole system composed by the state \( \varepsilon \)
and the timer variable \( \tau \) can be represented by the following
hybrid system:

\[
\begin{aligned}
\dot{\varepsilon} &= A\varepsilon \\
\dot{\tau} &= -1 \\
\varepsilon^+ &= (1 - LM)\varepsilon \\
\tau^+ &\in [T_1, T_2]
\end{aligned}
\]

with the flow set and the jump set defined as
\[
C = \{(\varepsilon, \tau) \in \mathbb{R}^{n+1}; \tau \in [0, T_2]\} \\
D = \{(\varepsilon, \tau) \in \mathbb{R}^{n+1}; \tau = 0\}
\]

For this system, we denote by \( \bar{x} = [\varepsilon^T \ \tau^T]^T \) the state and by \( f \) and \( G \), respectively, the flow map and the jump map,
\( i.e., \)
\[
\begin{aligned}
f(\bar{x}) &= \begin{bmatrix} A\varepsilon \\ -1 \end{bmatrix} \\
G(\bar{x}) &= \begin{bmatrix} (1 - LM)\varepsilon \\ [T_1, T_2] \end{bmatrix}
\end{aligned}
\]

Notice that to make the hybrid system (6) an accurate
description of the real time-triggered phenomenon, which
governs the feedback update process, the variable \( \tau \) needs
to belong to the interval \([0, T_2]\), property that is guaran-
teed by the definition of \( C \) and \( D \). Then, the stabilization
objective can be formalized by introducing the set \( A \)
\[
A = \{(\varepsilon, \tau) \in \mathbb{R}^{n+1}; \varepsilon = 0, \tau \in [0, T_2]\}
\]

Then, the problem we intend to solve can be formulated
as follows:

**Problem 1.** Given the matrices \( A, B, \) and \( M \) of appropriate dimensions and two positive scalars \( T_1 \) and \( T_2 \) such
that \( T_1 < T_2 \), compute a matrix \( L \in \mathbb{R}^{n \times n} \) such that
the set \( A \) defined in (8) is Uniform Global Asymptotically Stable (UGAS) for the hybrid system (6).

\( ^2 \) Since \( A \) is closed, given a vector \( x \in \mathbb{R}^{n+1} \), the distance of \( x \) from
\( A \) is defined as follows:
\[
|x|_A = \inf_{y \in A} \|x - y\|.
\]

It turns out that for every \( \bar{x} \in C \cup D \cup G(D) \), \( |\bar{x}|_A = \|\varepsilon\| \).

About the notion of UGAS of a given set for a generic
hybrid system \( \mathcal{H} \), we consider the definition provided in
(Goebel et al., 2012, Definition 3.6). Concerning the exist-
ence of solutions to system (6), relying on the concept of
solution proposed in (Goebel et al., 2012, Definition 2.6), it
is straightforward to check that for every initial condition
\( \bar{x}(0, 0) \in C \cup D \), every solution to \( \mathcal{H} \) is complete. In ad-
dition, we can characterize the domain of these solutions.

Indeed, the variable \( \tau \), acting as a timer, guarantees that
for every initial condition \( \bar{x}(0, 0) \in C \cup D \), at least for
\( j \geq 1, t_{j+1} - t_j \in [T_1, T_2] \). Therefore, the domain of a
solution \( \phi \) to \( \mathcal{H} \) can be written as follows:

\[
\text{dom} \phi = ([t_0, t_1] \times \{0\}) \cup \left( \bigcup_{j \in \mathbb{N}} ([t_j, t_{j+1}] \times \{j\}) \right)
\]

\[
T_1 \leq t_{j+1} - t_j \leq T_2 \quad \forall j \in \mathbb{N} \setminus \{0\} \\
0 \leq t_1 - t_0 \leq T_2
\]

where \( \text{dom} \phi \) is the domain of \( \phi \), which is a hybrid time
domain. It should be noticed that the structure of the
foregoing hybrid time domain implies that
\[
t \leq T_2(j + 1) \quad \forall (t, j) \in \text{dom} \phi.
\]

3. MAIN RESULTS

3.1 Conditions for Uniform Global Asymptotic Stability

The following result provides conditions for the UGAS
of the set \( A \) defined in (8) for system (6). These conditions
ensure that the assumptions of the Lyapunov result for hy-
brid systems presented in (Goebel et al., 2012, Proposition
3.24) hold.

**Theorem 1.** Given two positive scalars \( T_1 \) and \( T_2 \) such that
\( T_1 < T_2 \), if there exist a symmetric positive definite matrix
\( P \in \mathbb{R}^{n \times n} \) and a matrix \( L \in \mathbb{R}^{n \times n} \) such that
\[
(1 - LM)'e^{At}Pe^{At}(1 - LM) - P < 0, \quad \forall v \in [T_1, T_2],
\]
then the set \( A \) defined in (8) is UGAS for the hybrid system (6).

**Remark 2.** Notice that assuming relation (11) to hold
implies that the eigenvalues of \( e^{At}(1 - LM) \) are strictly
contained in the unit circle for every \( v \) belonging to \([T_1, T_2] \).

In Section 4, we provide a design procedure, including an
algorithm.

3.2 Effect of measurement noise

Until now, the measured output \( y \) was assumed to
be perfectly known at sampling times \( t_k \). However, in
real-world settings, the measured output is affected by
measurement noise. Hence, having some insight on the
robustness of hybrid system (6) with respect to a bounded
measurement noise is undoubtedly useful.

To this end, denoting the measurement noise as \( \eta : [0, +\infty) \rightarrow \mathbb{R} \), with \( \delta \geq 0 \) the measured output is defined by
\[
y = Mx + \eta.
\]

Then, the hybrid system (6) is rewritten as follows:
of the hybrid system only has perturbations on the jump map. On the other
hand, since by Theorem 1 we exhibit the ISS property guaranteed by Theorem 2
for the hybrid system (12). Hence, this result, we show that condition (11) actually suffices to
guarantee the ISS property for the hybrid system (12), which requires the matrix
$P$ to the perturbed system $H_\eta$ from $M$ satisfies, for all $(t,j) \in D_\phi$, $|\phi(t,j)|_{4K} \leq \kappa(\phi(t,j),t,j) + \omega$. It is
worthwhile to remark that getting a hybrid system exhibiting the above mentioned well-posedness property
may not be trivial and it actually derives from suitable choices done throughout the modeling stage.

4. NUMERICAL DESIGN PROCEDURE

In the previous section, a condition to establish the
UGAS and ISS properties, respectively, for systems (6)
and (12) was provided. However, due to its form, such a
condition is not computationally tractable to obtain a
solution to Problem 1. Indeed, from a numerical standpoint,
condition (11) has two drawbacks: it is not convex in
$v$, the relevance of the second drawback is evident
at a first sight, while the lack of convexity is a severe
constraint, since non-convex problems often lead to
NP-hard problems; see, for example, Boyd et al. (1997). Thus,
in order to make the problem numerically tractable, some
manipulations are needed. To this end, the following result
provides a first step toward a convex design procedure for
the proposed observer.

Proposition 1. Let $T_1$ and $T_2$ be two given positive scalars
such that $T_1 < T_2$. If there exist a symmetric positive
definite matrix $P \in \mathbb{R}^{n \times n}$, a matrix $J \in \mathbb{R}^{q \times n}$, and a matrix $F \in \mathbb{R}^{n \times n}$ such that for every $v \in [T_1,T_2]$

$$
\begin{bmatrix}
-He(F) & F - JM & e^{Av}P \\
* & -P & 0 \\
* & * & -P
\end{bmatrix} < 0
$$

(15)

then the matrices $P$ and $L = F^{-1}$ satisfy condition (11).

Remark 4. Notice that condition (15) is convex with respect to
the unknown matrices $F$, $L$, and $P$.

To efficiently design the observer, one needs to avoid
finding a solution to (15) for infinitely many values of $v$.
To overcome this issue, we propose to embed the term $e^{Av}$, with $v$ in the interval $[T_1,T_2]$, in a convex set, obtaining
in this way a convex design procedure composed by a
finite number of inequalities. This technique consists in
finding some matrices $X_1, X_2, \ldots, X_p \in \mathbb{R}^{n \times n}$, such that $e^{Av} \in \mathcal{C}(X_1, X_2, \ldots, X_p)$ whenever $v \in [T_1,T_2]$.

To this end, consider the following well known expression

$$
e^{Av} = \sum_{i=1}^{\sigma_c} \sum_{j=1}^{m_i} R_{ij} e^{\lambda_i v} \frac{v^{j-1}}{(j-1)!} + \sum_{i=1}^{m^*_c} 2e^{\Re(\lambda_i) v} \left( \Re(\lambda_i) \cos(\Im(\lambda_i) v) \right) v^{j-1} - \Im(\lambda_i) \sin(\Im(\lambda_i) v) \frac{v^{j-1}}{(j-1)!}
$$

(16)

where $\sigma_c$ is the number of distinct eigenvalues, $\sigma_c$ the number of distinct complex-conjugate eigenvalue pairs.
The constants $m_i$ and $m_e$ are, respectively, the multiplicity of the real eigenvalue $\lambda_i$ and of the complex-conjugate eigenvalue pair $\lambda_i, \lambda_i^*$ in the minimal polynomial of the matrix $A$. The matrices $R_{ij}$ are real $n \times n$ matrices corresponding to the residuals associated to the partial fraction expansion of $(sI - A)^{-1}$. Notice that several methods can be adopted to compute such matrices. In this work, we rely on the procedure proposed in Leyva-Ramos (1993). Once the value of the residuals are known, to build a polytopic embedding of $e^{At}$ one can proceed in a similar manner of Heemels et al. (2010). Namely,

$$
\gamma_{ij} \mathcal{M}(R_{ij}); \beta_{ij} \in \{\beta_{ij}, \beta_{ij}^*\}, \gamma_{ij} \in \{\gamma_{ij}, \gamma_{ij}^*\}
$$

(17)

where

$$
\beta_{ij} = \max_{v \in [T_i, T_j]} e^{\lambda_i v} v^{j-1} (j-1)!,
$$

$$
\beta_{ij}^* = \min_{v \in [T_i, T_j]} e^{\lambda_i v} v^{j-1} (j-1)!,
$$

$$
\gamma_{ij} = \max_{v \in [T_i, T_j]} 2e^{\Re(\lambda_i v)} \cos(\Im(\lambda_i v)) v^{j-1} (j-1)!,
$$

$$
\gamma_{ij}^* = \min_{v \in [T_i, T_j]} 2e^{\Re(\lambda_i v)} \cos(\Im(\lambda_i v)) v^{j-1} (j-1)!,
$$

(18)

The proposed technique leads to the following result.

**Corollary 1.** Let $T_1$ and $T_2$ be two given, positive scalars such that $T_1 < T_2$. Let $\{X_1, \ldots, X_n\}$ be the matrices obtained by (17). If there exist a symmetric, positive definite matrix $P \in \mathbb{R}^{n \times n}$, a matrix $J \in \mathbb{R}^{q \times n}$, and a matrix $F \in \mathbb{R}^{n \times n}$ such that, for every $i = 1, \ldots, \nu$,

$$
[ -\text{He}(F) F - JM \; X_i P \; \star \; \star \; -P ] < 0
$$

(19)

then the matrices $P$ and $L = F^{-1} J$ satisfy condition (11).

Corollary 1 represents an efficient solution to Problem 1, which finally can be solved by Algorithm 1, which is given below.

**Algorithm 1: Observer design**

1: Find the residual matrices $R_{ij}$ in (16)
2: Compute the scalars $\beta_{ij}, \beta_{ij}^*, \gamma_{ij}, \gamma_{ij}^*$ as in (18)
3: Compute the matrices $\{X_1, \ldots, X_n\}$ as in (17)
4: Solve (19) with respect to $J$, $P$ and $F$
5: $L \leftarrow F^{-1} J$
6: return $L$

5. ILLUSTRATIVE EXAMPLE

Consider the mass-spring system proposed by Geromel and de Oliveira (2001), which is defined by the following equations:

$$
A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -2 & 1 & -1 & 0 \\ 2 & -2 & 0 & -2 \end{bmatrix}, \quad M = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}
$$

(20)

$$
B' = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}
$$

(21)

Figure 2 depicts the projection onto ordinary time $t$ of the states $z(t, j)$ and $\tilde{z}$. In this simulation, the sampling instants are selected randomly in the interval $[T_1, T_2]$ according to a standard Gaussian distribution. Simulations show that the estimates appear to quickly converge toward the plant state $z$ since the estimate $\tilde{z}$ and the state $z$ are nearly overlapped after three jumps.

6. CONCLUSION

This paper proposed a methodology to model and design, through a convex problem, an event-triggered observer to estimate the state of a linear plant whenever the output is measured in an impulsive fashion. Moreover, the proposed observer is shown to be ISS with respect to measurement noise and having a degree of robustness with respect to small enough bounded perturbations. The results in this paper suggest several directions of research on event-triggered observers. For example, the setting allows to consider a design problem for the updating logic of $\tau$, in order to somehow schedule the sampling instants. Moreover, the design of an observer-based controller in the presence of impulsive output measurement represents certainly an interesting outlook.

REFERENCES


Fig. 2. The evolution of the states $z$ and $\dot{z}$ projected onto ordinary time $t$.

(a) Projection onto ordinary time $t$ of $z_1(t,j)$ (solid) and $\dot{z}_1(t,j)$ (dashed).

(b) Projection onto ordinary time $t$ of $z_2(t,j)$ (solid) and $\dot{z}_2(t,j)$ (dashed).

(c) Projection onto ordinary time $t$ of $z_3(t,j)$ (solid) and $\dot{z}_3(t,j)$ (dashed).

(d) Projection onto ordinary time $t$ of $z_4(t,j)$ (solid) and $\dot{z}_4(t,j)$ (dashed).


