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***Abstract*—In this paper the phase shift migration based Synthetic Aperture Radar (SAR) is described and applied on radar imaging for dual polarized ground penetrating radar system (GPR). Conventional techniques for SAR imaging focusing use the matched filter concept and convolve the measurement data with a filter impulse response (convolution kernel) which is modified by the range. In fact, conventional techniques for SAR imaging technique can be considered as ray-tracing based SAR imaging technique. It is an efficient technique to obtain focused radar images, when the medium is homogeneous so that the rays of EM wave propagation can be treated as straight lines. However, in case of layered materials the waves are refracted on the interface between two different materials. In contrast, phase shift migration based SAR imaging technique is through EM field extrapolation by phase shift to obtain the reflected EM intensity and radar imaging of scatterers' reflectivity in the radar illumination area. Although EM wave is refracted on the interface of different materials, the EM wave phase is unchanged. Compared to conventional SAR imaging technique, phase shift migration based SAR imaging approach is more suited to obtain focused radar subsurface imaging for GPR systems. In this paper the phase shift migration approach is applied to experimental data as well as practical measurement data. Meanwhile, a background removal algorithm and a spreading and exponential correcting technique is applied to improve the achievement. The presented focused subsurface and scatters radar imaging results show that phase shift migration based SAR technique is satisfied for radar subsurface imaging and inhomogeneous medium imaging.**

I. Introduction

Migration imaging technique originates from seismic imaging[1]. The first migration approaches are devised to image the seismic wave patterns. Due to the potential to image underground objects the migration algorithms for ground penetrating radar imaging were developed and applied on oil and other source exploration in last decades. These migration techniques are useful for increasing signal to noise ratio of the investigated radar images and improving the azimuth resolution. The application of migration imaging technique on radar EM wave subsurface imaging, e.g. GPR radar imaging, is evolved into the phase shift migration algorithm. The phase shift migration based SAR imaging method is based on the wave equations and its objective is to refocus the reflection signatures in recorded data back to the true positions and obtain shape of the reflection target.

Conventional synthetic aperture radar imaging techniques mainly achieving focused radar imaging is based on the physical-mathematical model that the radar received signal is sum of all reflection signals with corresponding phase delay. To derive the positions of the reflectors and achieve scatterers imaging inversely, as the delay-and-sum, conventional SAR imaging technique summarize the coherent backscattered data after proper phase compensation. Meanwhile, the received signal physical mathematical model can also be formulated simply as the convolution of the reflectors reflectivity function with a range reference signal and an azimuth reference signal. Such as range-doppler-SAR, conventional SAR can also achieve the focused radar imaging through range signal cross correlations (matched filter) and azimuth signal cross correlation (matched filter) with a certain range migration. Whatever the conventional SAR technique is applied, the received signal is formulated as the sum of the reflections signals with corresponding phase delay. When the radar EM wave propagates through the homogeneous medium, the trace of the EM wave transmitted by radar and reflected by scatterer can be seen as straight lines. The phase delay of the reflections signals can be built easily based on the geology relationship between the radar and scatterers. However, the phase delay of the reflections signals are difficult to be built, when the EM wave propagates through inhomogeneous medium and the EM wave tracings are not straight lines. Therefore, conventional SAR technique is feeble on focusing inhomogeneous material radar imaging. In fact the conventional synthetic aperture radar imaging techniques obtain the radar imaging by using signal processing way. However, the phase migration based SAR algorithm is mainly based on wave equation. Through EM field extrapolation, the phase shift migration approach obtains reflectivity function of scatters and achieve the objects radar imaging. Furthermore, phase migration based SAR algorithm has high computation efficiency because it reconstructs images iteratively using the fast Fourier transform (FFT).

In this paper, the phase shift migration based SAR imaging algorithm is discussed and applied on dual polarized GPR imaging. To simplify the discussion, the EM wave propagation loss in medium is ignored and the two dimension phase shift migration approach is described.

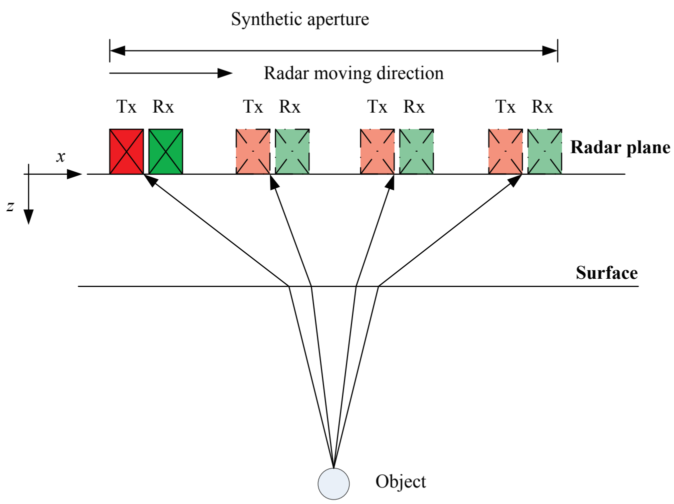

Fig. 1. Setting of the GPR system illuminating an object below the surface from different positions

II. The Principle of Phase Shift Migration for GPR Imaging

The configuration schematic diagram of the GPR system (radar subsurface imaging system) is as Fig. (1). The radar is on the plane $z_0 = 0$ and moves from left side to right side along x-axis. During the moving, radar transmits the EM wave and receives the reflected signal from the illuminated surface, subsurface and buried objects. The coordinate x is the radar aperture dimension (radar moving direction), z is the coordinate in the range dimension. Since we discuss 2-D radar subsurface imaging, the EM field distribution along y-axis is assumed to be constant.

According to the EM wave equation, an EM field $u(x,z,t)$ can be described by the following scalar wave equation:

$$\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}-\frac{1}{v_{\mathrm{m}}^2}\frac{\partial^2}{\partial t^2}\right]u(x,z,t)=0 \tag{1}$$

where v_{m} is the electromagnetic wave propagation velocity in the medium, t is the time. According to the property of Fourier transform, applying a three-dimensional Fourier transform on equation (1) over x domain, z domain and t domain yields

$$\left(k_{\mathrm{x}}^2+k_{\mathrm{z}}^2-\frac{\omega^2}{v_{\mathrm{m}}^2}\right)U(k_{\mathrm{x}},k_{\mathrm{z}},\omega)=0 \tag{2}$$

where k_{x}, k_{z} are the x direction and z direction components of wave number vector, ω is the angular frequency corresponding to time t. Since $U(k_{\mathrm{x}},k_{\mathrm{z}},\omega)\neq 0$, we have

$$k_{\mathrm{x}}^2+k_{\mathrm{z}}^2-\frac{\omega^2}{v_{\mathrm{m}}^2}=0 \tag{3}$$

Equation (3) is known as the dispersion relation for 2-D wave equations. Now applying a two-dimensional Fourier transform on the equation (1) over x domain and t domain generates

$$\left(k_{\mathrm{x}}^2+\frac{\partial^2}{\partial z^2}-\frac{\omega^2}{v_{\mathrm{m}}^2}\right)U(k_{\mathrm{x}},z,\omega)=0 \tag{4}$$

Substituting eq. (3) into eq. (4), the equation can be obtained as

$$\left(\frac{\partial^2}{\partial z^2}-k_{\mathrm{z}}^2\right)U(k_{\mathrm{x}},z,\omega)=0 \tag{5}$$

Considering equation (5) to be a homogeneous second-order linear constant coefficient ordinary differential equation, the solution is as

$$U(k_{\mathrm{x}},z,\omega)=C_0\mathrm{e}^{-\mathrm{j}k_{\mathrm{z}}z}+C_1\mathrm{e}^{\mathrm{j}k_{\mathrm{z}}z} \tag{6}$$

where C_0, C_1 are constant coefficients. $C_0\mathrm{e}^{-\mathrm{j}k_{\mathrm{z}}z}$ and $C_1\mathrm{e}^{\mathrm{j}k_{\mathrm{z}}z}$ are corresponding the upgoing wave and downgoing wave. Because there is only an upgoing EM wave in the measurement data. Equation (6) can be rewritten as:

$$U(k_{\mathrm{x}},z,\omega)=C_0\mathrm{e}^{-\mathrm{j}k_{\mathrm{z}}z} \tag{7}$$

Assigning $z=0$, we can obtain $C_0=U(k_{\mathrm{x}},0,\omega)$. Considering eq. (3), the solution of eq. (5) can be expressed as

$$U(k_{\mathrm{x}},z,\omega)=U(k_{\mathrm{x}},0,\omega)\,\mathrm{e}^{-\mathrm{j}z\sqrt{\frac{\omega^2}{v_{\mathrm{m}}^2}-k_{\mathrm{x}}^2}} \tag{8}$$

where $U(k_{\mathrm{x}},0,\omega)$ is the electromagnetic field in frequency wave number domain at range $z=0$, where is the radar plane. $U(k_{\mathrm{x}},0,\omega)$ can be obtained by applying two-dimensions Fourier transform on $u(x,z=0,t)$, which again represents the radar measurement data. Because $U(k_{\mathrm{x}},0,\omega)$ can be obtained and known, the electromagnetic field in frequency wave number domain can be determined and is unique based on equation (8). This equation shows that the extrapolation of electromagnetic field $U(k_{\mathrm{x}},z,\omega)$ in wave number-frequency domain along the z-axis is just a phase shift operation. By recursively extrapolating the wave field in wave number-frequency domain along the z-axis in steps Δz and using the resulting EM field of each step as input to the next iteration, the frequency-wave number domain distribution of the field $U(k_{\mathrm{x}},z,\omega)$ can be reconstructed. After the field distribution in frequency-wave number domain is reconstructed, the electromagnetic field distribution $u(x,z,t)$ in time-space domain can be calculated by using inverse Fourier transform. This electromagnetic field $u(x,z,t)$ stands for the received data at the range position z. Assigning $t=0$, $u(x,z,t=0)$ means the radar received data on the time equal to zero at the position (x,z), which also means the reflection intensity on the point (x,z). According to above discussion $u(x,z,t=0)$ is the focused radar image after phase migration procedure. The procedure of the 2-D phase shift migration is as:

Step 1. Fourier transform of the measurement data over x and t:

$$U(k_{\mathrm{x}},z_0=0,\omega)=\iint u(x,z_0=0,t)\,\mathrm{e}^{-\mathrm{j}k_{\mathrm{x}}x}\mathrm{e}^{-\mathrm{j}\omega t}dxdt \tag{9}$$

When the radar system uses stepped frequency signal, the received signal is as $U(x,z=0,\omega)$, and the Fourier transform operating is only needed over x:

$$U(k_{\mathrm{x}},z_0=0,\omega)=\iint u(x,z_0=0,\omega)\mathrm{e}^{-\mathrm{j}k_{\mathrm{x}}x}dx \tag{10}$$

Actually, the received signal is recorded along the x direction with limited discrete locations. Instead of Fourier transform (FT), the discrete Fourier transform (DFT) should be used. However, for computational efficiency, fast Fourier transform (FFT) is used in practice.

Step 2. Calculate the electromagnetic field at a new range location $z_1 = z_0 + \Delta z$ in frequency wave number domain by phase migration operation as eq. (8). Considering the radar signal as round trip signal, the electromagnetic field at a new range location can be expressed as

$$U\left(k_{\mathrm{x}}, z_1, \omega\right) = U\left(k_{\mathrm{x}}, z_0 = 0, \omega\right) \exp\left(\mathrm{j}\sqrt{\frac{4\omega^2}{v_{\mathrm{m}}^2} - k_{\mathrm{x}}^2}\Delta z\right) \tag{11}$$

Step 3. Using inverse Fourier transform operation along k_{x} direction to obtain the electromagnetic field into frequency-space domain

$$u\left(x, z = z_1, \omega\right) = \frac{1}{2\pi}\int U\left(k_{\mathrm{x}}, z = z_1, \omega\right) \mathrm{e}^{\mathrm{j}k_{\mathrm{x}}x} dk_{\mathrm{x}} \tag{12}$$

Step 4. Using inverse Fourier transform operation along ω direction to obtain the electromagnetic field into time-space domain and choose time equal to zero $t = 0$ to obtain the reflection intensity as local position.

$$u\left(x, z = z_1, t = 0\right) = \frac{1}{2\pi}\int u\left(x, z = z_1, \omega\right) \mathrm{e}^{\omega\times(t=0)} d\omega \tag{13}$$

Step 5. Repeat steps 2 to 4 to obtain the electromagnetic field distribution in frequency-wave number domain and reflection (scattering) intensity image.

The procedure steps of phase migration is as Fig. (2). In physical understanding, the procedure of phase migration is like parallel shifting the position of radar observation (measurement) plane from $z = 0$ to a position $z = z_1$. By phase shift migration processing, the electromagnetic field $u\left(x, z = z_1, t\right)$ can be calculated by practically measurement $u\left(x, z = 0, t\right)$, and $u(x, z = z_1, t)$ means the equivalent measured data by radar on the location (x, z_1). Selecting $t = 0$ value of the equivalent measurement data $u(x, z = z_1, t)$ obtains the local reflection, while $t = 0$ means no time delay return. Equivalently, phase migration processing shifts the location of radar measurement. When the location of radar measurement is same as the location of targets, the reflection of targets will appear on the measurement time equal to zeros $t = 0$.

It should be noted that it is difficult to have a focused radar subsurface imaging by applying the conventional synthetic aperture radar (SAR) technique on ground penetrating radar system because of the existence of electromagnetic refraction on the subsurface interfaces. When electromagnetic wave penetrated into the subsurface, the propagation velocity will be influenced by the subsurface medium and the propagation direction will also be changed due to the refraction appearing on the interface between two different dielectric constant mediums. In this case, the signal phase delays do not mean the direct distance between the radar and targets. However, phase

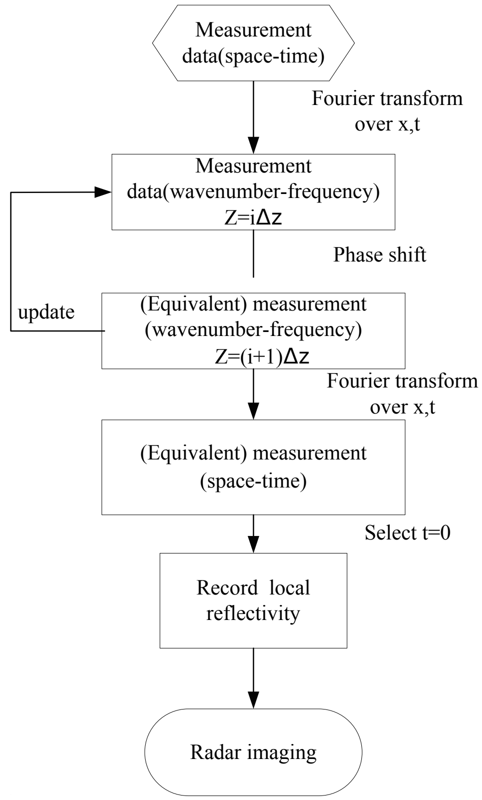

Fig. 2. Flow chart of phase migration

shift migration algorithm can easily deal with the propagation velocity variation in the subsurface medium by making the velocity v_{m} as a function of z and assuming v_{m} be constant inside each small interval Δz.

III. Application of Phase Shift Migration Approach

To detect buried objects and obtain the subsurface structure imaging precisely, the dual polarization GPR system is implemented. It operates over the frequency band from 0.3 GHz to 3 GHz with stepped frequency signal. Both of the transmitter and receiver consist of dual polarized double Vivaldi tapered slot antennas. By using the dual polarization antenna as transmitter and receiver, the radar system can simultaneously receive co and cross polarization echo (reflection signal). The measurement is made on a frozen pond. The measurement data and processed radar imaging of the most interesting track in time domain are shown as Fig. (3). Obviously, the phase shift migration approach processed radar imaging is more focused. The first strong return is the reflection by the ice surface. The reflection of pool bed, basalt bed and buried scatterers can also be seen.

A. Background Removal

For GPR systems one important objective is to find buried scatterers. For this aim, the reflection signals from surface and subsurface interfaces should be removed[2]. However, the reflections of buried objects appears only when the radar

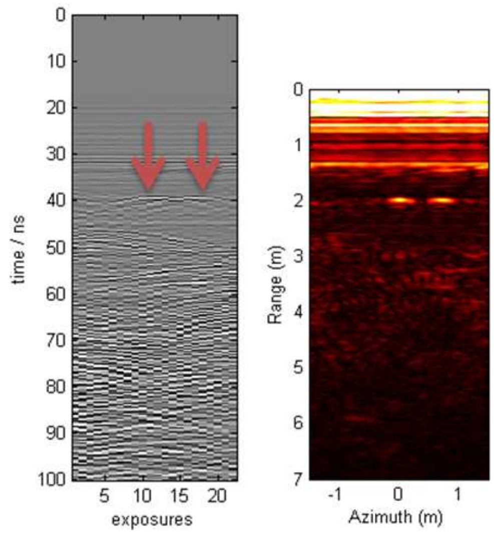

Fig. 3. Phase migration: the red arrows marks two hyperbola in the raw data (left) which are focused in the phase migration processed results (right)

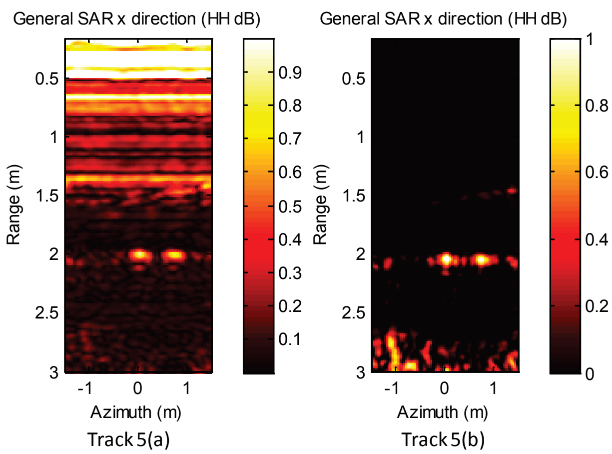

Fig. 4. Scatterers detection with background removal and SEC

illuminates them. The reflection signals of buried objects vary during the radar moving. Herein, the reflections from the subsurface and subsurface interfaces are seen as background reflections. Based on the difference of reflections from the buried objects and background during radar moving, a background removal algorithm is implemented on removing the reflections from the surface and subsurface interface. The background removal approach can be expressed as:

$$\hat{u}\left(x_i, z=0, t\right) = u\left(x_i, z=0, t\right) - \frac{1}{N_x}\sum_{j=1}^{N_x} u\left(x_j, z=0, t\right) \tag{14}$$

where N_x is the measurement times along x-axis.

B. Spreading and exponential correction

The waves transmitted by the GPR are subjected to the free-space path loss and losses in the material. Therefore, the reflections of scatterers in deep range become relatively weak. To compensate these attenuations and obtain relative accurate scatterers reflections, a spreading and exponential correction (SEC) algorithm is applied on the measurement data[2], [3]. The main idea of the SEC algorithm is increasing (or compensating) the reflection signal amplitude as the reflection signal time delay increasing. The compensation works as a gain function g(t), which is expressed as:

$$g\left(t\right) = \begin{cases} \left(\frac{t-t_0}{t_0}\right)^{sc} \cdot \mathrm{e}^{\frac{t-t_0}{t_{\mathrm{end}}-t_0}\cdot ec} & t \geq t_0 \\ 0 & t < t_0 \end{cases} \tag{15}$$

where t_0 is the start point of the SEC technique application, t is the time delay of the reflection signal. t_{end} represents the unique domain of the radar. The sprading correction is adjusted by sc, while by ec the medium losses are compensated. The values for sc and ec were adjusted, so that the phase migration yields the most interpretable results. After applying the background removal and SEC algorithm, the phase shift migration processed radar imaging is as Fig. (4b). Comparing the phase shift migration processed radar imaging without background removal and SEC algorithm as figure(4a), it is obvious to find that the background reflections disappeared and the scatter reflections become more clear.

IV. Conclusion

In this paper the phase shift migration based SAR imaging technique are described in detail. In the final paper we'll turn our attention to more measured and processed data, for instance the cross-polarized data.

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