A DIDACTIC INPUT-OUTPUT MODEL
FOR TERRITORIAL ECOLOGY ANALYSES

Garry McDONALD

FINAL DRAFT

A Didactic Input-Output Model for Territorial Ecology Analyses

(Un Modèle Didactique Input-Output en Ecologie Territoriale)

Prepared by Dr Garry McDonald[1]

with support from

Tania Armand[2], Richard Loiret[2], Samy Chabot[2], Mateo Cordier[2],

Johann Audrain[3] and Prof Martin O'Connor[2]

[1]Director, Market Economics Ltd and Honorary Research Fellow, Massey University

[2]REEDS, Universite de Versailles

[3]Fondaterra.

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1   Introduction

This report describes a didactic input-output modelling framework created jointly be the team at REEDS, Universite de Versailles and Dr Garry McDonald, Director, Market Economics Ltd. There are three key outputs associated with this framework: (i) a suite of didactic input-output models developed in Microsoft Excel, (ii) a technical report (this report) which describes the framework and the suite of models[1], and (iii) a two week intensive workshop dedicated to the training of REEDS researchers in conceptualisation, development and application of the didactic input-output framework and its models.

As part of the development of its multimedia tools, methods and case studies in territorial ecology, REEDS contracted Market Economics Ltd to develop and document a suite of didactic input-output models. These models are implemented at a national/sub-national scale[2] through the use of appropriate input-output data sets, covering economic sectors (agriculture, forestry and mining; manufacturing' construction and utilities; wholesale and retail trade; services), primary inputs (household income, imports other primary inputs), final demands (household consumption, exports and other final demands) and various (selectable) environmental resources/residuals (land use, water use, solid waste, recycling, energy, energy-related air emissions, ecosystem service appropriation, and so on).

The development of the didactic input-output modelling framework includes several operational input-output models in static and comparative static formulations which calibrate 'typical' patterns of economic activity in 'Western' industrial commodity economies, and demonstrate the principle of re-calibration with typical SNA and SEEA-style accounts. The focus of the didactic models is on economy-economy, environment-economy and economy-environment exchanges, but excludes environment-environment exchanges. Examples of specific applications of the models include: (1) pressure-based footprint type analyses, (2) sector interdependence and environment-economy tradeoffs, (3) and short-medium term comparative static projection modelling.

As noted, the didactic input-output models were developed collaboratively in Microsoft Excel format by Dr McDonald and the REEDS research team using clear and concise step-by-step methodological sequences. There are two key reasons supporting this collaborative and structured approach: (1) all of the REEDS researchers (with the exception of Prof Martin O'Connor) are undertaking, and mostly just beginning, PhD studies which may use input-output approaches – the development of a synergistic and collaborative networks between these researchers was sought; and (2) the use of a structured approach ensured that the models may be easily replicated in a variety of national, regional and territorial scale analyses – not only by the named researchers, but also by others that may follow (i.e. the models, along with this report, provide a teaching/training aid). It is, however, important to note that the models provide only a guide which may, or may not depending upon the inclination of the researchers, facilitate complementary or extensional work.

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[1] This includes supporting technical documentations which allow an experienced mathematical modeler/programmer to the didactic models, and their associated algorithmic implementations, into different functional forms.

[2] The models were implemented using input-output tables for both New Zealand and France. The models for France included applications both at the national and regional/department/aggregated commune scales.

The report is divided into five sections:

- *A Brief History of Input-Output Economics*. This section provides a brief history of Input-Output Analysis and its antecedents.
- *The Supply-Use Tables (SUTs)*. Underpinning the didactic input-output modelling framework is the Supply-Use Table framework. This section describes the basic structure of the SUT including the derivation of Symmetric Input-Output Tables (SIOTs). A 5 commodity by 5 sector example of the conversion is included in the Didactic Input-Output Model Excel workbook.
- *Symmetric Input-Output Tables (SIOTs) and Economic Multipliers*. This section includes description of the input-output transactions table, technical coefficients table, Leontief matrix, open and closed Leontief inverse matrices, and the derivation of input-output multipliers. The SIOTs developed focus on evaluation of not only the direct effects of changes in final demand, but also on the indirect (i.e. through backward linkage or upstream effects) and induced (i.e. resulting from increases in consumer spending associated with the direct and indirect effects) of such changes. The analytical derivation of the SIOT model is described in an accompanying 5 sector SIOT example included in the Didactic Input-Output Model Excel workbook.
- *Ecological Input-Output Table (EIOTs), Ecological Multipliers and the Pressure-Based Footprint Analyses*. This section includes an overview of the main types of ecological input-output tables, along with key analytical features of these tables, the calculation of direct and indirect resource/residual resources matrices using the Leontief Inverse Matrix, the calculation of Pressure-Based footprints, and associated economy-environment tradeoffs. This includes the developed of a Cumulative Effects Indicator which direct and indirect resource inputs/residual outputs to the amount of direct and indirect value added generated. The analytical derivation of the EIOT model is described in an accompanying 5 sector EIOT example included in the Didactic Input-Output Model workbook.
- *Comparative Static Analysis*. This section provides a brief introduction to comparative statics analysis using input-output tables. The SIOT and EIOT 5 sector analytical examples are extended to show how a comparative static analysis may be implemented – this is available in the Workshop Didactic Input-Output Model workbook.

A final note on extensions

It should be noted that the didactic input-output provides only a basic understanding of the applications of input-output analyses in relations to studying the economy and environment. There are many additional extensions which could be undertaken using the framework – these were discussed during the workshop. In particular, expansions of the SUT framework to include (1) investment and labour market feedbacks such as those seen in Social Accounting Matrices (SAMs), Computable General Equilibrium (CGE) and Dynamic Input-Output Tables (DIOTs) (for examples refer to Duchin and Szyld (1985) and Leontief and Duchin (1986)); (2) complete physical compilations – Physical Input Output Tables (PIOTs), (3) Multi-Regional Input-Output Tables (MRIO) and (4) Decomposition Analysis. For

further information the reader is directed to Miller and Blair (2009) for a comprehensive overview of possible extensions.

2   A Brief History of Input-Output Economics

The origins of input-output modelling may be traced back to the Physiocrats of the 18th Century. François Quesnay's Tableau Economique of 1758 traced successive rounds of wealth generated by agricultural expenditure. Although the Tableau Economique investigated the concepts of circular flow and general equilibrium, it was not until another Frenchman, León Walras in his *Elements d'Economie Politique Pure* of 1874, that a detailed theoretical framework for analysing economic interdependence was created. Walras developed a theory of general equilibrium that utilised production coefficients to relate the quantities of factors required to produce a unit of product to levels of total production of that product. Contemporary input-output economics is attributed to Wassily Leontief, a Nobel prize-winning American economist, who in 1936 published an input-output table for the American economy. Leontief simplified the Walras model to develop a linear approximation based on the general equilibrium concept of economic interdependence (Miller and Blair, 1985).

3 The Supply Use Framework

In this section the Supply and Use Table (SUTs) framework is described, along with the way in which SUTs may be converted into SIOTS. The SUT framework is a commodity-by-industry accounting framework originally proposed by Stone (1961, 1966), in which the number of commodities are typically greater than the number of industries. The matrix structure is compatible with the supply-use framework recommended for national accounting by the United Nations (1999) and followed by most countries in the derivation of their SUTs and SIOTs. The matrix, depicted in Figure 1, consists of nine sub-matrices (**R**, **S**, **T**, **U**, **V**, **W**, **X**, **Y** and **Z**) and five vectors (**α**, **β**, **δ**, **ε**, and **ζ**). Capital letters are used to denote matrices, while lower case letters are used to denote vectors and scalars. These components are described in detail below.

| | | Commodities 1 … B | Industries 1 … Γ | Final Demands | | | Total Use |
				Household Consumption 1 … Δ	Other Final Demands 1 … Θ	Exports 1 … Λ	
Commodities	1 … B		**U** Use	**S** Household Consumption	**T** Other Final Demands	**X** Exports	**α** Gross Commodity Outputs
Industries	1 … Γ	**V** Supply					**β** Gross Industry Outputs
Primary Inputs — Value Added	1 … Π		**W** Value Added	**R** Value Added to Household Consumption	**Z** Value Added to Other Final Demands		**δ** Value Added
Primary Inputs — Imports	1 … Φ	**Y** Imports					
Total Supply		**α'** Gross Commodity Inputs	**β'** Gross Industry Inputs	**ε** Household Consumption	**ζ** Other Final Demand		

Figure 1   Supply Use Table Framework

3.1 Component Matrices, Vectors, Scalars and Identities

The commodity accounts are comprised of matrices **U** (B×Γ), **S**, **T**, **X** and vector **α**. An element $u_{ij} \in$ **U** represents the value of commodity i used by industry j (i = 1 … **B**; j = 1 … **Γ**) within a given financial year.

Each column in matrix **U** shows the inputs used by industries classified according to the type of commodity used, while each row shows the inputs of each commodity according to the industries that use it. Matrix **U** is commonly referred to as the 'use', 'industry' or 'absorption' matrix.

Element $s_{ij} \in$ **S** represents the consumption of commodity i by household category j within a given financial year. This includes consumption of consumer durables and non-marketable governmental services[3] by households.

Element $t_{ij} \in$ **T** denotes the consumption of commodity i by other final demand category j within a given financial year. The following other final demand categories are covered by matrix **T**: gross fixed capital formation and changes in inventories.

Element $x_{ij} \in$ **X** denotes the consumption of commodity i by export region j within a given financial year. Depending on the destination, exports are grouped as either international (i.e. heading to other nations) or interregional (i.e. heading to other regions). Together matrices **S**, **T** and **X** represent a complete set of final demand categories.

Element $\alpha_i \in$ **α** gives the total value of commodity i output as supplied to all industries and final demand categories. Vector **α** is referred to as 'gross commodity output'. It is calculated by summing the elements of commodity i in matrices **U**, **S**, **T** and **X**. Thus,

$$\alpha_i = \sum_{j=1}^{\Gamma} u_{ij} + \sum_{j=1}^{\Delta} s_{ij} + \sum_{j=1}^{\Theta} t_{ij} + \sum_{j=1}^{\Lambda} x_{ij} \, , \; \forall \, i, i = 1 \ldots \text{B}. \tag{1}$$

Letting i denote an appropriately dimensioned column-summing vector, Equation 1 may be rewritten as,

$$\textbf{Ui} + \textbf{Si} + \textbf{Ti} + \textbf{Xi} \equiv \textbf{α}. \tag{2}$$

The production relationships within the economy are captured in matrix **V**. An element, $v_{ij} \in$ **V**, represents, in basic prices, the output of commodity j produced by domestic industry i within a given financial year. In this way, matrix **V** describes the sources of supply of products to the economy. Each row i shows the production of a particular industry classified according to the type of commodity produced, while each column j shows the production of a commodity according to the industries that produced it. This matrix is commonly referred to as the 'supply', 'production' or 'make' matrix.

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[3] This includes the value of the goods and services provided by the producers of government services for consumption by the community e.g. benefits and pensions, primary and secondary school education, and public health care. A convention is adopted that the government itself is the consumer on behalf of the community.

Element $y_{ij} \in \mathbf{Y}$ denotes imports of commodity j from region i. Depending on their origin, imports are grouped as international (i.e. from abroad) or interregional (i.e. from other regions).

By summing column j of matrices $\mathbf{V}$ and $\mathbf{Y}$ the total domestic commodity j output, $\mathbf{\alpha}_j$, may be derived, while summing row i of matrix $\mathbf{V}$ produces the total domestic industry i output, $\mathbf{\beta}_i$ (know as 'gross industry output'). Hence,

$$\mathbf{i'V} + \mathbf{i'Y} \equiv \mathbf{\alpha'} \tag{3}$$

and

$$\mathbf{Vi} \equiv \mathbf{\beta}. \tag{4}$$

Together Equations 2 and 3 fulfil the first key principle in balancing supply and use tables – the supply of a commodity, $\mathbf{\alpha i}$, must be equal to the use of that commodity, $\mathbf{\alpha'j}$, where $i = j$.

The components of value added are recorded in matrix $\mathbf{W}$. Value added components include compensation of employees, operating surplus, consumption of fixed capital, taxes on production and subsidies. The element $w_{ij} \in \mathbf{W}$ denotes the value added by component i to the economy in producing column j's industry output within a given financial year. Summing all commodity inputs made to an industry, $\mathbf{i'U}$, with the primary inputs made to that same industry, $\mathbf{i'W}$, derives an estimate of $\mathbf{\beta'}$ (known as 'gross industry input'),

$$\mathbf{i'U} + \mathbf{i'W} \equiv \mathbf{\beta'}. \tag{5}$$

This fulfils the second key principle in balancing supply and use matrices – that the total output of an industry, $\mathbf{\beta}_i$, must be equal to its cost of production, $\mathbf{\beta'}_j$, where $i = j$. Equations 1 to 5 form the key economic flow accounting identities of the commodity-by-industry model.

Definitions for matrices $\mathbf{R}$ and $\mathbf{Z}$, and the remaining three vectors $\mathbf{\sigma}$, $\mathbf{\epsilon}$ and $\mathbf{\zeta}$, are however required to complete the economic flow model. Element $r_{ij} \in \mathbf{R}$ represents the expenditure on value added

component *i* by household consumption category *j* within a given financial year.[4] This includes non-market transfers such as benefits and pensions. Element $z_{ij} \in \mathbf{Z}$ denotes the expenditure on value added component *i* by final demand category *j* within a given financial year. Matrix $\mathbf{Z}$ is a sparsely populated matrix consisting of commodity and non-commodity indirect taxes on products sold directly to capital formation or stored in stocks[5]. Closely associated with matrices $\mathbf{R}$ and $\mathbf{Z}$ are vectors $\mathbf{\sigma}$, $\mathbf{\varepsilon}$ and $\mathbf{\zeta}$.

Element $\sigma_i \in \mathbf{\sigma}$ represents the total value of value added component *i* supplied to all industries and final demand categories, thus,

$$\mathbf{Wi} + \mathbf{Ri} + \mathbf{Zi} \equiv \mathbf{\sigma}. \tag{6}$$

Element $\varepsilon_j \in \mathbf{\varepsilon}$ shows the total expenditure on commodities and value added components by household category *j*, hence,

$$\mathbf{i'S} + \mathbf{i'R} \equiv \mathbf{\varepsilon}. \tag{7}$$

Similarly, element $\zeta_j \in \mathbf{\zeta}$ gives the total expenditure on commodities of value added components by other final demand category *j*, thus,

$$\mathbf{i'T} + \mathbf{i'Z} \equiv \mathbf{\zeta}. \tag{8}$$

3.2   Gross Domestic Product and Expenditure

An accounting identity equating Gross Domestic Product (GDP) with Gross Domestic Expenditure (GDE) may also be formulated from Figure 1. First, GDP is derived by summing the elements of matrices $\mathbf{W}$, $\mathbf{R}$ and $\mathbf{Z}$[6], thus,

$$\text{GDP} = \sum_{ij} w_{ij} + \sum_{ij} r_{ij} + \sum_{ij} z_{ij}. \tag{9}$$

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[4] This includes commodities imported by wholesalers/retailers who add a margin and then on-sell to households.
[5] In the French SUT framework the matrix $\mathbf{Z}$ does not, in fact, exist.

[6] Double summations, such as $\displaystyle\sum_{i=1}^{\Pi}\sum_{j=1}^{\Gamma} w_{ij}$, are denoted here as $\displaystyle\sum_{ij} w_{ij}$.

Next, to derive GDE we must first augment several matrices together. Using | to denote the horizontal augmentation operator, household consumption matrix, **S**, other final demands matrix, **T**, exports matrix, **X**, and a transposed and negated imports matrix, -**Y'**, are augmented together to form a new matrix, $\overline{\mathbf{T}}$. Hence,

$$\overline{\mathbf{T}} = \mathbf{S} \mid \mathbf{T} \mid \mathbf{X} \mid \text{-}\mathbf{Y'}. \tag{10}$$

Element $\bar{t}_{ij} \in \overline{\mathbf{T}}$ denotes the value of commodity i sold to final demand category j. In turn, GDE may be derived by summing the elements of matrices $\overline{\mathbf{T}}$, **R** and **Z**, thus,

$$\text{GDE} = \sum_{ij} \bar{t}_{ij} + \sum_{ij} r_{ij} + \sum_{ij} z_{ij}. \tag{11}$$

3.3 Conversion to a Symmetric Input-Output Table

The SUT framework described above has the advantage of being able to account for multiple outputs per industry. However, for the purposes of economic and ecological multiplier analysis, using commodity-by-industry models are often problematic as:

- *Analytical ease of use*. Commodity-by-industry models are often rectangular matrices which means that straightforward matrix algebra (based on square matrices) cannot be used; and
- *Negative coefficients*. Commodity-by-industry models usually have multiple outputs per industry, which inevitably generates negative coefficients. Some commentators (e.g. Almon, 2000) argue that negative coefficients are problematic as they make no economic sense.

In order, therefore, to derive economic multipliers, ecological multipliers and pressure-based footprints, a procedure is required to convert the SUT commodity-by-industry framework into an SIOT industry-by-industry framework.

3.3.1 Commodity Technology and Industry Technology Assumptions

Under ideal circumstances an inter-industry matrix would be derived from source data describing the input structure of every type of activity producing a single commodity. This ensures homogeneity except in those cases where secondary products (i.e. by-products and joint products) are intrinsic to the

production process, e.g. it is physically impossible to separate meat (main product) and offal (by-product) production. Most statistical agencies, however, can only derive inter-industry matrices according to enterprise definitions which, in turn, may be complicated by the presence of secondary products. To minimise the bias resulting from joint/by-products, statistical agencies carefully craft business census questionnaires to utilise itemised cost accounting definitions which aid in separating secondary products. The compliers of SIOTs may utilise, in combination with census questionnaires, approaches such as the 'redefinition method', 'negative transfer method', 'aggregation' or 'positive transfer method' to deal with secondary products (United Nations, 1999). Outside national statistical agencies, however, data availability, confidentiality, cost/time constraints however typically prohibit the use of such techniques.

Crude SIOTS matrices may, however, be generated mathematically by assuming one of two possible production pathways: commodity-based or industry-based technology. Under the commodity technology assumption industries produce commodities in fixed proportions (United Nations, 1999). A given commodity therefore has the same input structure irrespective of the industry that produces it. By contrast, under the industry technology assumption, inputs are consumed in the same fixed proportions independently of the commodity being produced by an industry (United Nations, 1999). Primary and secondary products are therefore assumed to be produced using the same technology.

Various input-output analysts have critiqued each assumption's pros and cons – for example, refer to ten Raa *et al*. (1984), Kop Jansen and ten Raa (1990), Konijn and Steenge (1995) and the United Nations (1999). Kop Jansen and ten Raa (1990, p.214) argue that selection of one assumption over another is simply a "matter of judgement or taste". The United Nations (1999) provides strong theoretical justifications for the adoption of a commodity technology assumption, but acknowledges that due to the presence of joint/by-products negative coefficients may be produced in multiplier calculations, or when key balance identities are not adhered to. Smith and McDonald (2010) have developed a 'comprehensive model' which enables a combination of both the commodity-technology and industry-technology assumptions, with user given weights, while ensuring non-negative coefficients, to be applied to produce not only industry-by-industry, but also commodity-by-commodity SIOTs. Given that the industry technology assumption guarantees positive inter-industry coefficients and has been widely utilised, it is selected here as the approach for generating SIOTs.

3.3.2 Generating a SIOT Using the Industry Technology Assumption

Under an industry technology assumption, the relationship between commodity input and industry output, in matrix **V**, provides the basis for a transformation from commodity to industry space. Such a transformation, and the subsequent generation of an inter-industry matrix, requires several steps.

Step 1 Estimate Gross Commodity Outputs Required for Domestic Purposes

We begin by estimating gross commodity outputs required for domestic purposes, $\bar{\alpha}$, as

$$\bar{\alpha} = \alpha - (\mathbf{i}'\mathbf{Y})' .$$ (12)

Step 2   Calculate Domestic Industry Production of Commodities for Export

This requires construction of a matrix of commodity-by-industry direct requirements coefficients, **B**, by defining a fixed relationship between commodity input and industry output values. Thus,

$$\mathbf{B} = \mathbf{V}\,\hat{\bar{\alpha}}^{-1} ,$$ (13)

followed by estimation of a column vector, **Xi**, of total exports,

$$\gamma = \mathbf{Xi} .$$ (14)

Diagonalising γ and then premultiplying it by **B** yields domestic production of commodities for export,

$$\mathbf{M} = \mathbf{B}\hat{\gamma} .$$ (15)

Step 3   Calculate Total Supply for Domestic Use

Subtracting **M** from **V** gives the domestic supply, and augmenting this with a row for imports gives total supply for domestic use,

$$\mathbf{N} = \begin{pmatrix} \mathbf{V} - \mathbf{M} \\ \mathbf{i}'\mathbf{Y} \end{pmatrix} .$$ (16)

The values of matrix **N** represent the commodities, domestically produced and imported, that are required to support intermediate demand and final domestic demand. Standardising[7] **N** yields a matrix, $\ddot{\textbf{N}}$, that shows the composition of industry and import sources of commodity production for domestic demand with columns summing to one.

Step 4   Transform **U | T** *from Commodity to Industry Space*

Postmultiplying $\ddot{\textbf{N}}$ by **U | T** completes the transformation to industry space. Matrices **W**, **R** and **Z** may then be inserted; no mathematical manipulation of these matrices is required as they are already in the required form. In this way, the entire inter-industry framework may be generated,

$$
\begin{pmatrix}
\ddot{\textbf{N}}\textbf{U}\ \ddot{\textbf{N}}\textbf{T} & \textbf{B}\gamma \\
 & \textbf{0} \\
\textbf{W R Z} &
\end{pmatrix}. \tag{17}
$$

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[7] A … is placed above matrix **N** to indicate that it has been standardised. Mathematically, standardisation is achieved by the following formula $\textbf{N}(\textbf{i}'\textbf{N})^{-1}$.

4　Symmetric Input-Output Tables and Economic Multipliers

4.1　Symmetric Input-Output Tables

To construct a SIOT we must first develop the basic mathematics of input-output analysis (only a brief account is provided here, full details may be found in Richardson (1972), Leontief (1985) and Miller and Blair (2009)). The structure of an industry's production process is represented by a vector of structural coefficients that describe the relationship between the inputs (purchases) it consumes, and the outputs (sales) it produces. Interdependence between industries is described by a set of linear equations expressing the balance between total input, and the aggregate output of each commodity and service produced. For the sake of simplicity we now drop the SUT mathematical notation developed in Section 2; instead choosing to develop a notation dedicated specifically to SIOTs applications.

Thus, if an economy is separated into n industries, and if we denote X_i the total output (sales) of an industry i, and Y_i the final demand for industry i's production, then,

$$X_i = z_{i1} + z_{i2} + \ldots + z_{ij} + z_{in} + Y_i \tag{18}$$

for $i = 1 \ldots n$, and $j = 1 \ldots n$.

The z_{ij} terms represent inter-industry sales from industry i to industry j, and the Y_i term, sales to industry i's final demand (e.g. households, exports, capital formation and net increases in stocks). Taken together, the z terms and Y_i give the total output of industry i, X_i. We may then construct a system of equations for all n industries,

$$X_1 = z_{11} + z_{12} + \ldots + z_{1j} + \ldots + z_{1n} + Y_1$$

$$X_2 = z_{21} + z_{22} + \ldots + z_{2j} + \ldots + z_{2n} + Y_2$$

$$\vdots \tag{19}$$

$$X_i = z_{i1} + z_{i2} + \ldots + z_{ij} + \ldots + z_{in} + Y_i$$

$$\vdots$$

$$X_n = z_{n1} + z_{n2} + \ldots + z_{nj} + \ldots + z_{nn} + Y_n.$$

If we then consider the jth column of z's we have a column vector,

$$\begin{bmatrix} z_{1j} \\ z_{2j} \\ \vdots \\ z_{ij} \\ \vdots \\ z_{nj} \end{bmatrix}$$

(20)

The elements of this vector represent the inputs (i.e. purchases) by industry j, including the purchases of intermediate demands and primary inputs (e.g. wages and salaries, imports, operating surplus, capital depreciation, subsidies and taxes). We now have the basis for the SIOT depicted in Figure 2. A SIOT is conventionally presented in a matrix format with each industry assigned a row and column. The element z_{ij} in row i, column j, represents the volume of goods flowing from industry i to be used as inputs in industry j. Primary data for populating the input-output table are typically obtained from national economic accounts, which are, in turn, derived from a nation's census of production or similar.

	Industry 1　Industry … j …　Industry n	Sub-Total	House-holds　Govt. Expen-diture　Other Final Demand　Exports	Sub-Total	Total Gross Output
Industry 1 Industry … i … Industry n	Quadrant I z_{ij}		Quadrant II Y_i		X_i
Sub-Total					
Labour Operating Surplus Other Primary Inputs Imports	Quadrant III		Quadrant IV		
Sub-Total					
Total Gross Input	X_j				

Figure 2　　　**Symmetric Input-Output Table (SIOT)**

A SIOT may be divided vertically into two parts: the part on the left represents the inputs into the production process of the productive industries, while the part on the right represents the sales to the final demand categories. Each of these parts may be further subdivided horizontally into two sections

so as to distinguish between intermediate inputs and primary inputs. The resulting input-output table consists of quadrants (labelled I to IV in Figure 2).

Quadrant I, known as the processing or intermediate demand quadrant, represents the flows of transactions between "endogenous" industries used in the intermediate stages of production. A key characteristic of the intermediate demand quadrant is that there must be the same number of rows as columns.

Quadrant II displays the sales by each industry to final demand i.e. the part of an industry's output not used by another industry as an input. It describes the consumer behaviour of a number of important markets including household consumption and exports. The column categories are known as 'exogenous' as they are typically influenced by factors external to the economy.

Quadrant III describes the primary inputs used in each industry. These inputs are described as 'primary' because they do not form part of the output of intermediate production i.e. wages and salaries (representing labour), operating surplus, and imports. Summing the primary inputs, and in turn, subtracting imports, provides an estimate of the contribution made to GDP by each industry.

Quadrant IV displays the primary inputs that are directly used by final demand sectors. This includes non-market transfers such as benefits and pensions as well as imports of commodities for consumption by households and investors.

Embedded within the input-output table are several important accounting identities. Two of the most major are: (1) for each industry, $i=j$, total output, X_i, must equate to total input, X_j, and (2) the sum total of the final demand sectors must equate to the sum total of the primary inputs. Furthermore, the intimate relationship shared with national accounts enables standard economic aggregates such as Balance of Trade (i.e. exports less imports), and contribution to Gross Domestic Production (total primary inputs less imports) by each industry to be evaluated.

4.2 Technical Coefficients Table

A critical assumption of input-output analysis is that the inter-industry flow from i to j depends entirely on industry j total output, X_j. Say, for example, an industry j sells computer keyboards; it is assumed that with any increase in the sales of keyboards, there will be a corresponding increase in sales of plastics, metals etc required to create the keyboards. Based on this assumption a ratio of input to output may be formulated, commonly referred to as a technical coefficient. Thus, for a z_{ij}, the sale from row industry i to column industry j, and the total output of industry j, X_j, gives the technical coefficient a_{ij},

$$a_{ij} = \frac{z_{ij}}{X_j} \tag{21}$$

Thus, the a_{ij}'s can be thought of as representing the first round inputs from each row industry i following a unit increase in output of any row industry i per unit of output produced by column industry j. The a_{ij}'s represent fixed relationships between an industry's output and its inputs. Moreover, the relationship is linear – hence, there are no economics or diseconomies of scale, only constant returns to scale. Equation 19 may now be rewritten using the a_{ij}'s,

$X_1 = a_{11}X_1 + a_{12}X_2 + ... + a_{1j}X_i + ... a_{1n}X_n + Y_1$

$X_2 = a_{21}X_1 + a_{22}X_2 + ... + a_{2j}X_i + ... a_{2n}X_n + Y_2$

$\vdots$ (22)

$X_i = a_{i1}X_1 + a_{i2}X_2 + ... + a_{ij}X_i + ... a_{in}X_n + Y_i$

$\vdots$

$X_n = a_{n1}X_1 + a_{n2}X_2 + ... + a_{nj}X_i + ... a_{nn}X_n + Y_n.$

4.3   The Leontief Inverse

A common question answered by input-output analysis is: given a future projection of final demand, the Y_i's, how much output from each industry, the X_i's, would be required to meet the projection? This is a simple matter of solving a set of simultaneous equations where the Y_i's and a_{ij}'s are known, and the X_i's are unknown. In matrix terms, we define,

$$A = \begin{bmatrix} a_{11}a_{12}\cdots a_{1j}\cdots a_{1n} \\ a_{21}a_{22}\cdots a_{2j}\cdots a_{2n} \\ \vdots \\ a_{n1}a_{n2}\cdots a_{nj}\cdots a_{nn} \end{bmatrix}, \quad X = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{bmatrix}, \quad Y = \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{bmatrix}, \quad (23)$$

simplified in matrix notation as:

$(I - A)X = Y,$ (24)

where **I** is an identity matrix, and in turn, by rearrangement our question may then be answered by:

$$\mathbf{X} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{Y}, \text{ providing } |\mathbf{I} - \mathbf{A}| \neq 0, \tag{25}$$

and where $(\mathbf{I} - \mathbf{A})^{-1}$ is known as the Leontief Inverse (or Open Leontief Inverse). With $\mathbf{B} = (\mathbf{I} - \mathbf{A})^{-1}$, each b_{ij} represents not only the *direct*, but also the *indirect* (i.e. flow-on), requirements of industry *i* per unit of final demand for the output of industry *j*. The contribution made by an industry to an economy is thus not solely limited to the output it creates directly – an increase in final demand has repercussions throughout the entire economy, causing indirect increases in output beyond the initial change in final demand. In this way, the Leontief Inverse may be considered a powerful tool capable of capturing total effect (direct plus indirect) resulting from any exogenous final demand change.

4.4   Closed Leontief Inverse Matrix

Closing the Leontief matrix with respect to household income and expenditure permits the calculation of not only *direct* and *indirect* effects, but also *induced* effects caused by consumer spending. This is achieved by subtracting not only the intermediate demand technical coefficients from an identity matrix, but also a household income row and a household consumption column, and then inverting the result to gain $\mathbf{B^*} = (\mathbf{I} - \mathbf{A^*})^{-1}$. Each element in the closed inverse Leontief matrix indicates the total direct, indirect and induced requirements from row industry *i* arising from a unit increase in sales to final demand by column industry *j*. In this way the value added row and the household consumption column are treated as industries, producing revenue and requiring inputs from other industries.

4.5   Economic Multipliers

Economic multipliers, representing summary measures of the effects of interdependence, may be calculated using the Open and Closed Leontief Inverse matrices. Input-output practitioners generally categorise multiplier effects into three groups, each of which represents a different view of the economy under study: output, value added and employment.

4.5.1   Output Multipliers

Output multipliers relate a unit of spending to an increase in output in the economy. A Type I Output multiplier for industry *j* can be found by summing the *j*th column of the Leontief inverse matrix. The Type I Output multiplier tells us the direct and indirect requirements necessary to produce one additional unit of output in an economy. Similarly, a Type II (direct, indirect and induced effect) Output

multiplier for industry *j* can be found by summing the *j*th column of the closed Leontief inverse matrix. The Type II Output multipliers tells us the direct, indirect requirements necessary to produce one additional unit of output along with the additional induced impacts resulting from additional consumer spending in an economy.

4.5.2   Value Added Multipliers

Value added multipliers show the relationship between an additional unit of spending and changes in the level of value added (i.e. wages and salaries, operating surplus, subsidies and the like). Underpinning value added multipliers is the notion that if an industry experiences a change in output there will be an associated variation in labour inputs and, in turn, variations in household consumption. A direct value added multiplier for column industry *j* is calculated as the value added technical coefficient for industry *j*. A direct and indirect value added multiplier for column industry *j* may be computed by summing the products of the elements in the Leontief inverse matrix for industry *j* and the supplying industry's value added technical coefficient. It is necessary to sum household income changes over all industries as household consumption may vary in all industries to satisfy the change in demand for any industry's output. A direct, indirect, and induced value added multiplier for column industry *j* may be obtained by taken the wages and salaries row coefficient of the closed Leontief inverse matrix for column industry *j*.

Using the preceding value added multipliers it becomes possible to calculate Type I and Type II Value Added multipliers. These multipliers measure the value added generated following a unit change in household expenditure for a given industry. The Type I Value Added multiplier is expressed as the ratio of the direct and indirect value added multiplier to the direct value added multiplier – resulting from a unit increase in final demand for a given industry. The Type II Value Added multiplier attempts to explain induced effects initiated through consumer expenditure. This is measured by including the effect of household expenditure generated by value added made as the result of variations in demand in a given industry. The Type II Value Added multiplier is usually expressed as the ratio of the direct, indirect and induced value added multiplier to the direct value added multiplier – resulting from a unit increase in final demand for a given industry.

4.5.3   Employment Multipliers

Employment coefficients represent the number of people directly employed by the industry per unit of industry output. Mathematically, an employment coefficient may be expressed as:

$$\widehat{empcoeff}_j = \frac{\widehat{emp'}_j}{\beta'_j} \tag{26}$$

where $\widehat{empcoeff}_j$ represents the employment coefficient for industry *j*.

The substitution of the direct value added multiplier with an employment coefficient allows employment multipliers to be generated in the same fashion as value added multipliers. Type I and II Employment multipliers gauge respectively (1) the direct and indirect, and (2) direct, indirect and induced employment impact associated with a change in direct employment in an industry. Such employment multipliers allow employment levels associated with increases in final demand in a given industry to be estimated.

4.6   Assumptions of Input-Output Modelling

The assumptions of input-output analysis are documented in detail elsewhere (see for example, Dorfman *et al*. (1958), Richardson (1972), Leontief (1985), and Miller and Blair (2009)). In brief, the major assumptions may be summarised as:

- *Homogeneity*. Each industry in an input-output table produces only one output. Implicit in this assumption is the notion that all businesses that constitute an industry use the same product mix in production of this one output.
- *Additivity*. The total effect of carrying out several types of production is the sum of the separate effects. This implies the absence of any synergistic effects and external economies (diseconomies) of scale.
- *Linearity*. The ratio of inputs to outputs decreases and increases in a linear manner. This also infers that there are no external economies (diseconomies) of scale.
- *Fixed coefficients of production*. Inputs are required in fixed proportions to outputs in each industry. Inherently, this assumes that there are constant returns to scale in production, and that the elasticity of substitution between inputs is zero.
- *Temporal boundaries*. Input-output typically represents a snapshot of a financial year. All activity, direct or indirect, is assumed to be captured within the same year. In this way, activities planned for in advance must be internalised into the year of study i.e. assumed to have the same interdependencies as the study year.
- *Non-substitutability*. This means that within a given technology there is one preferred set of input ratios that will continue to be preferred regardless of final demand quantities.

5 Ecological Input-Output Tables

Ecological input-output tables (EIOTs) modify and extend the conventional input-output framework outlined above to include resource use and waste generation. A key feature of ecological input-output modelling is that it is principally concerned with the environment-economy interface; in particular, how changes in the economy might impact on the environment (e.g. resource provision/scarcity, waste/pollution generation, and the costs of substitutes/abatement etc) or vice versa. The long history associated with the development of environmental input-output tables has resulted in a plethora of applications, refinements and extensions. Nevertheless, much of this work has its roots firmly grounded in policy simulation frameworks developed in the late 1960s and early 1970s. This includes frameworks developed by *inter alia*, Daly (1968), Isard (1968), Ayres and Kneese (1969), Leontief (1970) and Victor (1972). Authors such as Lonergan and Cocklin (1985) and Miller and Blair (2009) have tentatively grouped environmental input-output frameworks into three categories:

- *Generalised or "augmented" input-output tables*. These are formed by adding rows and columns, representing pollution generation and abatement activities, to a technical coefficients matrix. A matrix of pollution or abatement coefficients, **P**, is defined where each element, P_{kj}, represents pollutant k generated per dollar of output of industry *j*. Multiplying **P** by the Leontief Inverse yields the direct and indirect pollution, **P***, produced per unit of final demand generated in industry j i.e. $\mathbf{P^*} = \mathbf{P(I-A)^{-1}Y}$. Although simple, this approach can provide valuable insight into the magnitude of the indirect environmental impacts associated with changes in economic activity.

- *Inter-industry economic-ecological input-output tables*. These models extend the basic inter-industry framework to include "ecosystem" sectors, with the use and production of ecological commodities (Miller and Blair, 1985). Several examples of inter-industry economic-ecological models, as discussed below, include Daly (1968), Ayres and Kneese (1969) and Leontief (1970).

- *Commodity-by-industry models*. Such models treat resource use and waste production as "commodities" in the form of a commodity-by-industry framework. A key trait of commodity-by-industry models is data compiled at a commodity level, independent of 'homogeneous' industry classification. Thus, multiple outputs per industry are permitted. Examples of commodity-by-industry models, as discussed below, include Isard (1968, 1972, 1975), Victor (1972, 1972a) and more recently the Physical Input-Output Tables (PIOTs) developed by Stahmer, Kuhn and Braun (1996, 1997, 1998), Gravgård (1998) and McDonald and Patterson (2008).

It is important to note that provided ecological accounts are developed using the same sector and commodity definitions as used in the SUT or SIOT, then any of the above forms can be relatively easily generated.

5.1   Inter-Industry Ecological Input-Output Models

5.1.1   The Daly Model

Daly (1968) constructed a highly aggregated model of the economy-environment interface based on an industry-by-industry framework (Figure 3). The model is divided into two domains: human and non-human. Conventional economic activities, such as agriculture, industry and households, are categorised under the human domain, while biophysical/ecological processes are classified within the non-human domain. The biophysical processes are further subdivided into living (animal, plant and bacteria) and non-living (atmosphere, hydrosphere and lithosphere) transformers of matter-energy. Interdependence between processes within the human and non-human sphere's of influence is portrayed respectively in quadrants I and IV. Quadrant III represents the reverse flow of 'free goods' (e.g. resource inputs) while quadrant III depicts the flow of 'externalities' (e.g. pollution and wastes) between the human and non-human spheres. Mixed monetary and physical units are utilised to describe the flows.

		Human			Non-Human							
		Agriculture	Industry	Household (final consumption)	Animal	Plant	Bacteria	Atmosphere	Hydrosphere	Lithosphere	Sink (final consumption)	Total
Human	Agriculture Industry Household (final consumption)	Quadrant I			Quadrant II							
Non-Human	Animal Plant Bacteria Atmosphere Hydrosphere Lithosphere Sun (primary services)	Quadrant III			Quadrant IV							

Figure 3   Daly Model (adapted from Daly (1968, p.402))

Not satisfied with a purely descriptive tool, Daly calculated technical coefficients by dividing each row element by its corresponding row total. This approach has however been criticised on the grounds that the economic and ecological commodities should not be totalled as they are expressed in different units. Furthermore, implicit in the calculation of row totals is the erroneous assumption that ecological outputs are distributed through market mechanisms. According to Victor (1972, p.41) these row totals are meaningless, "despite Daly's unsubstantiated claim that 'there appear to be no theoretical problems in extending the input-output model in this way'".

Daly's adoption of an industry-by-industry framework for analysing the economy has complications when considering the environment. Firstly, the homogeneity assumption is illogical when transferred to the non-human (ecological) domain because aggregation of different ecological commodities is not possible due to the absence of a consistent numeraire. Secondly, in the adoption of non-comparable units the model tries to commensurate ecological commodities, which have no price, with economic commodities which do. Thirdly, the linearity assumption converts many non-linear ecological interdependencies to a linear nature. And finally, the assumption of fixed proportions of inputs is not necessarily valid when considering ecological interrelationships.

5.1.2   The Ayres-Kneese Model

Ayres and Kneese (1969) developed an extended inter-industry model incorporating resource use, residuals, pollutant abatement and recycling. A key feature of the model is that it invokes the 'materials balance principle' i.e. mass and energy must be conserved across the model. In this way, the model abides by the first law of thermodynamics.

The Ayres-Kneese model is depicted in Figure 4. Coefficients in the extraction matrix I and production matrix II form a conventional inter-industry input-output table. These coefficients represent the fractional inputs per unit of output, as measured in pecuniary terms, of each industry. The coefficients in the abatement matrix III represent the actual costs of abatement. Additionally, matrices representing resource inputs, **R**, and residual outputs, **W**, are also incorporated.

	Extractive sectors	Production sectors	Abatement sectors
Resource Input Matrices	IV	*R* V	VI
Conventional Input-Output Table plus "Abatement" Sector	I	II	III
Residuals Output Matrices	VII	*W* VIII	IX

Figure 4   Ayres-Kneese Model (adapted from Ayres, 1978, p.118)

The resource inputs matrix R is further separated into three sub matrices IV, V and VI. Each sub-matrix has one row for each resource and one column of each industry identified in the input-output table. Each coefficient in matrix R records the resource input (in physical units) per unit of output (in pecuniary terms) of the industry (Ayres, 1978). Obviously, resource use would mostly be undertaken by the extraction sectors (sub-matrix IV), while the vast majority of entries in the upper sub-matrices V and VI would be zero – notable exceptions would include oxygen for combustion and wastes for reuse or recycling.

The resource output matrix, **W**, like the resource input matrix, **R**, has three sub-matrices VII, VIII and IX. Each sub-matrix has one row for each pollutant and one column for each industry of the input-output table. Each coefficient in the **W** matrix records residual output, in physical units per pecuniary unit of output in a given industry. Columns of the extraction sub-matrix, VII, represent gross residuals by industry, while the columns of the production matrix, VIII, record the gross production of wastes by industry. Abatement is recorded in sub-matrix IX. Generally, the entries in this matrix represent the net amount of residuals – normally negative.

Overall, the Ayres-Kneese model extends a conventional input-output table to include resource use, residual production and pollution and abatement. A key aspect of the model is that it instigates the 'materials balance principle' ensuring conservation of mass/energy for the system under study. The model also captures the flow across the resource use-economy interface, and vice versa across the economy-residual production interface. One additional advantage is that it can track abatement from one environmental medium to another.

5.1.3   The Leontief Model

Leontief (1970) has also attempted to include environmental factors into an inter-industry framework. His approach is to extend the input-output table by one additional industry representing pollution abatement – the column measuring pollution abatement in pecuniary units, the row recording pollutant output in physical terms (Figure 5).   As the economy generates pollution this additional industry absorbs the cost of the associated pollution abatement measures.   This allows the estimation of cost effects associated with mitigation technologies along with investigation of the effectiveness of possible policies that may be used to regulate pollution.

Inputs and Pollutants' Output	Industry 1	Industry 2	Pollution Abatement	Final Demand	Total Output
Industry 1	X_{11}	X_{12}	X_{1pa}	Y_1	X_1
Industry 2	X_{21}	X_{22}	X_{2pa}	Y_2	X_2
Physical Output of Pollutant	X_{1p}	X_{2p}	$-X_{pa}$	Y_p	X_p
Primary Inputs	V_1	V_2	V_{pa}	V_y	V
Total Inputs	X_1	X_2	X_{pa}	Y	X

Figure 5   **The Leontief Model (adapted from Richardson (1972, p.221))**

Since the pollutant row is measured in physical units, it is excused from any vertical summation. Instead, the gross output of the physical pollutants row is determined from the following calculation,

$$\mathbf{X}_p = \mathbf{X}_{1p} + \mathbf{X}_{2p}. \tag{27}$$

The net output of pollutants may be obtained in the following manner.

$$\mathbf{X}_p = \mathbf{X}_p - \mathbf{X}_{pa} \ . \tag{28}$$

By substitution

$$\mathbf{X}_p = \mathbf{X}_{1p} + \mathbf{X}_{2p} - \mathbf{X}_{pa} \ . \tag{29}$$

At the intersection of the pollutant abatement column industry and the physical pollutants row is an entry, $-\mathbf{X}_{pa}$, that represents the physical amount of pollutants eliminated by the pollution abatement column industry. This output is also expressed in monetary terms as the total input entry at the bottom of the pollution abatement column, $\mathbf{X}_{pa}$. This double valuation of the output of the pollution abatement industry allows direct estimation of the monetary cost associated with eliminating each unit of pollution.

Leaving aside the physical pollutants row allows simple accounting identities of the Leontief model to be represented algebraically,

$$\mathbf{X} = \mathbf{X}_1 + \mathbf{X}_2 + \mathbf{V} = \mathbf{X}_1 + \mathbf{X}_2 + \mathbf{X}_{pa} + \mathbf{Y}. \tag{30}$$

Thus $\mathbf{V} = \mathbf{Y} + \mathbf{X}_{pa}$ \hfill (31)

$$\mathbf{Y} = \mathbf{V} - \mathbf{X}_{pa}. \tag{32}$$

With the inclusion of the pollution abatement industry it can be seen from Equation 32 that the conventional input-output identity of final demand equating to value added is not preserved, i.e. $\mathbf{Y} \neq \mathbf{V}$. The absolute difference between final demand and value added is a measure of the pecuniary expenditure on pollution abatement. Furthermore, the absence of a pollution abatement row industry infers that this expenditure is absorbed completely by households. In other words, intermediate demand industries do not purchase inputs from the pollution abatement industry.

A primary criticism of the Leontief model is that the pollution abatement is recorded twice – physically in the pollutants row, and monetarily in the pollutants column. Without this double valuation of the pollution abatement industry, however, the monetary costs of eliminating each unit of pollution could not be estimated. A further limitation is the oversight of any consideration of the materials balance

principle. Adherence to the materials balance principle is impossible as only flows from the economy to the environment are modelled.

5.2　Commodity-by-Industry Ecological Input-Output Tables

5.2.1　The Isard Model

Between the late 1960s and mid 1970s Walter Isard and associates constructed several ecologic-economic input-output models. This included the notable Plymouth Bay, Massachusetts regional planning study of the environmental repercussion of marina development. The Isard model, like the Daly model, recorded interactions within and between the environment and economy in a comprehensive manner. The Isard model, however, relied on coefficients taken directly from scientific literature, while the Daly model attempted to derive such coefficients through accounting identities.

The Isard model is illustrated in Figure 6. The model is divided into quadrants with entries in coefficient format, negative coefficients representing inputs, and positive coefficients representing outputs. Quadrants I and IV describe flows respectively within the economy (i.e. goods and services) and the environment (i.e. energy and mass). Quadrant I, the inter-sector coefficients matrix, is a commodity-by-industry technical coefficients table. Unlike conventional industry-by-industry models, where only one homogeneous output per industry may be produced, Isard's commodity-by-industry model permits multiple outputs per industry.　Quadrant IV, the inter-process coefficients, records ecological interdependence between various ecological processes in terms of ecological commodities. Classification of the commodities and processes was based on an ecological taxonomy consisting of abiotic (i.e. meteorological, geological, physiological, hydrological and soil types) and biotic (i.e. plant and animal life) groupings.　In this way, detailed information on food chains, food webs and biogeochemical cycling was included in the model.

		Economic Industries			Ecologic Processes		
		Industry 1	Industry ...j...	Industry n	Process 1	Process ...p...	Process n
Economic Commodities	Commodity 1 Commodity ...i... Commodity n	Quadrant I - Economic System: Intersector Coefficients			Quadrant II - Ecologic Processes: Their Input and Output Coefficients re Economic Commodities		
Ecologic Commodities	Commodity 1 Commodity ...c... Commodity n	Quadrant III - Economic Sectors: Their Input and Output Coefficients re Ecologic Commodities			Quadrant IV - Ecologic System: Interprocess Coefficients		

Figure 6   **The Isard Model (Adapted from Isard (1968, p.87))**

Quadrants II and III depict flows between the economy and environment. The upper right hand corner, Quadrant II, shows the production and use of economic commodities by ecological processes. It is worth noting that coefficients in this quadrant generally reveal that very few economic commodities flow directly from the environment as delivered goods for consumption by final demand categories. Quadrant III records the use of ecologic commodities by industries as well as the export of ecologic commodities from industries to the environment. These coefficients, as in Quadrant II, are expressed in terms of the ecological inputs to, and outputs from, the economic system per unit of economic output.

Critics such as Victor (1972), Johnson and Bennett (1981) and Lonergan and Cocklin (1985) point to difficulties associated with obtaining appropriate data for the complex ecological interprocess coefficients in Quadrant IV. Isard (1975, p.343) recognised this, stating that "the set of data pertaining to the environmental system which we inherit today is tremendous in variety and amount ... We therefore confront difficulty in trying to develop a systematic input-output description of the ecologic system". Isard (1968) also comments on the restrictiveness of the linearity assumption suggesting that those ecological processes that are non-linear in nature should be considered outside of the input-output framework. A further concern is that it is implicitly assumed that environmental resources remain stable over time when, in actuality, changes in resource quality could affect the invariant nature of coefficient relationships (Kapp, 1970; Richardson, 1972). Despite these assumptions, the Isard model is conceptually very attractive. Steenge (1977, p.97) argues that the work of Isard "will remain

indispensable mainly because here the line separating theory and implementation was crossed definitively."

5.2.2   The Victor Model

Victor (1972, 1972a) developed a commodity-by-industry input-output model of the Canadian economy to analyse planning problems from an environmental perspective. Realising the conceptual strength of the Isard model, but also the difficulty associated with accurately populating the model's Quadrant IV, Victor sought out a compromise between theoretical ideal and empirical implementation. The resulting model, displayed in Figure 7, focuses on comprehensively recording economic-ecologic linkages, but ignores the within environment flows, arguing that data paucity would make a full implementation near impossible.

	Economic Commodities	Industries	Final Demand	Totals	Ecological Commodities Land · Air · Water
Economic Commodities		*A*	*B*	*c*	*G*
Industries	*D*			*e*	*F*
Primary Inputs		*H*	*K*	*I*	
Totals	*c'*	*e'*	*o'*	*p*	*q'*
Ecological Commodities — Land / Air / Water	*S*	*R*		*t*	

Figure 7   The Victor Model (adapted from Victor (1972, p.56))

The accounting framework used by Victor is essentially a commodity-by-industry table (in Stone's (1961, 1966) supply-use table form) appended with additional rows and columns representing respectively
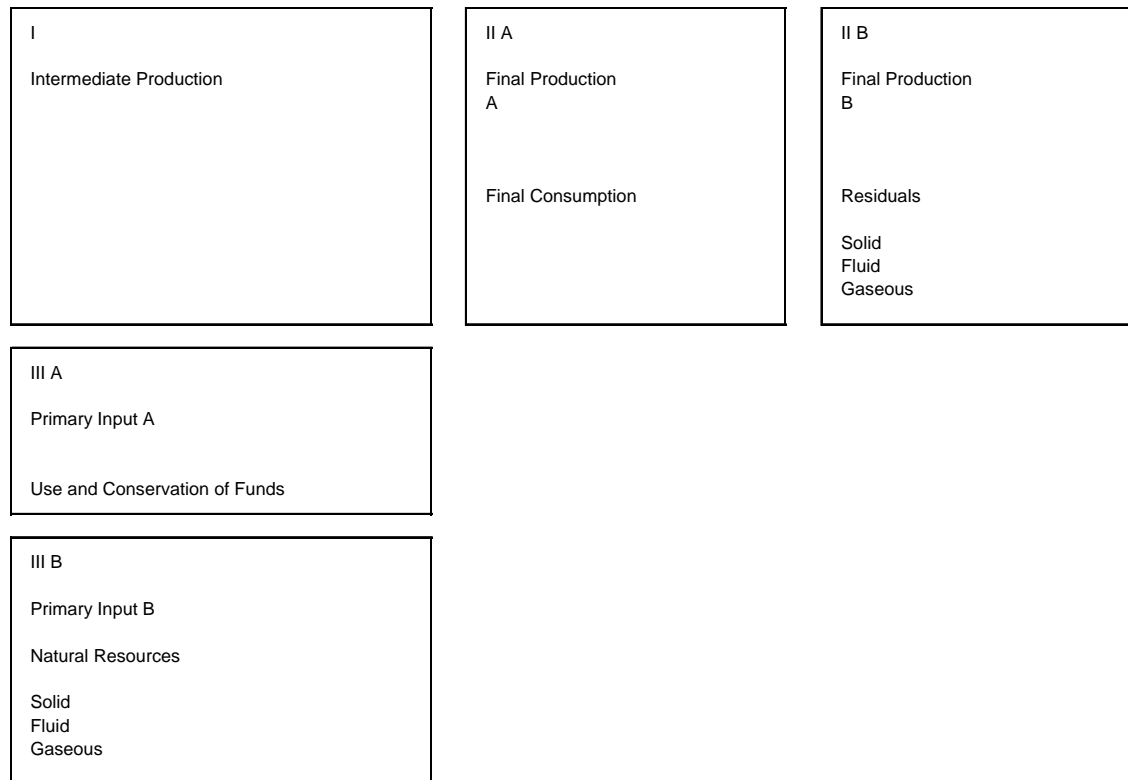
ecological inputs and outputs. Economic transactions are represented in monetary terms, while entries in ecological sectors are expressed in physical units. The ecological commodities that constitute the ecological sector are classified under three headings: land, air and water. In addition to the conventional input-output accounting identities, Victor defines several ecological accounting identities based on the materials balance principle i.e. the mass of material inputs of industry must equal the mass of material outputs of industry.

Using this framework Victor developed a series of analytical models relating economic production to effects on the environment in terms of resource use and waste generation. First, he created a set of ecological impact tables, with and without import leakages. Second, by using shadow prices to represent the social valuation of ecological commodities, he outlined a procedure for using the impact tables to derive estimates of ecological costs of producing and consuming economic commodities. And third, he disaggregated the estimates of ecological inputs and outputs by province, adding a valuable spatial dimension to his model. Overall, the major contribution made by Victor was to bridge the gap between an ideal solution and practical feasibility.

5.2.3 Physical Input-Output Tables

PIOTs not only trace the physical flow of commodities through the environment, but also between the environment and the economy and vice versa (Stahmer, Kuhn and Braun, 1997). A cornerstone of the PIOT framework is adherence to fundamental physical principles, particularly materials and energy balance as required by the first law of thermodynamics. PIOT accounting is a recent phenomenon. Katterl and Kratena (1990) are credited with pioneering the first PIOT – a partially complete table of the 1983 Austrian economy (Strassert, 2000). Old Länder, a PIOT of the 1990 West German economy, was the first complete and official table to be constructed (refer to Stahmer, Kuhn and Braun (1996, 1997, 1998). Several other PIOTs have followed, including an official table for the 1990 Danish economy (Gravgård, 1998), and less ambitious unofficial efforts for Italy (Nebbia, 1999), the United States (Acosta, 2000) and New Zealand (McDonald and Patterson, 2008).

A PIOT is typically presented in a tabular commodity-by-industry format with production processes (i.e. industries) described by their material inputs and outputs in physical units e.g. tonnes (Figure 8). Each input is described by its industrial origin (or as imports), while each output is explained by its destination i.e. industry, final consumption or exports (Strassert, 2000). Strassert (2000), in his description of the German PIOT, uses five matrices to describe physical flow. Matrix I, the intermediate production matrix, describes physical flow within the economic system. Matrix IIA describes the final consumption of physical commodities by households etc, while any residuals (i.e. waste, pollutants and emissions) produced in production or consumption are described in Matrix IIB. Similarly, Matrices IIIA and IIIB respectively describe the use or conservation of material funds, and the use of natural resources (i.e. renewable, non-renewable and recycled) supplied by the environment as an input into the production process.

I		II A		II B	
Intermediate Production		Final Production A		Final Production B	
		Final Consumption		Residuals Solid Fluid Gaseous	

III A
Primary Input A
Use and Conservation of Funds

III B
Primary Input B
Natural Resources
Solid Fluid Gaseous

Figure 8   **A Physical Input-Output Table (adapted from Strassert (2000, p.3))**

Unlike a conventional input-output table which focuses on the structural nature of economic transactions (Matrix I), final consumption by households and exports (Matrix IIA), and particularly the contribution made by each production process to value added (Matrix IIIA), a PIOT tends to focus instead on the structural nature of economic transactions (Matrix I), resource use (Matrix IIIB), residual production (Matrix IIB) and particularly on the completeness of material balance. Moreover, the PIOT is conceptually consistent with the ideas of authors such as Boulding (1966) and Daly (1991) who view economic production as a subsystem encapsulated within a finite and non-growing environment. This conceptualisation implicitly captures the role played by economics in extracting/harvesting low entropy matter-energy and ultimately producing high entropy matter-energy. Consequently, this one-way flow beginning with resources and ending with residuals can be thought of as the digestive tract of an open biological system connected by the environment at both ends (Daly, 1995).

5.3 Ecological Multipliers

Ecological multipliers, which measure the total 'embodied resource' or 'embodied residual' required to produce one additional unit's worth of a commodity in any given industry, are arguably an operational measurement of the eco-efficiency concept – this includes all the resource/residual requirements appropriated in producing and, possibly consuming, the commodity. Ecological multipliers may be

calculated for any resource (e.g. land use, water use, minerals, bio-mass) or any residual (e.g. solid waste, air emissions, water pollution). EIOTs have been used to generated ecological multipliers since the early 1970s - refer, for example, to Hite and Laurent (1971, 1972), Wright (1975) and Carter *et al*. (1981). The key advantage of EIOT is that it relies on production chain data routinely collected by statistical agencies rather than detailed bottom-up data on industrial processes to generate indirect (or embodied) requirements. The use of an EIOT may drastically reduce the need to collect primary process-level data. Essentially, the calculation of an ecological multiplier requires, as a minimum, a SIOT and a set of environmental accounts for a particularly resource or residual. These resource/residual accounts must be coded by the same industry definitions as employed in the SIOT. Although not covered here it is worth noting that methods also exist, based on the work of Costanza (1980) and Costanza and Neill (1981), for calculating ecological multipliers using ecologically extended SUTs – the key advantage of this approach is that accommodates multiple industry outputs, rather than assuming that there is only one output per industry (see also Patterson *et al*., 2010).

5.4 Pressure-Based Footprints

Since the mid 1990s pressure-based footprints have gained popularity within policy and decision-makers globally. This is largely due to the work of Wackernagel and Rees (1996) on Ecological Footprints (EFs). Essentially, an EF is a measure of the total amount of land required to support a given population. The EF, thus, includes not only the land directly used to house a population, but also the (indirect) land embodied in the goods and services required to support the existence of that same population – no matter where on Earth is may come from. A full critique of the Ecological Footprint concept is available in the International Journal of Ecological Economics Vol. 32, and in papers by Van den Bergh and Verbruggen (1999) and McDonald and Patterson (2004). In the context of the development of a didactic input-output framework, and its associated suite of models, we prefer to focus on pressure-based footprints for many different types of resources rather than simply the subset of resources/residuals relevant in EF analyses.

The calculation of a pressure-based footprint for any given study area may be defined according to the following accounting identity:

$$P \equiv \alpha + (\beta_1 + \beta_2 + ... + \beta_{n-1}) + \delta \tag{33}$$

where: α = resource/residual appropriation from within the study region; $\beta_1 + \beta_2 + ... + \beta_{n-1}$ = resource/residual appropriation from other regions (1 … n-1); and δ = resource/residual appropriation from other nations.

5.3.1 Calculation of the Resources/Residuals Appropriated Within the Study Region (α)

The resource/residual appropriated from industries within the study region, α, is calculated by pre-multiplying a diagonalised vector of resource/residual requirements[8], $\hat{r}$, by the Leontief Inverse matrix, **B**, to derive a matrix, **Q**, of the direct and indirect resource/residual requirements required to produce a unit of final demand:

$$\mathbf{Q} = (\mathbf{I} - \mathbf{A})^{-1} \hat{r} . \qquad\qquad (34)$$

To determine the resource/residual requirements of the domestic population, **D**, within the study area requires that matrix **Q** be multiplied by an appropriate diagonalised vector of domestic final demand, $\hat{f}$:

$$\mathbf{D} = \mathbf{Q}\, \hat{f} . \qquad\qquad (35)$$

5.3.2 Calculation of Resource/Residual Appropriation from Other Regions ($\beta_1 + \beta_2 + ... + \beta_{n-1}$) and nations ($\delta$)

Using the following two step process the resources/residuals appropriate by the study region population from within the study region, from other regions within the same nation, and from other nations may be calculated.

Step 1 Determination of the Interregional and International Imports

Each industry in the study region purchases commodities from various regions in the nation. For a given industry in the nation it is not known exactly from which region these commodities originate. This may be estimated by solving an optimisation problem. This typically involves assuming that each industry within the study area will seek to source commodities from supplier regions closest to them in terms of travel time. Thus, minimisation of travel time is to set the objective function, while known levels of industry imports (and exports) are used as the binding constraints. Solving this optimisation problem yields a matrix of interregional and international imports and exports to/from the study area.

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[8] This vector is expressed in resource/residual units per an appropriate unit of final demand for each economic sector i.e. ha/$m/year.

Step 2   Determination of the Resources/Residuals Embodied in Interregional and International Imports

The imports matrix quantifies the imports of commodities into a given industry (in the study area) from industries in Region 1. These values may then be placed into a Multi-Regional Input-Output (MRIO) table – providing EIOTs are available for the other regions and nations. The approach outlined in Section 5.3.1 may then be followed, except this time the ecological MRIO is used, generating not only estimates of the within-industry resource/residual requirements necessary to support domestic consumption, but also the appropriate inter-regional and international resource/residual requirements.

6 Comparative Static Analysis

A key feature of the input-output framework is that it may be used to forecast the future impacts associated with changes in final demand. There is no exact recipe as to how this should be done – and, therefore, no mathematical description is provided in this report. Nevertheless, it is worth exploring, at least in pragmatic terms, how a comparative static analysis may be performed using the SUT and IO frameworks.

6.1 Final Demand Projections

The starting point, and minimum requirement, for most SUT and IO based comparative static analyses is a set of final demand projections. The underpinning linear formulation of IO restricts any projections to principally the short-to-medium term analysis (<10 years forward). At a minimum the final demand projections will need to account for changes in household consumption, consumption of governmental services by households, the formation of capital, and exports.

- *Household consumption and the consumption of governmental services by households*. These categories are perhaps for which the easiest to develop future projections, as they are often dependent upon readily available information. Most statistical agencies, for example, regularly produce population projections by age-sex cohort along with age-sex cohort consumption equivalence factors which allow future projections of household consumption to be derived.
- *Gross fixed capital formation and international exports*. These categories are typically estimated using econometric techniques based on time series information.

It is important to note that in general most comparative static implementation assume that technical coefficients will remain the same across time. This, along with the linear nature of the SUT and IO frameworks, is a key limitation of using SUT and IO tables to generate future projections of gross output, value added, employment, resource inputs, and residual outputs. A further, and often overlooked, assumption is that the model is purely demand driven. In reality it is likely that both demand and supply will influence future economic activity. Although it is possible to create supply driven (based on the so-called 'Ghosh Inverse') models, most of the IO literature is critical of such approaches. The supply driven model, however, has the advantage that it enables investigation of labour and resource constraints.

6.2 Technical Change

If the comparative static analysis is to extend beyond 10 years then it is important that adjustments be made to the technical coefficients table for account for technical change. Unfortunately, this is not easily undertaken – involving a plethora of factors including understanding how existing technologies will be required, along with the replacement by new technologies, changes in the input mixes of the commodities that an industry requires to produce its outputs, changes in supply and demand, and so on.

For this reason it is recommended that sensitivity analyses be undertaken – possibly employing simulation techniques and Monte Carlo analysis.

Technical change is most commonly implemented through modifications to the technical coefficients table. Time series of technical coefficients may be produced and a regression analysis performed to estimate future technical coefficients. Alternatively, production (or utility) functions may be generated from time series data.

6.3   A Simple Comparative Static Projection Model

A simple comparative static projection model is described below. The model is formulated using a standard SIOT/EIOT formulation as outlined in Sections 4 and 5 above. The model requires the following two matrices as initial conditions: (1) a matrix of intermediate demand transactions figures, **U**, and (2) a diagonalised matrix of sector gross output figures, **V**.[9]

A net matrix (**V** − **U**) is then calculated with industry outputs shown as positive elements along the diagonal elements and industry inputs as negative elements in non-diagonal entries. Multiplication of the inverse of this matrix, (**V** − **U**)$^{-1}$, by a column vector of future total final demand for a given year, **f**, yields a column vector of gross output scalars, **g**, which allow an analyst to estimate future gross output for the same given year.

$$\mathbf{g} = (\mathbf{V} - \mathbf{U})^{-1}\,\mathbf{f} \tag{36}$$

The gross output scalars may also be used to estimate future value added/employment i.e. it is assumed that value added and employment grow linearly at the same rate as gross output. Adjustments are, however, typically made to account for labour/capital (or multi) factor productivity. Similarly, the gross output scalars may be used in conjunction with estimates of resource inputs/residuals outputs to generate estimate of future resource requirements or residuals generated. Adjustments of eco-efficiency for resource inputs and less/more residual output per $ output are also possible.

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[9] Note that the **U** and **V** matrices used here are synonymous with the **U** (use) and **V** (supply) matrices of the Supply-Use Framework. In this example, however, there are no joint products – only a single output per industry which appears along the diagonal of the **V** matrix.

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