A Robustness Analysis of Master-Model Based controller for stabilizing delayed haptic interaction: An LMI Approach

Hichem Arioui, Abderrahmane Kheddar, Saïd Mammar

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H. Arioui and A. Kheddar and S. Mammar*

Abstract

This paper extends previous results on stable haptic and force feedback interaction in the presence of constant and varying time-delay. The proposed method evolved from our previous earlier work where we apply Smith prediction principle on delayed haptic feedback interaction.

We will show that the proposed approach is easy to implement and it is original since it investigates another way to formulate stable haptic feedback algorithm and can be generalized for n-degree of freedom. It requires to know only the master interface

1. Introduction

There are many applications which require to feed back haptic cues to human operator: teleoperation, telepresence, virtual reality simulators such as virtual surgery training, driving simulators, etc. Commonly, haptic cues are fed back thanks to an actuated mechatronic device able to constraint the operator desired motion against the applied reaction forces.

Contrarily to the vision or auditory human modalities, haptic cues are collected through a direct and dynamic (i.e. active) human contact (touch) with its surrounding environment. In teleoperation and telepresence contexts, haptic informations are collected through a bilateral coupling between the human held or worn haptic display and a remote robotic system, able to replicate human desired motions and to collect haptic information during contact or constraint motions. Other applications based on virtual reality techniques, such as haptic feedback surgery training simulators, require also haptic devices to experience haptic interaction between the human operator and virtual environments (VE). This is made thanks to computer haptics algorithms which compute collision detection and sub-

sequent dynamic response motions and forces during contact between virtual objects.

Consequently, haptic devices are commonly active actuated mechatronic devices which must be controlled stably to be correctly used and t o avoid danger to operator.

In teleoperation technology, the remote executing machine could be far from the operator. Thus transmitted data (desired operator actions or trajectory and sensory feedback informations) could be delayed. Moreover, industry is also attracted by this technology in the frame of virtual prototyping. This extend also to the possibility to work at a distance and to concurrent engineering. Nevertheless, it is well known that even small time delay (experimentally reported to be $1/4$ sec in teleoperation) destabilizes any conventional-coupled haptic feedback architecture. This is an old problem in teleoperation and many solutions have been proposed to deal with this problem [3, 10, 15, 11]. Many solutions developed in the frame of teleoperation have been adapted to VE haptic feedback (i.e. the adaptation consists mostly in a clever discretization of the continuous ones). At the beginning, the proposed solutions where designed for constant time delay, and more recently extended to cope with time-varying delay.

In this paper, an original solution derived from Smith prediction [14] is proposed to design stable and transparent controllers for haptic feedback systems. The proposed controller requires only the master model [4, 8]. The solution is proven to be stable and also robust to master device parameters estimation. Simulation results are convincing and the implementation of the solution in any haptic system is very simple.

The other strong point of the solution is that it do not require to estimate the time delay or its behavior.

2. Previous work

In a recent previous work [7, 12], the principle of Smith prediction have been adapted for the synthesis of a stable haptic feedback controller to be used with constant time delay. Although Smith prediction method is

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*Authors are with Laboratoire Systmes Complexes, CNRS Fre 2494, University of Evry Val d'Essonne, 40 rue du Pelvoux, Evry, 91020, France. `arioui@iup.univ-evry.fr`

known since 1957 [14], it has not been implemented in the early time delayed force reflecting teleoperation systems. The reason which prohibits using the Smith prediction approach follows reasonably from the practical impossibility to predict mainly (i) the remote environment behavior and, (ii) the operator desired trajectories, since they are given on-line. Concerning virtual reality applications, since most controllers come from teleoperation experience, it was not surprising to notice that Smith prediction was not investigated as a potential solution for timedelayed VE haptic feedback controllers.

The originality of the proposed solution is in the some-how prediction of the master part within the remote part. Hence, the developed equations lead to a somehow prediction scheme where only the master model appears and also the estimation of the time delay is necessary. The term somehow prediction is used to signify that in fact the proposed solution is not really a prediction since only the master model is required, which means that no prediction of operator behavior or trajectory is needed. But, the upwards and forwards time delays must be known.

Figure 1. The master-model-based Smith prediction principle in the frame of a nominal LTI haptic feedback architecture.

Without loss of generality and to better understand the concept, a simple LTI model of a VE haptic interface has been taken. Figure 1 shows the implementation of the proposed controller (colored part of the block diagram representation) within the haptic architecture. $M(s)$ is the haptic device transfer function, s is the Laplace transform variable, $E(s)$ is a VE transfer function (assumed continuous with a high sampling frequency), x_m, x_c and x_e are respectively master, vir-

tual coupling and VE positions, F_e is the VE computed force, F_h is the operator applied force on the device, $C(s)$ is the commonly used virtual coupling [1] and [5], which guarantee unconditional stability of the haptic interaction system in the absence of time-delay, and τ_1, τ_2 are respectively upwards and forwards constant time delays. The closed loop transfer function of the haptic system without the proposed controller is given by:

$$\frac{F_e(s)}{F_h(s)} = \frac{M(s)E(s)e^{-s\tau_1}}{1 + e^{-s(\tau_1+\tau_2)}E(s)\left(M(s)+C(s)\right)} \quad (1)$$

the obtained closed loop system has an infinite eigenvalues because the time delay element is present in the characteristic equation. This may consequently imply an instability of the whole haptic interaction.

The proposed solution to overcome this instability is designed within the colored box of figure 1. The controller uses the process model of the haptic display like a local feedback loop within the local remote environment (real or virtual). The resulting transfer function of the global system is a stable haptic feedback with a delayed input F_h:

$$\frac{F_e(s)}{F_h(s)} = \frac{M(s)E(s)e^{-s\tau_1}}{1 + E(s)\left(M(s)+C(s)\right)} \quad (2)$$

In the final obtained result, and when using the proposed controller, we can notice that there is no more delay items in the characteristic equation of the closed loop system. As stated before, the main advantage of this control-prediction scheme is in using the model of the haptic reflecting device only. The latter is well know and its parameters well identified. However, the controller requires to estimate both upwards and forwards time-delay.

Obviously, this example can be easily generalized to any kind of haptic devices and mainly teleoperation systems. In this last case, $E(s)$ represents the linear model of the remote system interacting with its environment. Simulation results have been performed and discussed in [2, 4], the experimental one in [7].

3. Controller design

As previously stated, the proposed method needs:

- a good knowledge of the master model, and

- the estimation of time delay

Estimation of the model do not require complicated techniques. Well known model estimation methods (namely those developed in robotics) can be applied. Estimation of time delay is also easily made,

especially when the delay is constant. Simple networking commands (such as a ping command) achieves the matter. Practically, the time delay may fluctuates since neither public or usual networking access protocols nor the computer haptics algorithms (collision detection algorithms and dynamic force computation, in the frame of VE haptics) can guarantee time determinism.

3.1. Extension of the Controller design

When we look well to the proposed controller (figure 1) we notice that in fact, the desired remote position is corrupted by a local closed loop on the obtained contact force. Indeed, the actual x_e is not directly the desired master position x_m, but the delayed x_m minus the local closed controller based on the master model, the delayed master model and the obtained contact force.
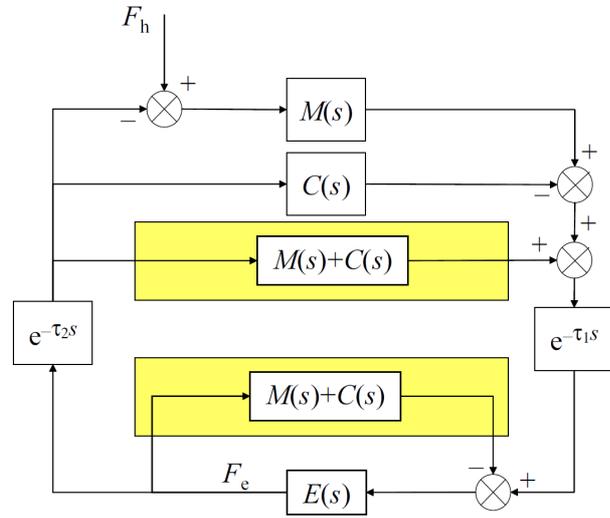
Figure 2. A novel implementation of the controller

At the beginning, the controller have been implemented as it is. But from a practical and a simple observation, the structure of the controller made an interesting extension which:

1. avoids the estimation of time-delay, and

2. make a straightforward extension to time varying time delay.

This is obtained simply as depicted in figure 2. The controllers in the fgures 1 and 2 are identical. Nevertheless, the second implementation highlights that it is no more necessary to estimate time delay, and more importantly: the behavior of time delay may have no effect on the stability of the system. The next section gives

a more generic discussion and proves the given assertions. The dynamic model of a haptic display can be approximated in a linear form, considering an apparent mass m and friction b:

$$F_h(t) - F_e(t - \tau_2(t)) = m\ddot{x}_m + b\dot{x}_m \qquad (3)$$

where x_m, $\dot{x}_m$ and $\ddot{x}_m$ are respectively the Cartesian space position, speed and acceleration, F_h and F_e denote respectively human and VE forces applied to the haptic device. Delays uses for the proof of the stability with the proposed control scheme are variable.

The fundamental idea is to emulate a passive behavior of the haptic device and the transmission channel. In the second comparator we have $x_h = x_m + x_c$ and knowing that $F_e(t - \tau_2(t)) = \hat{m}\ddot{x}_c + \hat{b}\dot{x}_c$ we obtain:

$$F_e(t - \tau_2(t)) = \hat{m}(\ddot{x}_h - \ddot{x}_m) + \hat{b}(\dot{x}_h - \dot{x}_m) \qquad (4)$$

Where $\hat{m}$ is the estimate apparent mass, $\hat{b}$ is the estimate friction. The previous equation is used to cancel the effect the delayed control $F_e(t - \tau_2(t))$ from the haptic device (master) position xm to be sent to the slave site.

Just after the transmission channel $\tau_1(t)$, at the third comparator we have:

$$F_e(t) = \hat{m}(\ddot{x}_h(t - \tau_1(t)) - \ddot{x}_{mr}) + \hat{b}(\dot{x}_h(t - \tau_1(t)) - \dot{x}_{mr}) \qquad (5)$$

where x_{mr} is the after-master position transmitted to the slave site. Substraction equation 4 delayed by $\tau_1(t)$ into the equation 5 leads to:

$$F_e(t - \tau_1(t) - \tau_2(t)) - F_e(t) = \hat{m}(-\ddot{x}_m(t - \tau_1(t)) + \ddot{x}_{mr}) + \hat{b}(-\dot{x}_m(t - \tau_1(t)) + \dot{x}_{mr}) \qquad (6)$$

At last, equation 1 is delayed also by $\tau_1(t)$, and the obtained $F_e(t - \tau_1(t) - \tau_2(t))$ is substituted in equation 5 leads to:

$$F_h(t - \tau_1(t)) - F_e(t) = \hat{m}\ddot{x}_{mr} + \hat{b}\dot{x}_{mr} + (m - \hat{m})\ddot{x}_m(t - \tau_1(t)) + (b - \hat{b})\dot{x}_m(t - \tau_1(t)) \qquad (7)$$

If we assume that the estimation error of the apparent mass and friction is zero ($m - \hat{m} = 0$ and $b - \hat{b} = 0$), then the equation 7 will have the following form:

$$F_h(t - \tau_1(t)) - F_e(t) = \hat{m}\ddot{x}_{mr} + \hat{b}\dot{x}_{mr} \qquad (8)$$

This last equation shows a passive behavior of the equivalent new master side. The correction is equivalent

to delay the input F_h. Assuming that the virtual environment passive, a fundamental property is that the feedback interconnection of passive systems is again passive(see [6] and [13] for more informations), it ensues from it, that the haptic interaction is stable.

4. Robustness Analysis

This section focuses on robustness analysis, which involves effects of error estimation in the model of the haptic device on stability of whole interaction. To simplify the robustness analysis of the control we consider the following linear model of the haptic interface (and a constant time delay), equation 7 becomes:

$$
\begin{aligned}
F_h(t-\tau_1) - F_e = &\hat{m}\ddot{x}_{mr} + \hat{b}\dot{x}_{mr} \\
&+ (m-\hat{m})\ddot{x}_m(t-\tau_1) + (b-\hat{b})\dot{x}_m(t-\tau_1)
\end{aligned} \quad (9)
$$

After slight calculation, we have:

$$
\begin{cases}
X_1 = x_{mr}, X_4 = x_m \\
X_2 = \dot{x}_{mr} = \dot{X}_1 \\
\dot{X}_2 = \frac{1}{m}F_h(t-\tau_1) + \frac{\tilde{m}}{\tilde{m}m}F_e(t-\tau_1-\tau_2) \\
\quad - \frac{1}{\hat{m}}F_e(t) - \frac{\hat{b}}{\hat{m}}X_1 - \frac{1}{\hat{m}(\tilde{b}-b\frac{\tilde{m}}{m})}X_4(t-\tau_1)
\end{cases} \quad (10)
$$

The virtual environment that we adopted is stiff virtual wall of K_e, the force $F_e(t)$ generated by the virtual environment is:

$$
F_e(t) = K_e(X_e - X_{wall}) \quad (11)
$$

The $C(s)$ is a proportional derivative controller (B_c: friction coefficient, K_c: stiffness coefficient) relating $F_e(t)$ to X_{mr} and X_e:

$$
F_e(t) = B_c(\dot{X}_{mr} - \dot{X}_e) + K_c(X_{mr} - X_e) \quad (12)
$$

this last equation can be rewritten as:

$$
\begin{aligned}
X_3 &= X_e \\
\dot{X}_3 &= \frac{K_c}{B_c}(X_1 - X_3) + X_2 + \frac{1}{B_c}F_e(t)
\end{aligned} \quad (13)
$$

Form the original haptic device, we have:

$$
\frac{1}{m}\left(F_h(t) - F_e(t-\tau_2) - b\dot{X}_m\right) = \ddot{X}_m \quad (14)
$$

In non nominal parameters conditions, the closed loop of the autonomous system takes the following state space form:

$$
\dot{X} = \begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & -\frac{\hat{b}}{\hat{m}} & 0 & 0 \\
\frac{K_c}{B_c} & 1 & -\frac{K_e+K_c}{B_c} & 0 \\
0 & 0 & 0 & -\frac{b}{m}
\end{bmatrix} X +
$$

$$
\begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\frac{b}{\hat{m}}(\frac{\hat{b}}{b} - \frac{\hat{m}}{m}) \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix} X(t-\tau_1) +
$$

$$
\begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & -\frac{\tilde{m}}{K_e\hat{m}m} & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix} X(t-\tau_1-\tau_2) +
$$

$$
\begin{bmatrix}
0 & 0 \\
\frac{1}{\hat{m}} & 0 \\
0 & 0 \\
0 & \frac{1}{m}
\end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \quad (15)
$$

where the state is $X = [x_{mr}, \dot{x}_{mr}, x_e, x_m]^T$, $u_1 = F_h(t-\tau_1) - F_e(t)$ is the control input of the new master side and $u_2 = F_h(t) - F_e(t-\tau_2)$ is nominal master control. The term $\frac{\tilde{m}}{K_e\hat{m}m}$ is neglected, because K_e is very important coefficient to simulate rigid wall. Stability of this system can be established on the basis of the autonomous system $\dot{X}(t) = A_0X(t) + A_dX(t-\tau_1)$. In [9], stability of such systems against delay values has been studied using combination of integral quadratic (IQCs) constraints and linear matrix inequalities (LMIs). The delay τ_1 is first modeled in an inverse uncertainty formulation of the form $\Delta(s) = \frac{1}{\tau_1 s}(e^{-\tau_1 s} - 1)$. The autonomous system can thus be transformed into an interconnection feedback configuration :

$$
\begin{cases}
\dot{X} = AX + Bu \\
y = CX + Du \\
u = \Delta(y)
\end{cases} \quad (16)
$$

where $A = A_0 + A_d$, $B = \tau_1 H$, $C = EA$, $D = \tau_1 EH$, H and E are full column and row rank respectively such that $A_d = HE$.

Assuming that the closed loop system is stable when there is delay, the autonomous system with delay is stable if there exist symmetric definite matrices P and Q and a skew matrix S such that

$$
\begin{bmatrix}
A^T P + PA + C^T QC & PB + C^T S + C^T QD \\
B^T P + S^T C + D^T QC & D^T QD + S^T D + D^T S - Q
\end{bmatrix} < 0 \quad (17)
$$

For given matrices A_0 and A_d, it is thus possible to determine the maximal acceptable delay by maximizing τ_1 under LMI using classical available software package. As our matrices are parameters varying, the stability region is determined as 3D plot function of b, m and τ_1.

5. Simulation results

This section presents simulation results of the developed controller. The haptic display is a one DOF ac-

tuated arm with apparent mass $m = 0.2kg$ and friction of about $b = 3Ns/m$. We took a virtual environment and a virtual probe to be the slave. The contact will be performed between the rigid probe and a virtual walls of stiffness $K_e \in [1000 - 10000]N/m$ (tosignify rigid contact interaction). In this first simulation, time delays are taken constant but different, indeed $\tau_1 = 1sec$ and $\tau_2 = 0.5sec$.
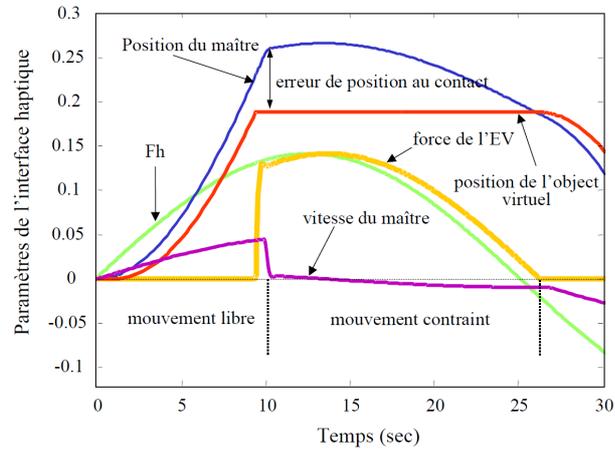

Figure 3. Simulation of delayed virtual contact with a stiff wall and force feedback.

Figure 3 shows the result of a simulation virtual contact. The operator apply a sinusoidal force F_h which drops the master and probe positions to increase until a contact is made between the probe and the wall.

This is done when the virtual probe position reach $20cm$. From this time the local VE controller $C(s)$ guarantees the local stability of the virtual interaction and the calculation of the virtual force to feed back to the operator. One can notice that when the contact is make, the mater velocity v_m drops to zero and the fed force F_e (the controller) increases accordingly to F_h during the contact. The position discrepancy, when the contact is made, is unavoidable whatever is the controller or the approach (unless a very prediction is made in the master side), this is due to the undergo physical time-delay. Nevertheless, the virtual probe position x_e is stably maintained by the operator during the whole contact time. We conducted many other simulation cases with multiple hard and viscous contacts that show that the behavior of the force feedback interaction is stable whatever the time delay. Obviously, one must not suspect that functional performances are acceptable for an actual use in the presence of important time delays.

Figure 4 shows the result of the previously set up system under similar parameters and time-varying delay. The variation of the delay τ_1 is represented on the same figure 4 and satisfies the causality constraints:
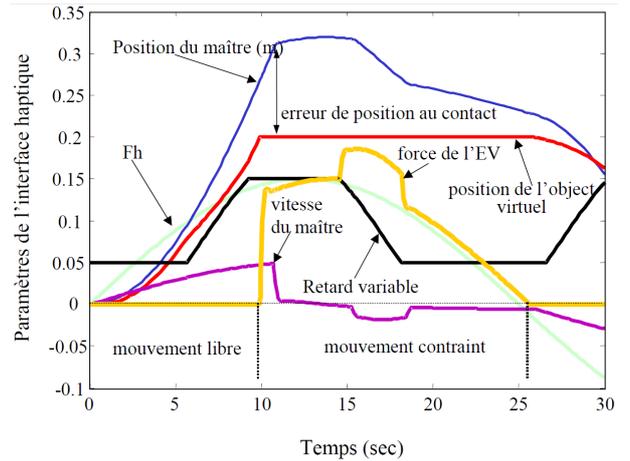
Figure 4. Similar simulation of haptic interaction under time-varying delay.

$t - \tau_1 > 0$ and $\dot{\tau}_1 < 1 \, \forall t$. In the simulation $0.5 \le \tau_1(t) \le 1.5sec$. We performed three different simulations: 1) time-varying $\tau_1(t)$ with $\tau_2(t)$ constant, 2) time-varying $\tau_2(t)$ with $\tau_1(t)$ constant and, 3) time-varying $\tau_1(t)$ and $\tau_2(t)$. Obtained simulation results shows that $\tau_2(t)$ dynamics has no effect on the stability of the overall force feedback system. This results make true the derived theoretical assumption stating that the adopted new implementation of the developed controllers cancels the effect of time delay. By analogy to the constant time delay case, since the time delay exponential are not present (after correction) in the characteristic equation. So in case 2, the simulation shows a stable behavior of the system, as if $\tau_2(t)$ was constant i.e. the dynamic of $\tau_2(t)$ is compensated by the controllers. For cases 1 and 3, clearly the dynamic of $\tau_1(t)$ affects the dynamic of the force reflecting system. Figure 4 shows clearly that in the transitions between two time delays behavior of $\tau_1(t)$ i.e. varying and constant, the force reflecting response switches between two over-all system corresponding behaviors. The transitions seem to be abrupt but do not affects the overall stability of the system. Similarly to the constant time delay linear case, this is related to the fact that the $\tau_1(t)$ exponential is still present in the closed loop transfer function numerator. Indeed the behavior of $\tau_1(t)$ affects the behavior of force feedback but not its stability, as proven in theory (equation 8).

Robustness results of the above system has been also performed according to the algorithm described in section (4). Figure 5 shows that within a determined margins, force reflection is still stable although some light oscillati ons app ear in t he master position x_m and the reflected force F_e. In this simulation in figure
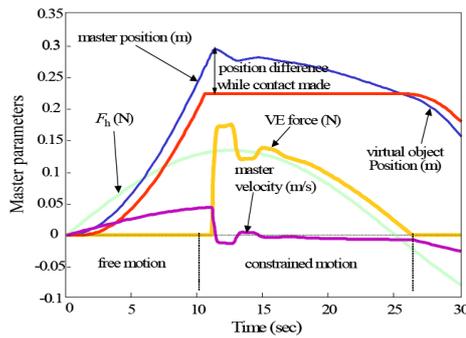
Figure 5. Simulation behavior with an error estimation of the master model parameters (mass and friction).

5 $\hat{m} = 0.3 kg$ and $\hat{b} = 5 Nm/s$ are the estimated parameters, we can notice that the behavior of the interaction is still stable with some oscillations which become more important when the error $-\frac{b}{\hat{m}}(\frac{\hat{b}}{b} - \frac{\hat{m}}{m})$ (the only term in the matrix A_d) is greater.

6. Conclusions

The paper presents a master-model based controller to stabilize delayed force feedback systems. The proposed method lies in an astute implementation of a somehow Smith prediction scheme, which requires only the haptic device model and do not necessitate the estimation of both (upwards and downwards) delays. The simulation results confirm a stable force reflection from the VE in presence of constant and also time-varying delays. A robustness analysis of the proposed controller has also been conducted. The error margins that guarantee the stability of fed back forces are found to be wide enough to allow using a linear model of the haptic interface based only on a apparent mass and friction estimation.

Comparing to wave-based approaches, this proposed solution is more transparent to the user, since there is no additional corrupting damping as engendered from the transformation of force and flux parameters into waves. The price to be paid is in the importance of the position discrepancy between the master and the slave when the contact is made (here the actual virtual object position) which may be more important in our case comparing to wave-based methods. In fact, in wave-based method, the artificial damping increases with speed (in free motion), which prevent important master-slave position discrepancies, but the price paid

is an additional felt force which is not directly related to actual remote contact forces, that is to say more stable but less transparent.

Future work is focused in the following :

- improve performances in virtual environment haptics. A prediction within the master site is possible based on computer haptics algorithms,

- extend the theory to a more generic non-linear case (in progress and results will be published soon elsewhere).

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