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Domain decomposition for Full-Wave simulation in a tokamak plasma

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Abstract
The aim of this work is to develop a numerical method for the full-wave simulation of electromagnetic wave propagation in a plasma. The propagation and the absorption of lower hybrid (LH) electromagnetic waves is a powerful method to generate current in tokamaks by Landau wave particle resonance. Full-wave calculations of the LH wave propagation is a challenging issue because of the short wave length with respect to the machine size. We propose a Fourier finite element method for solving the Maxwell equations based on a mixed augmented variational formulation. In order to develop a parallel version of the simulation and consider non homogenous plasma response, a nonoverlapping domain decomposition approach is presented.

Introduction
Let the domain \(\Omega\) be a torus (tokamak plasma volume) with strong external time-invariant magnetic field \(B_{\text{ext}}\). We study a second order partial differential equation for the time-harmonic electric field \(E\) arising from Maxwell equations:

\[
\begin{align*}
\text{curl} \, \text{curl} \, E - \frac{\omega^2}{c^2} KE & = f \quad \text{in } \Omega, \\
\text{div} \, (KE) & = g \quad \text{in } \Omega
\end{align*}
\]

where \(\omega > 0\) is the excited wave frequency and \(c\) denotes the speed of light in free space. The plasma response is described by the matrix \(K\), in Stix frame (third coordinate parallel to \(B_{\text{ext}}\)). It includes a cold plasma approximation of the relative dielectric permittivity tensor and Landau damping:

\[
K(x) = \begin{pmatrix}
S(x) & -iD(x) & 0 \\
 iD(x) & S(x) & 0 \\
 0 & 0 & P_L(x)
\end{pmatrix}
\]

Expressions of the entries \(S, D\) and \(P_L\) involve plasma frequencies, cyclotron frequencies of each species (ion and electron) and also the collision frequency. In general, the matrix \(K\) is complex-valued and non-hermitian. Let \(\Gamma\) be the boundary of the domain \(\Omega\) and \(\Gamma_A \subset \Gamma\) be an antenna on the tokamak, then several boundary conditions are possible:

Neumann: \(\text{curl} \, E \times n = i\omega \mu_0 j_s\) on \(\Gamma_A\)

Dirichlet: \(E \times n = E_A \times n\) on \(\Gamma_A\).

On the other part of the boundary \(\Gamma_C = \Gamma \setminus \Gamma_A\), we assume a perfectly conducting condition:

\(E \times n = 0\) on \(\Gamma_C\).

1 Finite element method
1.1 Variational formulation and well-posedness
Taking the divergence condition (2) as constraint, we use a mixed augmented variational formulation (MAVF) [3], which gives rise to a \(H^1\) conforming variational space, \(X_N(K, \Omega) := H_0(\text{curl}, \Omega) \cap H(\text{div} \, K, \Omega)\). We obtain the following variational formulation of the Dirichlet problem:

Find \((E, p) \in X_N(K, \Omega) \times L^2(\Omega)\) such that

\[
a_s(E, F) + b(F, p) = L_s(F) \quad \forall F \in X_N(K, \Omega) \\quad \text{and} \quad \forall q \in L^2(\Omega),
\]

where

\[
a_s(E, F) := (\text{curl} \, E \mid \text{curl} \, F) - \frac{\omega^2}{c^2} (KE \mid F)
\]

\[+ s (\text{div} \, KE \mid \text{div} \, KF)\]

\[L_s(F) := (f \mid F) + s (g \mid \text{div} \, KF)\]

\[b(E, q) := (\text{div} \, KE \mid q)\]

\[l(q) := (g \mid q)\],

with parameter \(s \in \mathbb{C}\). Here, \((\cdot \mid \cdot)\) denotes the standard \(L^2\) inner product in \(\Omega\).

The well-posedness of the considered formulation follows from the Babuska-Brezzi theorem. Thanks to spectral properties of the dielectric tensor, the sesquilinear form \(a_s\) is coercive if \(\Re(s) > 0\) and \(\Im(s) \leq 0\).

1.2 Dimension reduction and discretization
The 3D problem can be reduced to a series of 2D one by using cylindrical coordinates \((R, Z, \phi)\) and by
expanding all functions \( f(R, Z, \phi) \) as Fourier series in the angular coordinate \( \phi \)

\[
f(R, Z, \phi) = \frac{1}{\sqrt{2\pi}} \sum_{\nu \in \mathbb{Z}} f_{\nu}(R, Z) e^{i\nu\phi}
\]

where the coefficients \( f_{\nu}(R, Z) \) are defined on a cross section of \( \Omega \) [4]. Then the sesquilinear forms of the variational formulation can be written as sum of modal forms

\[
a_s(u, v) = \sum_{\nu \in \mathbb{Z}} a_{s,\nu}(u_{\nu}, v_{\nu}), \quad b(v, p) = \sum_{\nu \in \mathbb{Z}} b_{\nu}(v_{\nu}, p_{\nu})
\]

The modal variational formulation is then discretized using a Taylor-Hood \( P_2 \)-iso-\( P_1 \) finite element.

1.3 Numerical results

![Figure 1: Real part of a component of the electric field for \( \omega = \omega_{LH} = 1.3 \times 10^{10} \text{ rad/s} \)](image)

2 Domain decomposition

Consider a nonoverlapping decomposition \( \overline{\Omega} = \bigcup_k \overline{\Omega}_k \). In the domain decomposition method considered here, we solve the original problem in each subdomain \( \Omega_i \); the equivalence with the one-domain formulation is obtained by continuity conditions

\[
[E \times n]_{\Sigma_{ij}} = 0 \quad \text{and} \quad [KE \cdot n]_{\Sigma_{ij}} = 0 \quad (3)
\]

which ensure the \( X(K, \Omega) \) regularity of the electric field and

\[
\text{curl} E \times n]_{\Sigma_{ij}} = 0, \quad (4)
\]

which implies that the one-domain formulation holds in the sense of distributions. We have denoted, as usual, \([f]_{\Sigma_{ij}}\) the jump of a quantity \( f \) across the interface \( \Sigma_{ij} = \partial \Omega_i \cap \partial \Omega_j \). The conditions (3) are dualized by introducing the associated Lagrange multipliers \( \lambda_{ij} \in H^{1/2}(\Sigma_{ij}) \), while (4) is treated as a natural condition. The existence and uniqueness of the solution \((E_i, p_i, \lambda_{ij})\) to the multidomain formulation was proved and:

\[
E_i = E|_{\Omega_i} \quad \text{and} \quad p_i = p|_{\Omega_i}
\]

where \((E, p)\) is the solution to the one-domain formulation.

The full linear system involving all subdomains (the outer system) is a generalized saddle-point problem:

\[
\begin{pmatrix}
Q & G^H \\
G & 0
\end{pmatrix}
\begin{pmatrix}
E \\
\lambda
\end{pmatrix}
=
\begin{pmatrix}
F \\
0
\end{pmatrix}
\quad (5)
\]

where \( Q \) is a block sparse non-hermitian matrix. Each block corresponds to a problem in one subdomain. The sparse matrix \( G \) expresses the interactions between subdomains. The outer system (5) is solved using a preconditioned GMRES algorithm. The inner problem on each subdomain is also a generalized saddle-point problem, and is solved using a direct method.

References


