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Domain decomposition for Full-Wave simulation in a tokamak plasma

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Abstract

The aim of this work is to develop a numerical method for the full-wave simulation of electromagnetic wave propagation in a plasma. The propagation and the absorption of lower hybrid (LH) electromagnetic waves is a powerful method to generate current in tokamaks by Landau wave particle resonance. Full-wave calculations of the LH wave propagation is a challenging issue because of the short wave length with respect to the machine size. We propose a Fourier finite element method for solving the Maxwell equations based on a mixed augmented variational formulation. In order to develop a parallel version of the simulation and consider non homogenous plasma response, a nonoverlapping domain decomposition approach is presented.

Introduction

Let the domain $\Omega$ be a torus (tokamak plasma volume) with strong external time-invariant magnetic field $B_{\text{ext}}$. We study a second order partial differential equation for the time-harmonic electric field $E$ arising from Maxwell equations:

\begin{align}
\text{curl}\text{curl} \, E - \frac{\omega^2}{c^2} K E &= f \quad \text{in } \Omega, \quad (1) \\
\text{div}(KE) &= g \quad \text{in } \Omega \quad (2)
\end{align}

where $\omega > 0$ is the excited wave frequency and $c$ denotes the speed of light in free space. The plasma response is described by the matrix $K$, in Stix frame (third coordinate parallel to $B_{\text{ext}}$). It includes a cold plasma approximation of the relative dielectric permittivity tensor and Landau damping:

$$K(x) = \begin{pmatrix}
S(x) & -iD(x) & 0 \\
iD(x) & S(x) & 0 \\
0 & 0 & P_L(x)
\end{pmatrix}$$

Expressions of the entries $S$, $D$ and $P_L$ involve plasma frequencies, cyclotron frequencies of each species (ion and electron) and also the collision frequency. In general, the matrix $K$ is complex-valued and non-hermitian. Let $\Gamma$ be the boundary of the domain $\Omega$ and $\Gamma_A \subset \Gamma$ be an antenna on the tokamak, then several boundary conditions are possible:

Neumann: $\text{curl} E \times n = i\omega\mu_0 j$ on $\Gamma_A$

Dirichlet: $E \times n = E_A \times n$ on $\Gamma_A$.

On the other part of the boundary $\Gamma_C = \Gamma \setminus \Gamma_A$, we assume a perfectly conducting condition:

$$E \times n = 0 \quad \text{on } \Gamma_C.$$

1 Finite element method

1.1 Variational formulation and well-posedness

Taking the divergence condition (2) as constraint, we use a mixed augmented variational formulation (MAVF) \cite{3}, which gives rise to a $H^1$ conforming variational space, $X_N(K, \Omega) := H_0(\text{curl}, \Omega) \cap H(\text{div} K, \Omega)$. We obtain the following variational formulation of the Dirichlet problem:

Find $(E, p) \in X_N(K, \Omega) \times L^2(\Omega)$ such that

$$a_s(E, F) + b(F, p) = L_s(F) \quad \forall F \in X_N(K, \Omega)$$

$$b(E, q) = l(q) \quad \forall q \in L^2(\Omega).$$

where

$$a_s(E, F) := \langle \text{curl} E \mid \text{curl} F \rangle - \frac{\omega^2}{c^2} \langle KE \mid F \rangle$$

$$+s \langle \text{div} KE \mid \text{div} KF \rangle$$

$$L_s(F) := \langle f \mid F \rangle + s \langle g \mid \text{div} KF \rangle$$

$$b(E, q) := \langle \text{div} KE \mid q \rangle$$

$$l(q) := \langle g \mid q \rangle,$$

with parameter $s \in \mathbb{C}$. Here, $\langle \cdot \mid \cdot \rangle$ denotes the standard $L^2$ inner product in $\Omega$.

The well-posedness of the considered formulation follows from the Babuska-Brezzi theorem. Thanks to spectral properties of the dielectric tensor, the sesquilinear form $a_s$ is coercive if $\mathbb{R}(s) > 0$ and $\Im(s) \leq 0$.

1.2 Dimension reduction and discretization

The 3D problem can be reduced to a series of 2D one by using cylindrical coordinates $(R, Z, \phi)$ and by
expanding all functions \( f(R, Z, \phi) \) as Fourier series in the angular coordinate \( \phi \)

\[
f(R, Z, \phi) = \frac{1}{\sqrt{2\pi}} \sum_{\nu \in \mathbb{Z}} f_\nu(R, Z) e^{i\nu\phi}
\]

where the coefficients \( f_\nu(R, Z) \) are defined on a cross section of \( \Omega \) [4]. Then the sesquilinear forms of the variational formulation can be written as sum of modal forms

\[
a_s(u, v) = \sum_{\nu \in \mathbb{Z}} a_{s, \nu}(u_\nu, v_\nu), \quad b(v, p) = \sum_{\nu \in \mathbb{Z}} b_\nu(v_\nu, p_\nu)
\]

The modal variational formulation is then discretized using a Taylor-Hood \( P_2\)-iso-\( P_1 \) finite element.

1.3 Numerical results

![Figure 1: Real part of a component of the electric field for \( \omega = \omega_{LH} = 1.3 \times 10^{10} \text{ rad/s} \)](image)

2 Domain decomposition

Consider a nonoverlapping decomposition \( \Omega = \bigcup_k \Omega_k \). In the domain decomposition method considered here, we solve the original problem in each subdomain \( \Omega_i \); the equivalence with the one-domain formulation is obtained by continuity conditions

\[
[E \times n]_{\Sigma_{ij}} = 0 \quad \text{and} \quad [KE \cdot n]_{\Sigma_{ij}} = 0 \quad (3)
\]

which ensure the \( X(K, \Omega) \) regularity of the electric field and

\[
[\text{curl} E \times n]_{\Sigma_{ij}} = 0, \quad (4)
\]

which implies that the one-domain formulation holds in the sense of distributions. We have denoted, as usual, \( [f]_{\Sigma_{ij}} \) the jump of a quantity \( f \) across the interface \( \Sigma_{ij} = \overline{\Omega_i} \cap \overline{\Omega_j} \). The conditions (3) are dualized by introducing the associated Lagrange multipliers \( \lambda_{ij} \in H^{1/2}(\Sigma_{ij}) \), while (4) is treated as a natural condition. The existence and uniqueness of the solution \( (E_i, p_i, \lambda_{ij}) \) to the multidomain formulation was proved and:

\[
E_i = E|_{\Omega_i} \quad \text{and} \quad p_i = p|_{\Omega_i}
\]

where \((E, p)\) is the solution to the one-domain formulation.

The full linear system involving all subdomains (the outer system) is a generalized saddle-point problem:

\[
\begin{pmatrix}
Q & G^H \\
G & 0
\end{pmatrix}
\begin{pmatrix}
E \\
\lambda
\end{pmatrix} =
\begin{pmatrix}
F \\
0
\end{pmatrix}
(5)
\]

where \( Q \) is a block sparse non-hermitian matrix. Each block corresponds to a problem in one subdomain. The sparse matrix \( G \) expresses the interactions between subdomains. The outer system (5) is solved using a preconditioned GMRES algorithm. The inner problem on each subdomain is also a generalized saddle-point problem, and is solved using a direct method.

References


