Electre Methods: Main Features and Recent Developments

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Abstract. We present main characteristics of Electre family methods, designed for multiple criteria decision aiding. These methods use as a preference model an outranking relation in the set of actions – it is constructed in result of concordance and non-discordance tests involving a specific input preference information. After a brief description of the constructivist conception in which the Electre methods are inserted, we present the main features of these methods. We discuss such characteristic features as: the possibility of taking into account positive and negative reasons in the modeling of preferences, without any need for recoding the data; using of thresholds for taking into account the imperfect knowledge of data; the absence of systematic compensation between "gains" and "losses". The main weaknesses are also presented. Then, some aspects related to new developments are outlined. These are related to some new methodological developments, new procedures, axiomatic analysis, software tools, and several other aspects. The paper ends with conclusions.

Les Méthodes Electre : Traits Caractéristiques et Développements Récents

Résumé On commence par rappeler que les méthodes Electre ont été conçues pour l'aide multicritère à la décision dans une optique constructiviste. Elles font intervenir des relations de surclassement construites à partir des concepts de concordance et de non discordance. On examine dans la seconde partie les traits caractéristiques de ces méthodes : aucune nécessité de recoder les critères tels qu'ils ont été concrètement définis ; possibilité de prise en compte des raisons positives et négatives dans la modélisation des préférences sans qu'un recodage des données soit nécessaire ; utilisation possible de seuils pour prendre en compte la part d'arbitraire dans la modélisation des critères ; agrégation des critères ne reposant pas sur une compensation systématique entre "gains" et "pertes". Cette deuxième partie se termine par une discussion des points forts et des points faibles de ces méthodes. Ces méthodes ont connus d'importants développements récents. On les passe en revue dans la troisième partie : développements méthodologiques, nouvelles procédures, analyse axiomatique, outils informatiques et nombreux autres aspects.

1 Introduction

Since their conception, which started in the sixties of the last century, ELECTRE methods have been widely used for Multiple Criteria Decision Aiding (MCDA) in many real-world decision problems, ranging from agriculture to environment and water management, from finance to project selection, from personnel recruiting to transportation, and many others. The theoretical research on the foundations of ELECTRE methods has also been intensive all this time. In this paper, we present the main features of ELECTRE methods, we characterize their strengths, as well as their weaknesses, paying a particular attention to the developments of the last decade.

In the next sub-section, we explain what we mean by constructivist or "European" conception of MCDA. The term "European" does not mean, however, that this conception was only developed and followed by Europeans. A large number of researchers all over the Globe are being working in this area and are applying the techniques in real-world problems, for example, in Canada, Tunisia, Poland, Switzerland, Italy, Spain, Portugal, Germany, New Zealand, and many other countries.

In the sub-section 2.1, the basic notation are introduced.

1.1 The Constructivist Conception of MCDA

This sub-section is based on the speech of Roy (Roy 2010), delivered on the 30th of January 2009, on receiving a honorary doctoral degree from the *Università degli Studi di Catania*, Italy.

Before introducing the constructivist or "European" conception of an MCDA methodology, we should present the meaning of a decision aiding situation and its key elements, and three fundamental pillars that support such a conception. In what follows, the term "decision aiding", rather than "decision support", "decision making", or "decision analysis", will be adopted for escaping from simplistic assimilations.

1.1.1 Decision Aiding Situation/Decision Aiding Process

Consider a company or a public institution, where a manager and/or a group of people are confronted with a decision situation or "problem" that requires them to make a decision. They call on an in-house operational research service or an outside consultant or even a university research team to get help in making "the best possible" decision. This aspect allows to characterize a *decision aiding situation*, where two key actors are relevant for co-interaction that will lead to build and make evolve the *decision aiding process*; a process that comprises several phases, see (Roy 1996). The two key actors will be designated in what follows as the *analyst*, who is appointed to give this decision aiding, and as the *decision maker*, in whose name or for whom this decision aiding is to be given.

1.1.2 Three Fundamental Pillars

In the operational research and decision aiding community, to which we belong, the *decision-aiding activity* (which is meant to be scientific) is founded on three pillars:

1) The *actions* (formal definition of the possible actions or alternatives),

2) The *consequences* (aspects, attributes, characteristics, ... of the actions, that allow to compare one action to another), and

3) The *modeling of one or several preference systems* (it consist of an implicit or explicit process, that for each pair of actions envisioned, assigns one and only one of the three possibilities (see sub-section 2.1): *indifference*, *preference*, or *incomparability*).

The last pillar needs further explanation. When given two possible actions, any individual, whoever he/she may be, based on the actions' consequences, and his/her value system, can state: "I prefer the first to the second" or *vice-versa*, "I am indifferent between the two", or "I am unable to compare these two actions". *Modeling a preference system* means to specify a process that will provide this type of results based on a pre-established model of the action consequences. These consequences are most often complex and inadequately known. They can be modeled in quantitative or qualitative terms, in a deterministic or stochastic manner, with a part of arbitrariness or ill determination. We will designate by $C(a)$ the model of the consequences of action a.

1.1.3 The "European" Conception of MCDA

According to the "European" conception, the analyst must seek for obtaining *a coherent and structured set of results*. These results should be sought in order to guide the decision aiding process and facilitate communication about the decisions. To do so, the analyst must use an approach that aims at *producing knowledge from working hypotheses*, taking into account the objectives and the value systems involved in a particular decision context. This approach should be based on models that are, at least partially, *co-constructed through interaction* with the decision maker. This co-construction first concerns the way the considered actions are taken into account, as well as the consequences on which these actions will be judged. Secondly, the *co-construction process* concerns the way that certain characteristics (notably the values attributed to the different parameters) of the preference model were judged the most appropriate given the specificities of the decision context and the working hypotheses retained. In this conception, it is no longer necessary to assume that there exists, in the mind of the decision maker, a stable procedure capable of defining the decision maker's preference system completely, before even beginning the decision aiding process.

To elaborate results likely to make things more clear to the decision maker (e.g., if..., then... results), in the "European" conception, the analyst must propose working hypotheses which will allow the co-construction of the preference model to play an appropriate role in the decision aiding process. The co-constructed model must be a tool for looking more thoroughly into the subject, by exploring, interpreting, debating and even arguing. To guide this process of co-construction, the analyst must also interact with the decision maker assuming that he/she understands the questions that are asked. Nevertheless, in the "European" conception, it is not necessary to assume that the given responses are produced through a stable pre-existing process, but only that these responses are made up through interaction with the decision maker's value system, which is *rarely free of ambiguity or even contradiction*. In particular, the analyst must make sure that the person who responds to the questions is able to place these questions in the context of the current study. The analyst must also admit that these questions can bring the person thus questioned to revise certain pre-existing preferences momentarily and locally.

According to the "European" conception, the knowledge produced does not aim to help the decision maker to discover a good approximation of a decision which would objectively be one of the best, taking into account his/her own value system, but rather more humbly to provide the decision maker with a set of results derived from the reasoning modes and working hypotheses. The decision maker will better understand the results produced and will appropriate them (and potentially share with others) if the analyst makes sure that understanding of the underlying reasoning modes and working hypotheses is integrated into the model co-construction process.

In this "European" conception, the analyst does not need to accept either of the following two postulates (Roy 2010):

- *Postulate of the decision maker's optimum.* In the decision context studied, there exists at least one optimal decision, or, in other words, there exists one decision for which it is possible (if sufficient time and means are available) to establish objectively that there are no strictly better decisions with respect to the decision maker's preference system.

- *Postulate of the decision context reality.* The principal aspects of the reality on which the decision aiding is based (particularly the decision maker's preferences) are related to objects of knowledge that can be seen as data (i.e., existing outside of the way they are modeled); these objects can also be seen as sufficiently stable over time and for the questions asked, such that it is possible to refer to the exact state or the exact value (deterministic or stochastic) of given characteristics judged to accurately portray an aspect of that reality.

He/she may find these postulates as totally unrealistic, or may even have good reasons for accepting the existence of *incomparabilities* in the preference models used.

1.2 Notation

For a suitable description of the main features and recent developments of ELECTRE methods (sections 2 and 3, respectively) it is necessary to introduce a few notation related to the basic data.

It should be noticed that numbers used to code preferences have an ordinal meaning only. Consequently, the difference between the preferences of two actions on any criterion must not be considered as an intensity of preferences (for more details see 3.3.4).

The basic data needed for any MCDA problem can be represented as follows:

– $A = \{a_1, a_2, \ldots, a_i, \ldots, a_m\}$ is the set of m *potential actions*; this set is, possibly, only partially known *a priori*, which is common in sorting problems (see 2.3),

– $F = \{g_1, g_2, \ldots, g_j, \ldots, g_n\}$ is a *coherent family of criteria* with $n \geqslant 3$,

– $g_j(a_i)$ is the *performance* of action a_i on criterion g_j, for all $a_i \in A$ and $g_j \in F$; an $m \times n$ *performance matrix* M can thus be built, with $g_j(a_i)$ in row i and column j ($i = 1, \ldots, m$; $j = 1, \ldots, n$).

In the following, we assume without loss of generality that the higher the performance $g_j(a)$ is, the better it is for the decision makers (*increasing direction of preference*).

Since the recent appearance of new ELECTRE methods for sorting problems, in what follows we rename the well-known ELECTRE TRI method by ELECTRE TRI-B, where B means "bounds".

2 Main Features

The distinctive features of ELECTRE methods, to which analysts should pay special attention on, when dealing with real-world decision aiding situations, are presented in this section. These are: the four preference situations handled by ELECTRE methods, the preference modeling through outranking relations, the concepts of concordance and non-discordance, the structure of the methods, the main strengths as well as the weaknesses of ELECTRE methods.

2.1 Modeling Four Main Preference Situations

The ELECTRE methods are based on the following four preference situations concerning the comparison of two actions (Roy 1996):

I (*Indifference*): it corresponds to a situation where there are clear and positive reasons that justify an equivalence between the two actions (it leads to a reflexive and symmetric binary relation);

P (*Strict Preference*): it corresponds to a situation where there are clear and positive reasons in favor of one (identified) of the two actions (it leads to a nonreflexive and asymmetric binary relation);

Q (*Weak Preference*): it corresponds to a situation where there are clear and positive reasons that invalidate strict preference in favor of one (identified) of the two actions, but they are insufficient to deduce either the strict preference in favor of the other action or indifference between both actions, thereby not allowing either of the two preceding situations to be distinguished as appropriate (it leads to a nonreflexive and asymmetric binary relation);

R (*Incomparability*): it corresponds to an absence of clear and positive reasons that would justify any of the three preceding relations (it leads to a nonreflexive and symmetric binary relation).

2.2 Preference Modeling Through Outranking Relations

This sub-section presents four fundamental aspects of preference modeling in ELECTRE methods: modeling hesitation (partial and comprehensive) situations, modeling comprehensive incomparabilities, the concept of concordance and the relative importance of criteria, and the concept of non-discordance and the veto thresholds.

Let us observe that aggregation procedures used in ELECTRE methods are better adapted to situations where decision models include more than five criteria (up to twelve or thirteen).

2.2.1 The Concept of Pseudo-criterion

Definition 1 (pseudo-criterion) *A pseudo-criterion is a function g_j associated with two threshold functions, $q_j(\cdot)$ and $p_j(\cdot)$, satisfying the following condition: for all ordered pairs of actions $(a, a') \in A \times A$, such that $g_j(a) \geqslant g_j(a')$, $g_j(a) + p_j(g_j(a'))$ and $g_j(a) + q_j(g_j(a'))$ are non-decreasing monotone functions of $g_j(a')$, such that $p_j(g_j(a')) \geqslant q_j(g_j(a')) \geqslant 0$ for all $a \in A$.*

For more details about the concept of pseudo-criterion see (Roy 1991) and (Roy and Vincke 1984). Here we consider the thresholds as variables, but they can also be defined as constant values. Moreover, not necessarily all the criteria are subject to a definition of these discriminating thresholds. It should also be noted, that the way a pseudo-criterion was defined above, takes into account only *direct thresholds*, since the arguments of the threshold functions are the worst of the two performances $g_j(a)$ and $g_j(a')$. When the thresholds are expressed as a function of more preferred of the two values, we call them *inverse thresholds*. In the case of constant thresholds, there is no distinction between direct and inverse thresholds.

According to the above definition,

- $q_j(g_j(a'))$ is the greatest performance difference for which the situation of indifference holds on criterion g_j between two actions a and a', where $q_j(g_j(a')) = g_j(a) - g_j(a')$,
- $p_j(g_j(a'))$ is the smallest performance difference for which the situation of preference occurs on criterion g_j between two actions a and a', where $p_j(g_j(a')) = g_j(a) - g_j(a')$.

The reader can find more details about the discrimination thresholds in (Roy 1985; Roy 1996).

2.2.2 The Definition of the Partial Binary Relations

Consider an ordered pair of actions $(a, a') \in A \times A$, and the two thresholds associated with the pseudo-criterion $g_j \in F$, which is used to model the following situations (note that no assumption is made here about which one of the two actions is better on criterion g_j):

1) $g_j(a) - g_j(a') > p_j(g_j(a')) \quad \Leftrightarrow \quad aP_ja'$,
2) $q_j(g_j(a')) < g_j(a) - g_j(a') \leqslant p_j(g_j(a')) \quad \Leftrightarrow \quad aQ_ja'$ (hesitation between aI_ja' and aP_ja'),
3) $-q_j(g_j(a)) \leqslant g_j(a) - g_j(a') \leqslant q_j(g_j(a')) \quad \Leftrightarrow \quad aI_ja'$.

The above three binary relations can be grouped into one partial outranking relation S_j comprising the three corresponding situations. $S_j = P_j \cup Q_j \cup I_j$, where aS_ja' means that "a is at least as good as a'" on criterion g_j. When aS_ja', the voting power of criterion g_j, denoted by w_j (assume w.l.g. that $\sum_{\{j \mid g_j \in F\}} w_j = 1$), is taken in total. Figure 1 illustrates the different zones of the partial binary relations previously defined, i.e. the situations aP_ja', aQ_ja', aI_ja', $a'Q_ja$, and $a'P_ja$, as well as the fraction φ_j of the voting power associated with each one of these situations.

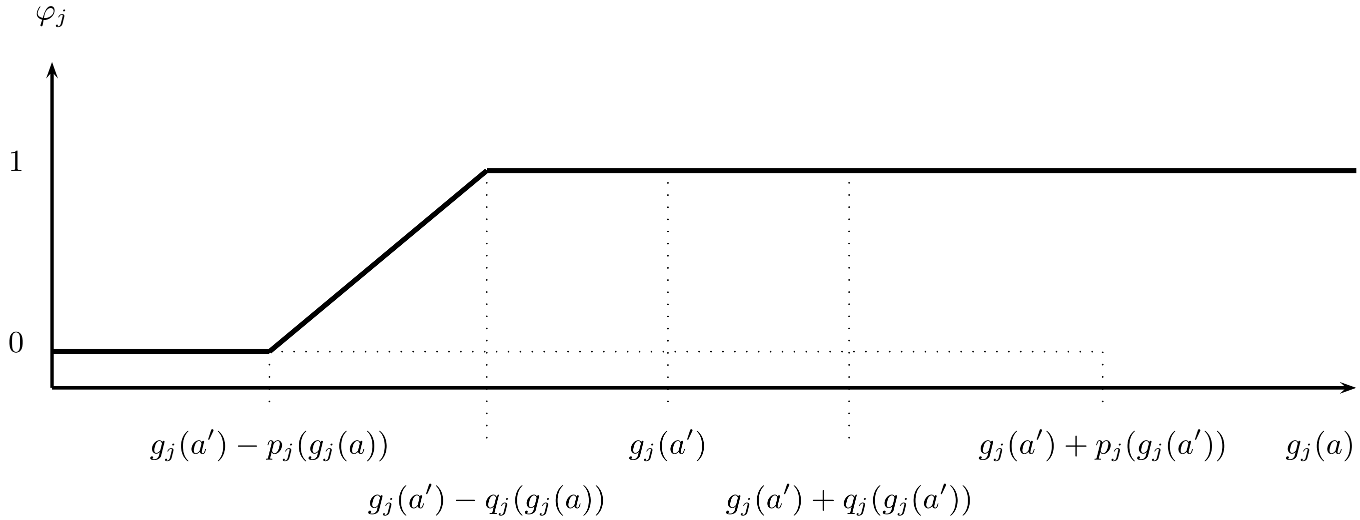

Figure 1: Variation of φ_j for a given $g_j(a')$ and variable $g_j(a)$

From the definition of the partial binary relations and from Figure 1 it is easy to see that the two types of thresholds, direct and inverse, have to be taken into account.

2.2.3 The Comprehensive Outranking Relation

Preferences in ELECTRE methods are modeled by the comprehensive binary outranking relation S, whose meaning is *"at least as good as"*; in general, $S = P \cup Q \cup I$. Consider two actions $(a, a') \in A \times A$. Modeling comprehensive preference information leads to the four cases ($\succ = Q \cup P$):

1) aSa' and not$(a'Sa)$, i.e., $a \succ a'$ (a is *preferred* to a');

2) $a'Sa$ and not(aSa'), i.e., $a' \succ a$ (a' is *preferred* to a);

3) aSa' and $a'Sa$, i.e., aIa' (a is *indifferent* to a');

4) not(aSa') and not$(a'Sa)$, i.e., aRa' (a is incomparable to a').

2.3 The Concepts of Concordance and Discordance

All outranking based methods rely on the concepts of concordance and discordance which represent, in a certain sense, the raisons *for* and *against* an outranking situation.

2.3.1 Concordance

Definition 2 *To validate an outranking aSa', a sufficient majority of criteria in favor of this assertion must occur.*

The comprehensive concordance index The *comprehensive concordance index* can be defined as follows:

$$c(a,a') = \sum_{\{j \mid g_j \in \mathcal{C}(aSa')\}} w_j + \sum_{\{j \mid g_j \in \mathcal{C}(a'Qa)\}} w_j \varphi_j,$$

where

$$\varphi_j = \frac{g_j(a) - g_j(a') + p_j(g_j(a))}{p_j(g_j(a)) - q_j(g_j(a))},$$

and $\mathcal{C}(a\{Rel\}a')$ is the coalition of criteria in which relations $\{Rel\}$ hold for a, a'.

The concordance index is used to model the concept of concordance. It permits to build an $m \times m$ concordance matrix C composed of elements $c(a,a')$, for all $a, a' \in A$.

One can see that the criteria can be classified in three groups:

1) Those that are in favor of the assertion aSa' with no reservation,

2) Those that hesitate between the indifference and the opposition,

3) Those that are in the opposition.

The element $c(a,a')$ is the comprehensive concordance index with the assertion aSa'. This index results from a summation of the voting power of criteria from the first group, and of the fraction φ_j of the voting power of criteria from the second group (see also 2.4.1).

2.3.2 Discordance

Definition 3 *The assertion aSa' cannot be validated if a minority of criteria is strongly against this assertion.*

The concept of veto threshold When criterion g_j opposes strongly to the assertion aSa', g_j puts its veto to the assertion aSa'. This occurs if $g_j(a') - g_j(a) > v_j(g_j(a))$. The value $v_j(g_j(a))$ is called the veto threshold of g_j.

For each criterion $g_j \in F$, an opposition power is determined, for each $(a,a') \in A \times A$.

Partial discordance indices

$$d_j(a,a') = \begin{cases} 1 & \text{if } g_j(a) - g_j(a') < -v_j(g_j(a)), \\ \frac{g_j(a) - g_j(a') + p_j(g_j(a))}{p_j(g_j(a)) - v_j(g_j(a))} & \text{if } -v_j(g_j(a)) \leqslant g_j(a) - g_j(a') < -p_j(g_j(a)), \\ 0 & \text{if } g_j(a) - g_j(a') \geqslant -p_j(g_j(a)). \end{cases}$$

where $d_j(a,a')$ is the partial discordance index of criterion g_j.

It permits to build an $m \times m$ discordance matrix D_j composed of elements $d_j(a,a')$, for all $(a,a') \in A \times A$ and for each criterion $g_j \in F$.

2.4 Structure

Each ELECTRE method comprises two main procedures: an aggregation procedure and an exploitation procedure.

2.4.1 Multiple Criteria Aggregation Procedures (MCAPs)

By definition, MCAP is a procedure that builds one or possibly several outranking relations on the basis of the performances of each action on each criterion, which leads to assign to each ordered pair of actions one and only one of the four situations presented in sub-section 2.1. Let us notice that the decision maker does not make any pairwise comparison; all the comparisons are done by the procedure itself.

The MCAP has to take into account the role played by the criteria: some of them can play a "very important" role, while others can play a "totally secondary" role. For this purpose, ELECTRE methods make use of intrinsic weights and possible veto thresholds (Figueira, Roy, and Mousseau 2005, chap. 4). The intrinsic weights can be interpreted as the voting power of each criterion. The higher the intrinsic weight, the more important the criterion is. Note that the voting power neither depends on the range of the criterion scale nor on the encoding chosen (in particular the unit selected) to express the evaluation (score) on this scale. In ELECTRE methods it is not assumed that the weights, as well as the veto thresholds, have a real existence in the mind of the decision maker. They do not have a "true value". Such parameters are artifacts, co-constructed abstract "objects" (Roy 2010).

For example, in ELECTRE III, its MCAP associates for each ordered pair of action $(a, a') \in A \times A$ a credibility index of the assertion aSa'. This credibility index is a fuzzy measure denoted by $\sigma(a, a') \in [0, 1]$, which combines $c(a, a')$ and $d_j(a, a')$:

$$\sigma(a, a') = c(a, a') \prod_{j \in \mathcal{J}(a,a')} \frac{1 - d_j(a, a')}{1 - c(a, a')},$$

where $j \in \mathcal{J}(a, a')$ if and only if $d_j(a, a') \geqslant c(a, a')$.

2.4.2 Exploitation Procedures (EPs)

By definition, EP is a procedure used to obtain *adequate results* from which *recommendations* can be derived.

2.4.3 The Nature of the Results

The *nature of the results* depends on the specific *problematic*. The three major problematics in MCDA can be stated as follows:

Choosing Selecting a restricted number of the most interesting potential actions, as small as as possible, which will justify to eliminate all others.

Sorting Assigning each potential action to one of the categories among a family previously defined; the categories are ordered, in general, from the worst to the best one. An example of a family of categories suitable for assignment procedures is given below:

C_1: actions whose implementation is not advised
C_2: actions whose implementation could only be advised after significant modifications,
C_3: actions whose implementation could only be advised after slight modifications,
C_4: actions whose implementation is always advised without any reservation.

Ranking Ranking of actions from the best to the worst, with the possibility of ties (*ex æquo*) and incomparabilities.

The way in which MCDA is envisaged (the problematic) will constrain the form of the results to be obtained.

Remark

1) In *sorting problematic* the result depends on *absolute evaluation* of actions: the assignment of an action takes into account, *only its intrinsic evaluation on all the criteria*, and it *neither depends on nor influences* the category to be selected for the assignment of another action.

2) As in the *remaining problematics* the actions are compared against each other, the result depends in these cases on *relative evaluation* instead of absolute one as in the previous case.

2.4.4 Software

The family of Electre methods includes several variants of methods designed for the three main problematics defined above. Below, we list these variants, together with an associated software available at LAMSADE.

1) Choosing: Electre I, Electre Iv, and Electre IS. Electre IS is a generalization of Electre Iv. Electre IS software runs on an IBM compatible PC with Windows 98 or higher.

2) Ranking: Electre II, Electre III, and Electre IV. Electre III-IV software runs on a PC with Windows 3.1, 95, 98, 2000, Millennium and XP.

3) Sorting: Electre Tri-B, Electre Tri-V, and Electre Tri-nC. Electre Tri-B software runs on Microsoft Windows 3.1, 95, 98, Me, 2000, XP, and NT. It incorporates an Assistant tool which enables the user to define the weights and the λ-cutting level indirectly, from an elicitation process based on a set of assignment examples. The weights are thus inferred through a certain form of regression. Due to the indirect elicitation, Electre Tri-B Assistant reduces the cognitive effort required from the decision makers.

2.5 Strong Features

This sub-section goes through major strong features of Electre family methods. They include the possibility of dealing with the qualitative as well as the quantitative nature of criteria. The heterogeneity of scales and the non-relevance of compensatory effects are also discussed here. The imperfect knowledge of data and some arbitrariness when building criteria can be taken into account in Electre methods, and, finally, they can deal with the reasons for and against an outranking.

2.5.1 The Qualitative Nature of Some Criteria

Electre family methods have the provision for taking into account the *qualitative nature of some criteria*. They allow to consider original data, without the need of recoding them. In fact, all the criteria are processed as qualitative criteria, even if some are quantitative by their very nature.

2.5.2 The Heterogeneity of Scales

The Electre family methods can deal with *heterogeneous* scales to model such diversified notions as noise, delay, aesthetics, cost, ... Whatever the nature of scales, every procedure can run with preserved original performances of the actions on the criteria, without the need of recoding them, for example, by using a normalization technique or the assessment of the corresponding evaluations through the use of a utility or a value function.

Table 1: Performance matrix

	g_1	g_2	g_3	g_4
a_1	9.500	9.500	8.100	5.400
a_2	8.300	8.300	7.300	8.500

2.5.3 The Non-relevance of Compensatory Effects

The MCAP of Electre methods are not based on the notion of compensation. Such MCAPs where conceived such that they do not allow for compensation of performances among criteria, i.e. the degradation of performances on certain criteria cannot be compensated by improvements of performances on other criteria.

The weights of criteria do not mean substitution rates as it is the case in many other methods. The limited possibility of compensation can be brought into light through the concordance and discordance indices:

- Concerning the concordance index, when comparing action a to action a', with the exclusion of the ambiguity zone, only the fact that a outranks or does not outrank a' with respect to criteria from F is relevant, while it is not relevant how much the performance of a is better or worse than the performance of a' on criteria from F;

- The existence of veto thresholds strengthening the non-compensatoiriness effect is yet another reason of the possibility of non compensation in Electre methods. For example, when $d_j(a, a') = 1$, no improvement of the performance of a and no deterioration of the performance of a', with respect to the other criteria than g_j, can compensate this veto effect.

Consider the following example with 4 criteria and only 2 actions (scales: [0,10]). The performance matrix for this example is given in Table 1. Suppose that the weighted-sum model was chosen, i.e, $V(a) = w_1 g_1(a) + \ldots + w_j g_j(a) + \ldots + w_n g_n(a)$. In the considered example, the weights w_j are equal for all criteria ($w_j = 0.250$, for all $j = 1, \ldots, 4$):

$V(a_1) = 8.125 > V(a_2) = 8.100$ (notice that $V(a_1) - V(a_2) = 0.025$), and so $a_1 P a_2$ (a_1 is strictly preferred to a_2). The difference between the performances of the two actions is small on the first 3 criteria, while this difference on the fourth criterion (3.100) is very big in favor of a_2. The compensatory effect led a_1 to be strictly preferred to a_2. This example shows, in an obvious way, the possibility that a big preference difference not favorable to a_1 on one of the criteria (g_4) can be compensated by 3 differences of small amplitude on the remaining criteria, in such a way that a_1 becomes finally strictly preferred to a_2.

In Electre methods the type of compensatory effect shown in the above example, does not occur in a systematic way (see 2.5.4 and 2.5.5). Thus, contrarily to many other methods, for example, Choquet integral, Sugeno integral, and MACBETH, there is no need in Electre methods to use identical and commensurable scales, which by their very nature, do not have, in general, the property called commensurability.

2.5.4 Taking into Account the Imperfect Knowledge of Data and Some Arbitrariness when Building Criteria

Electre methods are adequate to take into account the *imperfect knowledge* of the data and the *arbitrariness* related to the construction of the family of criteria. This is modeled through the indifference and preference thresholds (discrimination thresholds). Consider the same example with the (constant) discrimination thresholds, given in Table 2.

Table 2: Performances and discrimination thresholds

	g_1	g_2	g_3	g_4
a_1	9.500	9.500	8.100	5.400
a_2	8.300	8.300	7.300	8.500
q_j	1.000	1.000	1.000	1.000
p_j	2.000	2.000	2.000	2.000

Table 3: Performances, discrimination and veto thresholds

	g_1	g_2	g_3	g_4
a_1	9.500	9.500	8.100	5.400
a_2	8.300	8.300	7.300	8.500
q_j	1.000	1.000	1.000	1.000
p_j	2.000	2.000	2.000	2.000
v_j	3.000	3.000	3.000	3.000

On the one hand, it should be noticed that any small variation of some performance will not affect in a significant way the preference difference resulting from the MCAP used in ELECTRE methods, but it will modify the weighted-sum value. For example, if on criterion g_3 we would change the performance of action a_2 from 7.300 to 7.100, then the weighted-sum score $V(a_2)$ would move from 8.100 to 8.050 ($V(a_1) - V(a_2) = 0.050$). Consequently, there would be a reinforcement of the preference in favor of a_1.

On the other hand, in ELECTRE methods, $c(a_1, a_2)$ and $c(a_2, a_1)$ remain unchanged as it will be shown hereafter. Since the weighted-sum based models do not allow for the inclusion of thresholds, a_1 is still better than a_2. Now, if we consider 7.500 instead of 7.300, then $V(a_2) = 8.150$, and consequently a_2Pa_1. This slight variation is really too small to invert the preference between a_1 and a_2, but since the weighted-sum based models do not allow for the inclusion of thresholds, a_2 became preferred to a_1. This phenomenon shows the sensitivity of the weighted-sum with respect to non significant variations of the performances, due to the compensatory character of the model.

The performances of the actions can be affected by the imperfect knowledge coming from different sources. At the same time, the way the criteria are built or conceived contains some part of arbitrariness. These are the two major reasons that led to define the discrimination thresholds in ELECTRE methods. When considering the discrimination thresholds and using ELECTRE methods, $c(a_1, a_2) = 0.250 + 0.250 + 0.250 = 0.750$ and $c(a_2, a_1) = 0.200 + 0.200 + 0.250 + 0.250 = 0.800$ (see 2.3.1).

2.5.5 Reasons *for* and Reasons *against* an Outranking

The ELECTRE methods are based, in a certain sense, on the *reasons for* (concordance) and the *reasons against* (discordance) of an outranking between two actions. Consider the same example and a veto threshold $v_j = 3$, for all $j = 1, \ldots, 4$ (see Table 3).

If the concordance threshold $s = 0.800$, then a_2Sa_1 and $not(a_1Sa_2)$. But, if $s = 0.700$, then a_2Sa_1 and a_1Sa_2, i.e., a_2Ia_1.

The discordance index (see 2.3.2) of g_4, $d_4(a_2, a_1) = 1$, and whatever the value of concordance threshold s, we get $not(a_1Sa_2)$. This means that g_4 imposes its veto power on the assertion a_1Sa_2. Weighted-sum based models do not allow for the inclusion of veto effects.

The above shows, moreover, that the consideration of a veto threshold reinforce the non-compensatory character of the ELECTRE methods.

2.6 Weaknesses

This sub-section gives account of the main drawbacks or weaknesses of ELECTRE methods, notably when the quantitative nature of the family of criteria requires the use of a different method, when a score should be assigned to each action, when the independence with respect to irrelevant actions and the possible instability of results is required, or the possible and frequent occurrence of intransitivities would make a problem.

2.6.1 Scoring Actions

In certain contexts it is required to assign a *score* to each action. When the decision makers require that each action should get a score, the ELECTRE methods are not adequate for such a purpose and the scoring methods should be applied instead. The decision makers should be aware, however, that using a score method they cannot provide information that leads, for example, to intransitivities or to incomparabilities between some pairs of actions. Moreover, the score is very fragile.

For the time being, there is no outranking-like method allowing to assign a score to different actions in a convincing manner. This seems, however, a very difficult task to accomplish, because it is assumed to take into account a measure of preference difference (or intensity of preference). In PROMETHEE (Brans and Mareschal 2005, chap. 5) there was an attempt to define a measure of preference differences, but the way in which it was presented seems to contain matter for some criticism (see 6.4.1 in (Roy and Bouyssou 1993)).

2.6.2 The Quantitative Nature of the Family of Criteria

When all the *criteria are quantitative* it is "better" to use some other methods. But, if we would like to take into account a completely or even a partially non-compensatory method, as well as the reasons for and against, then, even if the criteria would be all quantitative, we should use ELECTRE methods. Assume that all the criteria are quantitative and defined on the same scale with the same unit. Also, then, if we are dealing with imperfect knowledge with respect to at least one criterion, ELECTRE methods are suitable.

2.6.3 The Independence with Respect to Irrelevant Actions

Except ELECTRE TRI-B, ELECTRE TRI-C, and ELECTRE TRI-nC, the remaining ELECTRE methods do not fulfill the property of *independence with respect to irrelevant actions*, which says that when comparing two actions, the preference relation should not depend on the presence or absence of other actions. Roy (Roy 1973) shows that rank reversal may occur and, consequently, the property of independence w.r.t. irrelevant actions can be violated when dealing with outranking relations. Notice that rank reversal may occur only when the set of potential actions is subject to evolve, which is quite a natural assumption, however, it is not present in many real-world decision aiding processes where the number of actions is rather small and easily identified. Roy (Roy 1973) presents an example illustrating that such phenomena can be interpreted quite naturally and the author also suggests that allowing the independence property is not realistic in many real-world decision aiding situations. Other works devoted to the same kind of concern include, for example, Perny (Perny 1992), Roy and Bouyssou (Roy and Bouyssou 1993), Simpson (Simpson 1996), and Wang and Triantaphyllou (Wang and Triantaphyllou 2008).

In fact, the *instability* of the results in ELECTRE methods was recently re-analyzed by Wang and Triantaphyllou (Wang and Triantaphyllou 2008) with respect to ELECTRE II and III. When the decision makers feel more comfortable and confident with an evaluation model that provides a stable result, they might be a little bit surprised by the results provided by ELECTRE methods in certain circumstances. In our perspective, a stable result is not necessarily the evidence of an adequate processing of data because some aggregation procedures assume that the data have a meaning, but very often they do not really have it. For example, this is often the case of the weighed-sum based methods, where the results may be stable but not necessarily meaningful (Martel and Roy 2006). Moreover, if one uses different normalization procedures (as is the case when one deals with multiple units of measurement) such methods may alter the derived results (Triantaphyllou 2000). What the ELECTRE methods show is related to the poorly determined margins on the results, very often related to the poor quality of data since the scales are processed as ordinal ones.

Regarding the rank reversal it is important to underline the following aspects:

1) It is quite natural that MCAPs based on pairwise comparisons violate the principle of independence with respect to irrelevant actions. The possibility of what is called rank reversal is a consequence of this violation.

2) In ELECTRE methods when there exists a phenomenon of rank reversal between action a and action a', this shed some light on the fact that the way a and a' are compared is not robust. This is due to the following two reasons:

   a) the existence of discriminating thresholds and the values that should have been assigned to them,

   b) the fact that such a comparison is conditioned by the way the actions a and a' are compared to the remaining actions (Wang and Triantaphyllou 2008; Figueira and Roy 2009).

2.6.4 Intransitivities

Intransitivities may also occur in ELECTRE methods (Roy 1973). It is also well-known that methods using outranking relations (not only the ELECTRE methods) do not need to satisfy the transitivity property. This aspect represents a weakness only if we impose *a priori* that preferences should be transitive. There are, however, some reasons for which the transitivity should not be imposed:

1) It is quite natural that the binary relation of indifference should be considered intransitive (see an example illustrating this phenomenon in (Luce 1956)); there is also no reason to avoid defining indifference thresholds for certain criteria.

2) It is also possible to have insensitivities with respect to the binary relation of preference; we would say that it is possible and rather frequent to have a majority of the criteria in favor of a over b, and majority of the criteria of b over c, without necessarily implying that there is a majority of the criteria in favor of a over c; we can also have a majority of criteria in favor of c over a; this is the well-known Condorcet Paradox, described, e.g., in (Bouyssou, Marchant, Pirlot, Perny, Tsoukiàs, and Vincke 2000). In fact, Gerhlein (Gerhlein 1983) proved that for 25 voters and 11 candidates, the probability that the Condorcet Paradox occurs is 50%.

Let us notice that there is no such intransitivity phenomena in ELECTRE TRI-B and ELECTRE TRI-C methods.

2.7 A Discussion of the Weak and the Strong Points of Electre Methods

A *discussion on the weak and the strong points* of ELECTRE methods can be found in (Figueira and Roy 2009). The objective of this discussion was also to draw the attention of the readers to a particular philosophy of interpreting the results of ELECTRE methods, which is different from philosophies of interpreting the results obtained by other methods, notably the ones adopted in the paper by Wang and Triantaphyllou (Wang and Triantaphyllou 2008). In (Figueira and Roy 2009), as well as in (Roy 1996), the authors try to show, that the objective of decision aiding is not to discover an absolute truth or, a pre-existing “real” best action, ranking, or assignment. The modifications that may occur when adding or removing an action, emphasizing the limitations of the conclusions that can be derived by using ELECTRE methods, without any robustness concern in mind. Clearly, this is also what decision aiding is designed to do: to show how the conclusions can be tenuous and do not reveal a pre-existing truth.

3 Recent Developments

The recent developments presented in this section are mainly methodological. They concern new approaches, an axiomatic analysis of Electre methods, as well as some aspects related to the meaningfulness of the methods.

3.1 Methodological Developments

This section is devoted to the presentation of the inference based approaches and some related issues, the inference-robustness based approaches, the pseudo robustness based approaches, and the new concepts of robustness that can be applied to Electre methods.

3.1.1 Pure-inference Based Approaches

Mousseau and Słowiński (Mousseau and Słowiński 1998) proposed the first general algorithm for inferring the values of the model parameters of Electre Tri-B method from assignment examples given by the decision maker, i.e., from holistic judgments. The assignment examples serve to build a set of mathematical constraints and the inference of the model parameters consists in solving a mathematical programming problem. This approach represents the paradigm of disaggregation-aggregation of preferences (Jacquet-Lagrèze and Siskos 1982), which aims at extracting implicit information contained in holistic statements given by a decision maker. In this case, the statements to be disaggregated are assignment examples. Such an indirect elicitation of preferences requires from the decision maker a much smaller cognitive effort than direct elicitation of the model parameters.

The proposed interactive disaggregation-aggregation procedure finds values of the model parameters that best restore the assignment examples provided by the decision maker. Finding values of all the model parameters at once, i.e., the weights, all thresholds, category bounds and the cutting level λ used in Electre Tri-B, requires, however, solving a hard non-linear programming problem. In order to overcome this difficulty, one can decompose the inference procedure into a series of linear programs specialized in finding values of subsets of these parameters. A computer implementation of this inference method with respect to weights and the $\lambda-$cutting level gave birth to a software tool called Electre Tri Assistant (Mousseau, Słowiński, and Zielniewicz 2000). The tool is also able to identify "inconsistent judgements" in the assignment examples.

Let us notice that in all inference procedures concerning the Electre Tri-B method, the "pessimistic" version of the assignment procedure was considered only (the "optimistic" version is even more difficult to model in terms of mathematical programming because it requires binary variables).

The *inference-based approaches* proposed after the work of Mousseau and Słowiński (Mousseau and Słowiński 1998) are the following:

1) Inferring the *weights* and the $\lambda-$cutting level of Electre Tri-B by linear programming (the discrimination and the veto thresholds as well as the category bounds being fixed) (Mousseau, Figueira, and Naux 2001). In this work, the authors consider the linear programming model of Mousseau et al. (Mousseau, Słowiński, and Zielniewicz 2000), and perform several numerical experiments related to checking the behavior of this inference disaggregation tool. These experiments show that $2n$ (n being the number of criteria) assignment examples are sufficient to infer adequately the weights and the $\lambda-$cutting level.

2) Inferring the *bounds of categories* (Ngo The and Mousseau 2002). This work deals with the possibility of inferring the bounds of categories of Electre Tri-B. After making a slightly simplifying assumptions, the authors developed linear programming and 0-1 linear programming models to infer the bounds.

3) Inferring *veto thresholds* (Dias and Mousseau 2006). This work is a complement of the previous ones. The authors proposed mathematical programming models to assess veto thresholds for the original outranking relation and its two other variants, which may be used in ELECTRE methods, including ELECTRE III. In this case, the inference tools make use of linear programming, 0-1 linear programming, or separable programming.

4) Some *manageable disaggregation procedures* for valued outranking relations were proposed (Mousseau and Dias 2006). The authors used a modified definition of the valued outranking relation, preserving the original discordance concept. This modification makes easier to solve inference problem via mathematical programming. These procedures can be used within ELECTRE III and ELECTRE TRI methods.

5) For some decision examples given by decision makers, there may be no feasible values of model parameters which would permit the model to represent these examples. We then say that the preference information is *inconsistent with respect to the model*. Resolving inconsistency is a problem of utmost importance, as shown in (Mousseau, Figueira, Dias, Gomes da Silva, and Clímaco 2003; Mousseau, Dias, and Figueira 2006). The authors proposed algorithms for resolving inconsistency, where the decision makers must choose between different options of withdrawing or relaxing inconsistent examples. It should be noted, however, that unless inconsistency does not come from violation of dominance, it is not a fault of the decision maker but a deficiency of the preference model to restore the decision examples. Thus, instead of withdrawing or relaxing inconsistent examples, one should also consider the possibility of using a more adequate preference model (Figueira 2009).

3.1.2 Inference-robustness Based Approaches

The *disaggregation-aggregation* approach for *inferring weights* and deriving *robust conclusions* in sorting problems was proposed in (Dias, Mousseau, Figueira, and Clímaco 2002). This work presents a new interactive approach that combines two different approaches, the inference based approach with the robustness based approach. It is also applied to ELECTRE TRI-B. The first approach was described in the previous sub-section. The second approach considers a set of constraints with respect to the parameter values (weights and $\lambda-$cutting level), used to model the imperfect character of the information provided by the decision maker. Then, for each action, the best and worst categories compatible with the constraints are determined. This type of results allows to derive some robust conclusions about the assignments. The robustness analysis is used in this study to guide the decision maker through an interactive inference of weights and $\lambda-$cutting level.

3.1.3 Pseudo-robustness Based Approaches

Stability analysis or pseudo-robust conclusions based on Monte Carlo simulation methods, mainly for ranking and sorting problems (Tervonen, Figueira, Lahdelma, Almeida Dias, and Salminen 2009). The authors proposed a new method SMAA-TRI based on stochastic multiple criteria acceptability analysis (SMAA), for analyzing the stability of some parameters of the ELECTRE TRI-B method. The method consists of analyzing finite spaces of arbitrarily distributed parameter values. Then a Monte Carlo simulation is applied in these spaces for describing each action in terms of the share of parameter values that have been assigned to different categories. This is a kind of stability analysis that can be used to derive pseudo-robust conclusions. For each action, the result obtained is the share of parameter values for each category (in terms of percentage).

3.1.4 New Concepts for Robustness Concerns

Although having a more general range of applicability, the works that will be described below should be able to bring answers to the robustness concerns, when applied to decision aiding using ELECTRE methods.

In (Aissi and Roy 2010, section 3.4), the authors propose a measure of robustness, which is applied to ranking of potential actions $a \in A$ obtained when using ELECTRE III or ELECTRE IV, in the case where it is necessary to take into account a family $\hat{S}$ of scenarios (or of "variable settings"). Let P_s denote a (partial or complete) order provided by ELECTRE with scenario $s \in \hat{S}$, and let $P = \{P_s \mid s \in \hat{S}\}$. First, the authors consider the following measure of robustness:

$$r_\alpha(a) = \text{Proportion of pre-orders } P_s \in \hat{S},$$

in which a occupies a position in the ranking at least equal to α; where α denotes an *a priori* fixed position. Under such basis we can judge that action "a is at least as robust as action a'", when $r_\alpha(a) \geqslant r_\alpha(a')$. Then, the authors proposed to improve this measure by taking into account another position in the ranking β (also defined *a priori*) in order to penalize the actions with a very bad position in certain scenarios. Thus, they propose the following robustness measure:

$$r_{\alpha\beta}(a) = r_\alpha(a) - \text{ Proportion of } P_s \in \hat{S},$$

in which action a occupies a position in the ranking greater than or equal to β.

The results obtained with this robustness measure (possibly supplemented by a sensitivity analysis with respect to the reference positions α and β) must be able to be synthesized in the form of robust conclusions (concept with which the authors deal in section 5) easily understandable by the decision maker (for more details on this subject see section 6 in Chapter 1 in (Zopounidis and Pardalos 2010)).

Still in (Aissi and Roy 2010, section 5.3), the authors propose two frameworks intended to generalize an approach that was successfully used in two concrete cases by one of the authors. In these formal frameworks (using different ELECTRE methods), the approach allows to work out some conclusions and then recommendations answering to certain robustness concerns. The approach mainly aims at restricting the number of combinations of the options to be explored. This restriction is supported by making in clear positions those combinations of options, which appear to have the most significant effect for answering robustness concerns.

In Chapter 1 of (Zopounidis and Pardalos 2010), B. Roy introduces in section 5, various suggestions and proposals for answering to certain robustness concerns by weakening the role of the worst case. These suggestions and proposals do not concern in particular the ELECTRE methods but, at least for some of them, they can be useful.

3.2 Improvements and New Approaches

This section presents the main novelties of ELECTRE-like methods, such as a concept of bi-polar outranking relations implemented in the RUBIS method, the modeling of three different types of interaction among criteria, the research done to modify the credibility index through the use of the reinforced preference thresholds and the counter-veto thresholds, the ELECTRE TRI-C and ELECTRE TRI-NC methods, and the ELECTRE^{GMS} method.

3.2.1 Bi-polar Outranking Based Procedures

The concept of *bi-polar outranking relations* was proposed by Bisdorff et al. (Bisdorff, Meyer, and Roubens 2008) and implemented in the RUBIS software. The RUBIS method is a progressive multiple criteria decision aiding method for choice problems. It is also an outranking based method. It is, however, based on a new concept of bi-polar outranking relation.

The bipolar outranking index $\widetilde{S} : A \times A \rightarrow [-1, 1]$ is defined as follows: for $(a, a') \in A \times A$,

$$\widetilde{S}(a, a') = \min \left\{ \widetilde{C}(a, a'), -V_1(a, a'), \ldots, -V_n(a, a') \right\}$$

where

$$\widetilde{C}(a, a') = \sum_{\{j \mid g_j \in \mathcal{C}(a\{P,Q,I\}a')\}} w_j - \sum_{\{j \mid g_j \in \mathcal{C}(a'Pa)\}} w_j$$

and for all $g_j \in F$,

$$V_j(a, a') = \begin{cases} 1 & \text{if } g_j(a) - g_j(a') \leqslant -v_j(g_j(a)), \\ -1 & \text{if } g_j(a) - g_j(a') > -wv_j(g_j(a)), \\ 0 & \text{otherwise} \end{cases}$$

where $wv_j(g_j(a))$ and $p_j(g_j(a)) \leqslant wv_j(g_j(a)) \leqslant v_j(g_j(a))$ is a weak veto threshold.

The maximum value +1 of the bipolar outranking index is reached in the case of unanimous concordance, whereas the minimum value −1 is obtained either in the case of unanimous discordance, or if there exists a strong veto situation on at least one criterion. The median situation 0 represents a case of indetermination: either the arguments in favor of an outranking are compensated by those against it, or a positive concordance in favor of the outranking is outbalanced by a potential (weak) veto situation.

The semantics linked to this bipolar outranking index is the following:

- $\widetilde{S}(a, a') = +1$ means that assertion "aSa'" is *clearly validated*,
- $\widetilde{S}(a, a') > 0$ means that assertion "aSa'" is *more validated than non-validated*,
- $\widetilde{S}(a, a') = 0$ means that assertion "aSa'" is *undetermined*,
- $\widetilde{S}(a, a') < 0$ means that assertion "aSa'" is *more non-validated than validated*,
- $\widetilde{S}(a, a') = -1$ means that assertion "aSa'" is *clearly non-validated*.

On the basis of the bipolar outranking index, a recommendation for choice problems is given by a procedure based on five pragmatic principles ($\mathcal{P}_1$: non-retainment for well motivated reasons; $\mathcal{P}_2$: minimal size; $\mathcal{P}_3$: efficient and informative refinement;$\mathcal{P}_4$: effective recommendation; $\mathcal{P}_5$: maximal credibility) and the theoretical concepts of hyper-kernel and augmented cordless circuits in a digraph.

3.2.2 Taking into Account the Interaction Between Criteria

The interaction between criteria is modeled through the *weights of the interaction coefficients* and the modifications in the concordance index (Figueira, Greco, and Roy 2009). This work presents an extension of the comprehensive (overall) concordance index of Electre methods, which takes the interaction among criteria into account. Three types of interactions have been considered: mutual strengthening, mutual weakening, and antagonism. The new concordance index correctly takes into account these three types of interactions by imposing such conditions as boundary, monotonicity, and continuity. The following types of interactions were considered (let us notice that the cases $a - b$ are mutually exclusive, but cases $a - c$ and $b - c$ are not). Let $\bar{\mathcal{C}}(a'Pa)$ denote the coalition of criteria that strongly opposes to the assertion "a outranks a':

a) *Mutual strengthening effect*
If both criteria g_i and g_j strongly, or even weakly, support the assertion aSa' (more precisely, $g_i, g_j \in \bar{\mathcal{C}}(a'Pa)$), we consider that their contribution to the concordance index must be greater than the sum of $k_i + k_j$, because these two weights represent the contribution of each of the two criteria to the concordance index when the other criterion does not support aSa'. We suppose that the effect of the combined presence of both g_i and g_j among the criteria supporting the assertion aSa' can be modeled by a mutual strengthening coefficient $k_{ij} > 0$, which intervenes algebraically in $c(a, b)$. Note that $k_{ij} = k_{ji}$.

b) *Mutual weakening effect*
If both criteria g_i and g_j strongly, or even weakly, support the assertion aSa' (more precisely, $g_i, g_j \in \bar{\mathcal{C}}(a'Pa)$), we consider that their contribution to the concordance index must be smaller than the sum of $k_i + k_j$, because these two weights represent the contribution of each of the two criteria to the concordance index when the other criterion does not support aSa'. We suppose that this effect can be modeled using a mutual weakening coefficient $k_{ij} < 0$, which intervenes algebraically in $c(a, a')$. Note that $k_{ij} = k_{ji}$.

c) *Antagonistic effect*
If criterion g_i strongly, or weakly, supports the assertion aSa', and criterion g_h strongly opposes to this assertion, we consider that the contribution of criterion g_i to the concordance index must be smaller than the weight k_i that was considered in the cases in which g_h does not belong to $\mathcal{C}(a'Pa)$. We suppose that this effect can be modeled by introducing an antagonism coefficient $k'_{ih} > 0$, which intervenes negatively in $c(a, a')$. Note that the presence of an antagonism coefficient $k'_{ih} > 0$ is compatible with both the absence of antagonism in the reverse direction ($k'_{hi} = 0$) and the presence of a reverse antagonism ($k'_{hi} > 0$).

The antagonistic effect does not double the influence of the veto effect; in fact, they are quite different. If criterion g_h has a veto power, it will always be considered, regardless of whether g_i belongs to the concordant coalition or not. The same is not true for the antagonistic effect, which occurs only when criterion g_i belongs to the concordant coalition. Let us notice that a veto threshold expresses the power attributed to a given criterion g_j to be against the assertion "a outranks a'", when the difference between performances $g_j(a')$ and $g_j(a)$ is greater than this threshold.

The authors demonstrated that the generalized index is able to take satisfactorily into account the three types of interactions or dependencies among criteria, and they also examined the links between the new concordance index and the Choquet integral. Nevertheless, this extension is appropriate only when the number of pairs of interacting criteria is rather small. Otherwise, we consider that the family of criteria should be rebuilt, since it contains too many interactions and (possibly) incoherencies.

3.2.3 The Reinforced Preference and the Counter-veto Effects

The credibility index $\sigma(a, a')$ of the outranking relation aSa' (see sub-section 2.2) involves preference scales which are purely ordinal. For this reason, as soon as on criterion g_j, the difference of performances $g_j(a) - g_j(a')$ becomes greater than the preference threshold, the value of this difference does not influence the credibility of outranking of action a over action a'. If one would judge that a very large value of this difference gets the meaning of "very strong preference", then one could wish to take this judgment into account in the definition of the credibility of outranking of a over a'. To satisfy such a wish, Roy and Słowiński (Roy and Słowiński 2008) proposed two complementary ways:

- The first one involves a new threshold called reinforced preference threshold: it corresponds to the value of the difference of performances $g_j(a) - g_j(a')$ which is "judged meaningful" for considering criterion g_j as more important in the concordant coalition (by increasing its weight), comparing to the situation where (all things equal elsewhere) the difference of performances is smaller than this threshold (however, not smaller than the preference threshold);

- The second one involves another threshold called counter-veto threshold (it is not necessarily equal to the previous one, as it has a different meaning and it plays a different role): it corresponds to the value of the difference of performances $g_j(a) - g_j(a')$ which is "judged meaningful" for weakening the mechanism of veto against the credibility of outranking (from the side of discordant criteria), comparing to the situation where (all things equal elsewhere) the difference of performances is smaller than this threshold (however, not smaller than the preference threshold).

After defining some principles and requirements for the new formula of the credibility index $\sigma(a, a')$ giving account of the two ways above, Roy and Słowiński gave the following proposal which satisfies these requirements.

Let $rp_j(g_j(a))$ denote the reinforced preference threshold for criterion g_j. When this threshold is crossed, the importance coefficient w_j in the formula for concordance index $c(a, a')$ should be replaced by $\omega_j w_j$, where $\omega_j > 1$ is called reinforcement factor. Let $\mathcal{C}(aRPa')$ denote the set of criteria for which $g_j(a) > g_j(a') + rp_j(g_j(a))$. The new concordance index in then defined as follows,

$$\hat{c}(a,a') = \frac{\sum_{\{j \mid g_j \in \mathcal{C}(aRPa')\}} \omega_j w_j + \sum_{\{j \mid g_j \in \mathcal{C}(aSa') \setminus \mathcal{C}(aRPa')\}} w_j + \sum_{\{j \mid g_j \in \mathcal{C}(a'Qa)\}} w_j \varphi_j}{\sum_{\{j \mid g_j \in \mathcal{C}(aRPa')\}} \omega_j w_j + \sum_{\{j \mid g_j \in F \setminus \mathcal{C}(aRPa')\}} w_j}$$

Let $cv_j(g_j(a))$ denote the counter-veto threshold for criterion g_j, and k the number of criteria for which this threshold has been crossed.

In order to give account of the reinforced preference and the counter-veto effects, the credibility index $\sigma(a, a')$ of the assertion aSa' has to be adequately adapted. For example, the credibility index $\sigma(a, a')$ defined in point 2.4.1, takes the following form:

$$\hat{\sigma}(a,a') = c(a,a') \left[\prod_{j \in \mathcal{J}(a,a')} \frac{1 - d_j(a,a')}{1 - c(a,a')} \right]^{(1-k/n)},$$

where $j \in \mathcal{J}(a, a')$ if and only if $d_j(a, a') \geqslant c(a, a')$. Again, $\hat{\sigma}(a, a') \in [0, 1]$.

For any criterion g_j, $g_j \in F$, the two thresholds $rp_j(g_j(a))$ and $cv_j(g_j(a))$ can be chosen equal, and, moreover, one may wish to consider only one of the two effects; deleting an effect means giving to the corresponding threshold an infinite or very large value. Consequently, no order relation is imposed between these two thresholds.

The new formula for the index of the credibility of outranking $\hat{\sigma}(a, a')$ can be substituted to similar formulae used in original versions of Electre III, Electre Tri-B, Electre Tri-C, and Electre Tri-nC.

The assignment of values to the new thresholds $rp_j(g_j(a))$ and $cv_j(g_j(a))$ can be done in a constructive way of thinking about the model of decision problem at hand. One can use for this some protocols of inquiry similar to those proposed for assigning appropriate values to indifference and preference thresholds (Roy and Bouyssou 1993), or to the weights (Figueira and Roy 2002). These protocols involve few easy questions which do not require from the addressee to speculate about completely unrealistic situations. Another way could be to proceed via disaggregation-aggregation approach, so as to get thresholds $rp_j(g_j(a))$ and $cv_j(g_j(a))$ as compatible as possible with some exemplary pairwise comparisons of few real actions (Mousseau and Słowiński 1998).

The way of introducing the two new effects is consistent with the handling of purely ordinal preference scales. Each of the two new thresholds is like a frontier representing a qualifier without any reference to a notion of quantity. The weights remain intrinsic weights, and do not become substitution rates, the indifference and preference thresholds play exactly the same role as before.

The new formula could also be used outside Electre methods, for example, as similarity or closeness index (Słowiński and Stefanowski 1994; Słowiński and Vanderpooten 1997; Słowiński and Vanderpooten 2000), or as a filtering operator (Perny 1998).

3.2.4 The Electre Tri-C and Electre Tri-nC methods for sorting problems

Electre Tri-C (Almeida-Dias, Figueira, and Roy 2009) is a new method for sorting problems designed for dealing with decision aiding situations where each category from a completely ordered set is defined by a single characteristic reference action. The characteristic reference actions are co-constructed through an interactive process involving the analyst and the decision maker. Electre Tri-C has been also conceived to verify a set of natural structural requirements (conformity, homogeneity, monotonicity, and stability). The method makes use of two joint assignment rules, where the result is a range of categories for each action to be assigned.

Set A of the considered actions is either completely known *a priori* or may appear progressively during the decision aiding process. The objective is to assign these actions to a set of completely ordered categories, denoted by $C_1, \ldots, C_h, \ldots, C_q$ with $q \geqslant 2$. The two joint rules, called descending rule and ascending rule, can be presented as follows:

Descending rule
Choose a credibility level $\lambda \in [0.5, 1]$. Decrease h from $(q+1)$ until the first value t, such that $\sigma(a, b_t) \geqslant \lambda$:

a) For $t = q$, select C_q as a possible category to assign action a.

b) For $0 < t < q$, if $\rho(a, b_t) > \rho(a, b_{t+1})$, then select C_t as a possible category to assign a; otherwise, select C_{t+1}.

c) For $t = 0$, select C_1 as a possible category to assign a.

Ascending rule
Choose a credibility level $\lambda \in [0.5, 1]$. Increase h from 0 until the first value k, such that $\sigma(b_k, a) \geqslant \lambda$:

a) For $k = 1$, select C_1 as a possible category to assign action a.

b) For $1 < k < (q + 1)$, if $\rho(a, b_k) > \rho(a, b_{k-1})$, then select C_k as a possible category to assign a; otherwise, select C_{k-1}.

c) For $k = (q + 1)$, select C_q as a possible category to assign a.

Each one of the two joint rules requires the selecting function $\rho(a, b_h)$, which allows to choose between the two consecutive categories where an action a can be assigned to. The results appear in one of the following forms, and the decision maker may choose:

1) A single category, when the two selected categories are the same;

2) One of the two selected categories, when such categories are consecutive;

3) One of the two selected categories or one of the intermediate categories, when such categories are not consecutive.

In (Almeida-Dias, Figueira, and Roy 2010), Electre Tri-C method was generalized to Electre Tri-nC method where each category is defined by a set of several reference characteristic actions, rather than one. This aspect is enriching the definition of each category and allows to obtain more narrow ranges of categories to which an action can be assigned to, than the Electre Tri-C method. The joint assignments rules are similar to the previous ones.

3.2.5 The Possible and the Necessary Approach: Electre^{GKMS} method

The inference based approaches to Electre methods presented in sub-section 3.1 have been recently extended to handle in a special way the robustness concerns. More precisely, in (Greco, Mousseau, and Słowiński 2009; Greco, Kadziński, Mousseau, and Słowiński 2010), the authors considered the inference based approach to Electre methods using the *robust ordinal regression* (ROR) (Greco, Słowiński, Figueira, and Mousseau 2010). In ROR, the preference parameters of a decision model are inferred from holistic preference comparisons of some reference actions made by the decision maker. In consequence, one gets in general many sets of values of preference model parameters representing this preference information, however, in previous inference based approaches, only one specific set was selected and used to work out a recommendation. Since the selection of one among many sets of parameter values compatible with the preference information provided by the decision maker is rather arbitrary, the ROR approach proposes taking into account all these sets in order to give a recommendation in terms of necessary and possible consequences of applying all the compatible preference models on the considered set of actions (Greco, Mousseau, and Słowiński 2008; Figueira, Greco, and Słowiński 2009). With respect to Electre methods, the ROR approach was applied in the method Electre^{GKMS}, where the possible and the necessary outranking relations are calculated as follows. Given an ordered pair of actions $(a, a') \in A \times A$, *a necessarily* outranks a', which is denoted by $aS^N a'$, if for all compatible sets of parameter values, a outranks a', while *a possibly* outranks a', denoted by $aS^P a'$, if for at least one compatible set of parameter values, a outranks a'. After exploiting the necessary outranking relation in the similar way as in the original Electre methods, one gets a robust recommendation, because it is supported by all outranking models compatible with the holistic preference information. The Electre^{GKMS} method has been adapted also to the case of group decision making, and called Electre^{GKMS}-Group method (Greco, Mousseau, and Słowiński 2009; Greco, Kadziński, Mousseau, and Słowiński 2010). In this case, several decision makers cooperate in a decision problem to make a collective decision. Decision makers share the same "description" of the decision problem (the same set of actions, evaluation criteria, and performance matrix). Each decision maker provides his/her own preference information, composed of pairwise comparisons of some reference actions. The collective preference model accounts for preferences expressed by each decision maker.

Let us denote the set of decision makers by $\mathcal{D}=\{d_1, d_2 \ldots, d_p\}$. For each decision maker $d_r \in D' \subseteq D$, we consider all compatible outranking models. Four situations are interesting for an ordered pair $(a, a') \in A \times A$:

- $a\ S^{N,N}(D')\ a'$: $aS^N a'$ for all $d_r \in D'$, $a\ S^{N,P}(D')\ a'$: $aS^N a'$ for at least one $d_r \in D'$,
- $a\ S^{P,N}(D')\ a'$: $aS^P a'$ for all $d_r \in D'$, $a\ S^{P,P}(D')\ a'$: $aS^P a'$ for at least one $d_r \in D'$.

3.3 Axiomatic and Meaningfulness Analysis

This section is devoted to theoretical foundations of Electre methods, concerning their axiomatization and the meaningfulness of statements they provide with respect to different types of scales of considered criteria.

3.3.1 Axiomatic Analysis

Concerns about axiomatic basis of Electre methods have been described in a long series of papers started in the last millennium (Bouyssou 1986; Bouyssou and Vansnick 1986; Pirlot 1997; Bouyssou, Pirlot, and Vincke 1997). The works on this topic were continued in this millennium (Dubois, Fargier, Perny, and Prade 2003; Bouyssou and Pirlot 2002a). We will not review in detail all the works on the axiomatic analysis of Electre methods, but we will concentrate our attention on contributions related to conjoint measurement analysis of Electre methods done by Bouyssou and Pirlot on one hand, and Greco, Matarazzo and Słowiński, on the other hand.

Greco et al. (Greco, Matarazzo, and Słowiński 2001a) introduced the first conjoint measurement model of an ELECTRE method, namely, ELECTRE IV. Let $X = X_1 \times X_2 \times \ldots \times X_n$ be a product space, where X_j is the value set of criterion $j = 1, 2, \ldots, n$. Let (x_j, z_{-j}), $x_j \in X_j$ and $z_{-j} \in X_{-j} = \prod_{i=1, i\neq j}^{n} X_i$, denote an element of X equal to z except for its j^{th} coordinate being equal to x_j. Analogously, let $(x_{\bar{A}}, z_{-\bar{A}})$, $x_{\bar{A}} \in X_{\bar{A}} = \prod_{j\in\bar{A}}$ and $z_{-\bar{A}} \in X_{-\bar{A}} = \prod_{j\notin\bar{A}} X_j$, $\bar{A} \subseteq \{1, 2, \ldots, n\}$, denote an element of X equal to $x_{\bar{A}}$ for coordinates $j \in \bar{A}$ and to $z_{-\bar{A}}$ for coordinates $j \notin \bar{A}$. A comprehensive outranking relation $\succsim$ is defined on X such that $x \succsim y$ means that "x is at least as good as y". The symmetric part of $\succsim$ is the indifference relation denoted by $\sim$, while the asymmetric part of $\succsim$ is the preference relation denoted by $\succ$. The only minimal requirement imposed on $\succsim$ is its reflexivity. In the following, for each $j = 1, \ldots, n$, we consider a marginal outranking relation $\succsim_j$, such that $x_j \succsim_j y_j$ means "criterion j is in favor of the comprehensive outranking of x over y".

For each ordered pair $(x, y) \in X$, let $S(x, y) = \{j \,| x_j \succsim_j y_j\}$.

We say that a comprehensive outranking relation $\succsim$ on X and the marginal outranking relations $\succsim_j$, $j = 1, \ldots, n$, constitute a concordance structure if and only if for all $x, y, w, z \in X$:

$$[S(x, y) \supseteq S(w, z)] \Rightarrow [w \succsim z \Rightarrow x \succsim y].$$

Greco et al. (Greco, Matarazzo, and Słowiński 2001a) proposed the following result.

Theorem 1 *(Greco, Matarazzo, and Słowiński 2001a) The three following propositions are equivalent:*

1) *for each* $x_j, y_j, u_j, v_j, w_j, z_j \in X_j, a_{-j}, b_{-j}, c_{-j}, d_{-j} \in X_{-i}, j = 1 \ldots, n,$

$$(\mathbf{A}) \quad [(x_j, a_{-j}) \succsim (y_j, b_{-j}) \;\; \textit{and} \;\; (u_j, c_{-j}) \succsim (v_j, d_{-j})]$$

$$\Rightarrow$$

$$[(x_j, c_{-j}) \succsim (y_j, d_{-j}) \;\; \textit{or} \;\; (w_j, a_{-j}) \succsim (z_j, b_{-j})],$$

   and

$$(\mathbf{B}) \quad (x_j, a_{-j}) \succsim (y_j, b_{-j}) \Rightarrow (x_j, a_{-j}) \succsim (x_j, b_{-j});$$

2) *there exists a marginal outranking relation* $\succsim_j$ *for each criterion* $j = 1, \ldots, n$, *such that the comprehensive outranking relation* $\succsim$ *on* X *is a concordance structure;*

3) *there exists*

   - *a marginal outranking relation* $\succsim_j$ *for each criterion* $j = 1, \ldots, n$,
   - *a set function (capacity)* $v : 2^{\{1,\ldots,n\}} \to [0, 1]$, *such that* $v(\emptyset) = 0$, $v(\{1, \ldots, n\})$ *and for each* $\bar{A} \subseteq \bar{B} \subseteq \{1, \ldots, n\}$, $v(\bar{A}) \leqslant v(\bar{B})$, *and*
   - *a threshold* $t \in]0, 1[$ *such that* $v(S(x, y)) \geqslant t \Leftrightarrow x \succsim y$.

ELECTRE methods are based not only on the concordance relation but also on the discordance relation. For each criterion $j = 1, \ldots, n$, there is defined a veto relation V_j, such that for each $x_j, y_j \in X_j$, $x_j V_j y_j$ means that "the preference of y_j over x_j is so strong that, for all $a_{-j}, b_{-j} \in X_{-j}$, it is not true that $(x_j, a_{-j}) \succsim (y_j, b_{-j})$", i.e. (x_j, a_{-j}) cannot be as good as (y_j, b_{-j}).

We say that a comprehensive outranking relation $\succsim$ on X is a concordance structure with veto if and only if for all $x, y, w, z \in X$:

$$[S(x, y) \supseteq S(w, z) \text{ and } non(x_j V_j y_j) \text{ for all } j = 1, \ldots, n] \Rightarrow [w \succsim z \Rightarrow x \succsim y]\,.$$

Greco et al. (Greco, Matarazzo, and Słowiński 2001a) proposed the following result.

Theorem 2 *(Greco, Matarazzo, and Słowiński 2001a) The three following propositions are equivalent:*

1) for each $x_j, y_j, u_j, v_j, w_j, z_j \in X_j, a_{-j}, b_{-j}, c_{-j}, d_{-j}, e_{-j}, f_{-j} \in X_{-j}, j = 1, \ldots, n,$

$$(\mathbf{A}') \quad [(x_j, a_{-j}) \succsim (y_j, b_{-j}) \ \text{ and } \ (u_j, c_{-j}) \succsim (v_j, d_{-j}) \ \text{ and } \ (w_j, e_{-j}) \succsim (z_j, f_{-j})]$$

$$\Rightarrow$$

$$[(x_j, c_{-j}) \succsim (y_j, d_{-j}) \ \text{ or } \ (w_j, a_{-j}) \succsim (z_j, b_{-j})],$$

*and above condition (**B**) holds;*

2) there exists a marginal outranking relation $\succsim_j$ *and a veto relation* V_j *for each criterion* $j = 1, \ldots, n$*, such that the comprehensive outranking relation* $\succsim$ *on* X *is a concordance structure with the veto relation;*

3) there exists,

- *a marginal outranking relation* $\succsim_j$ *for each criterion* $j = 1, \ldots, n$*,*
- *a set function (capacity)* $v : 2^{\{1,\ldots,n\}} \to [0, 1]$*, such that* $v(\emptyset) = 0$*,* $v(\{1, \ldots, n\})$ *and for each* $\bar{A} \subseteq \bar{B} \subseteq \{1, \ldots, n\}$*,* $v(\bar{A}) \leqslant v(\bar{B})$ *and*
- *a threshold* $t \in]0, 1[$ *such that,*

$$v(S(x, y)) \geqslant t \text{ and } V(x, y) = \emptyset \Leftrightarrow x \succsim y.$$

Bouyssou and Pirlot (Bouyssou and Pirlot 2002b) introduced another axiomatic analysis of Electre I that proposed a certain number of results aiming at presenting the Electre I method as a special case of their non-additive non-transitive model.

Theorem 3 *(Bouyssou and Pirlot 2005) The above Theorem 1 holds also when Proposition 1) is replaced by the following one:*

1′) for each $x_j, y_j, w_j, z_j \in X_j, a_{-j}, b_{-j}, c_{-j}, d_{-j} \in X_{-j}, j = 1 \ldots, n,$

$$(\mathbf{RC2}) \quad [(x_j, a_{-j}) \succsim (y_j, b_{-j}) \ \text{ and } \ (y_j, c_{-j}) \succsim (x_j, d_{-j})]$$

$$\Rightarrow$$

$$[(z_j, a_{-j}) \succsim (w_j, b_{-j}) \ \text{ or } \ (w_j, c_{-j}) \succsim (z_j, d_{-j})],$$

$$(\mathbf{UC}) \quad [(x_j, a_{-j}) \succsim (y_j, b_{-}) \ \text{ and } \ (z_j, c_{-j}) \succsim (w_j, d_{-j})]$$

$$\Rightarrow$$

$$[(y_j, a_{-j}) \succsim (x_j, b_{-j}) \ \text{ or } \ (x_j, c_{-j}) \succsim (y_j, d_{-j})],$$

$$(\mathbf{LC}) \quad [(x_j, a_{-j}) \succsim (y_j, b_{-j}) \ \text{ and } \ (y_j, c_{-j}) \succsim (x_j, d_{-j})]$$

$$\Rightarrow$$

$$[(y_j, a_{-j}) \succsim (x_j, b_{-j}) \ \text{ or } \ (z_j, c_{-j}) \succsim (w_j, d_{-j})].$$

The axioms of the first result, however, interact with the axioms of their non-additive and non-transitive model (Bouyssou and Pirlot 2002a), and, therefore, they produced another result.

Theorem 4 *(Bouyssou and Pirlot 2007) The above Theorem 1 holds also when Proposition 1′) is replaced by the following one:*

1″) for each $x_j, y_j, w_j, z_j \in X_j, a_{-j}, b_{-j}, c_{-j}, d_{-j} \in X_{-j}, j = 1, \ldots, n$,

$$(\mathbf{RC1}) \quad [(x_j, a_{-j}) \succsim (y_j, b_{-j}) \;\; \textit{and} \;\; (z_j, c_{-j}) \succsim (w_j, d_{-j})]$$

$$\Rightarrow$$

$$[(x_j, c_{-j}) \succsim (y_j, d_{-j}) \;\; \textit{or} \;\; (z_j, a_{-j}) \succsim (w_j, b_{-j})],$$

$$(\mathbf{M1}) \quad [(x_j, a_{-j}) \succsim (y_j, b_{-j}) \;\; \textit{and} \;\; (z_j, c_{-j}) \succsim (w_j, d_{-j})]$$

$$\Rightarrow$$

$$[(y_j, a_{-j}) \succsim (x_j, b_{-j}) \; \textit{or} \; (w_j, a_{-j}) \succsim (z_j, b_{-j}) \;\; \textit{or} \;\; (x_j, c_{-j}) \succsim (y_j, d_{-j})],$$

$$(\mathbf{M2}) \quad [(x_j, a_{-j}) \succsim (y_j, b_{-j}) \; \textit{and} \; (y_j, c_{-j}) \succsim (x_j, d_{-j})]$$

$$\Rightarrow$$

$$[(y_j, a_{-j}) \succsim (x_j, b_{-j}) \; \textit{or} \; (z_j, a_{-j}) \succsim (w_j, b_{-j}) \; \textit{or} \; (z_j, c_{-j}) \succsim (w_j, d_{-j})],$$

*and above condition (***RC2***) holds.*

Finally, Bouyssou and Pirlot (Bouyssou and Pirlot 2009) considered also the veto condition, proposing the following result.

Theorem 5 *(Bouyssou and Pirlot 2009) The above Theorem 2 holds also when Proposition 1) is replaced by the following one:*

for each $x_j, y_j, w_j, z_j \in X_j, a_{-j}, b_{-j}, c_{-j}, d_{-j}, e_{-j}, f_{-j} \in X_{-j}, j = 1, \ldots, n$,

$$(\mathbf{M3}) \quad [(x_j, a_{-j}) \succsim (y_j, b_{-j}) \;\; \textit{and} \;\; (y_j, c_{-j}) \succsim (x_j, d_{-j}) \;\; \textit{and} \;\; (z_j, e_{-j}) \succsim (w_j, f_{-j})]$$

$$\Rightarrow$$

$$[(y_j, a_{-j}) \succsim (x_j, b_{-j}) \;\; \textit{or} \;\; (z_j, a_{-j}) \succsim (w_j, b_{-j}) \;\; \textit{or} \;\; (z_j, c_{-j}) \succsim (w_j, d_{-j})],$$

*and above conditions (***RC1***), (***RC2***), and (***M1***) hold.*

The approach of Bouyssou and Pirlot (Bouyssou and Pirlot 2009) has the merit of putting the axiomatic basis of Electre methods in the larger context of their general non-additive and non-transitive model. However, their conditions are more numerous and complex than the conditions proposed by Greco et al. (Greco, Matarazzo, and Słowiński 2001a).

3.3.2 Representing Preferences by Decision Rules

In Greco et al. (Greco, Matarazzo, and Słowiński 2002), an equivalence of preference representation by conjoint measure and decision rules induced using the Dominance-based Rough Set Approach (DRSA) (Greco, Matarazzo, and Słowiński 2001b) was demonstrated for choice and ranking problems. One of the most important conclusions in this context is that Electre Iv method can be represented in terms of DRSA. In this case, for all $a \in A$ and for all $g_j \in F$, $q_j(g_j(a)) = p_j(g_j(a))$, such that $Q_j = \emptyset$, and $d_j(a, a') \in \{0, 1\}$. Then, the set of decision rules describing the aggregation procedure of Electre Iv has the following form:

$$\text{if } aS_{j_1}a' \text{ and } \ldots aS_{j_p}a' \text{ and } \ldots aV^c_{j_p+1}a' \text{ and } \ldots aV^c_{j_n}a', \text{ then } aSa'$$

where $aV_j^c a'$ means that $d_j(a,a')=0$ (i.e., there is no veto with respect to criterion $g_j \in F$) and

$$w_{j_1} + \ldots + w_{j_p} \geqslant s$$

with s being a specific concordance threshold. Not all the above decision rules are necessary to obtain a representation of the outranking relation S on A, because it is enough to consider only those decision rules that involve subsets $\widetilde{F} = \{g_{j_1}, \ldots, g_{j_p}\} \subseteq F$ including no $g_i \in \widetilde{F}$ for which

$$w_{j_1} + \ldots + w_{j_p} - w_i \geqslant s.$$

Using this result, Greco et al. (Greco, Prędki, and Słowiński 2002) proposed a methodology to infer preference parameters (weights and veto thresholds) of Electre methods on the basis of a set of decision rules obtained by DRSA.

3.3.3 A Conjoint Measurement Analysis of a Simplified Version of Electre Tri-B

An axiomatic analysis of a simplified variant of Electre Tri-B has been proposed in (Bouyssou and Marchant 2007a) and in (Bouyssou and Marchant 2007b), in the framework of conjoint measurement theory. This variant only takes into account the "pessimistic" assignment rule, and does not make use of veto thresholds; preference and indifference thresholds are considered equal.

From a technical point of view, the authors make use of conjoint measurement techniques to work with partitions, instead of binary relations. This aspect of dealing with the problem was first proposed by Goldstein (Goldstein 1991) and after generalized by Greco et al. (Greco, Matarazzo, and Słowiński 2001a). Based, moreover, on the concepts of conjoint measurement theory, these authors analyze a certain type of "non-compensatory sorting methods" close to the "pessimistic" version of Electre Tri-B, and make a comparison with other sorting methods. They proved that the simplified version of Electre Tri-B is non-compensatory. This result does not hold, however, for the "optimistic" version of Electre Tri-B with the same simplifications.

Some hints to elicit parameters from assignment examples within the framework of the studied version of Electre Tri-B were also provided in their work.

To give an axiomatic basis to Electre Tri-B, they considered the following simplified model. Consider a twofold partition $\langle \mathcal{A}, \mathcal{U} \rangle$ of X, which means that the two sets $\mathcal{A}$ and $\mathcal{U}$ are non-empty and disjoint, and that their union makes the entire set X. For the sake of simplicity, one can imagine $\mathcal{A}$ as a set of all good actions, and $\mathcal{U}$ as a set of all bad actions. In Electre Tri-B, the sorting of action $x \in X$ is based on comparison of x with profile p separating the categories, using outranking relation S. Then, in the "pessimistic" version of Electre Tri-B, for all $x \in X$,

$$x \in \mathcal{A} \Leftrightarrow xSp,$$

while in the "optimistic" version of Electre Tri-B,

$$x \in \mathcal{A} \Leftrightarrow not(pPx),$$

where P is the asymmetric part of S, i.e. xSp and $not(pSx)$. A partition $\langle \mathcal{A}, \mathcal{U} \rangle$ has a representation in the non-compensatory sorting model if:

- for all $j = 1, \ldots, n$, there is a set $\mathcal{A}_j \subseteq X_j$,
- there is a subset $\mathcal{F}$ of 2^N, such that, for all $I, J \in 2^N$, $N = \{1, \ldots, n\}$,

$$[I \in \mathcal{F} \text{ and } I \subseteq J] \Rightarrow J \in \mathcal{F},$$

such that, for all $x \in X$,

$$x \in \mathcal{A} \Leftrightarrow \{j \in N \mid x_j \in \mathcal{A}_j\} \in \mathcal{F}.$$

Bouyssou and Marchant (Bouyssou and Marchant 2007a) proposed the following result.

Theorem 6 *(Bouyssou and Marchant 2007a) A partition* $\langle \mathcal{A}, \mathcal{U} \rangle$ *has a representation in the non-compensatory sorting model if and only if*

$$\text{for each } x_j, y_j \in X_j \text{ and all } a_{-j}, b_{-j} \in X_{-j}, j = 1 \ldots, n,$$

$$(\textbf{Linear}) \quad [(x_j, a_{-j}) \in \mathcal{A} \ \textit{and} \ (y_j, b_{-j}) \in \mathcal{A}] \Rightarrow [(y_j, a_{-j}) \in \mathcal{A} \ \textit{or} \ (x_j, b_{-j}) \in \mathcal{A}],$$

$$(2 - \textit{graded}) \ [(x_j, a_{-j}) \in \mathcal{A} \textit{ and } (y_j, a_{-j}) \in \mathcal{A} \textit{ and } (y_j, b_{-j}) \in \mathcal{A}]$$

$$\Rightarrow$$

$$[(x_j, b_{-j}) \in \mathcal{A} \textit{ and } (z_j, a_{-j}) \in \mathcal{A}].$$

It is interesting to note that the same axioms have been given by Słowiński et al. (Słowiński, Greco, and Matarazzo 2002) as an axiomatic basis to the sorting procedure based on the Sugeno integral (Sugeno 1974). Therefore, the non-compensatory sorting model is equivalent to the sorting model based on the Sugeno integral.

Bouyssou and Marchant (Bouyssou and Marchant 2007a) considered also a non-compensatory sorting model with veto, that augment the above non-compensatory sorting model by consideration of sets $\mathcal{V}_j \subseteq X_j, j = 1, \ldots, n$, such that for all $x \in X$,

$$x \in \mathcal{A} \Leftrightarrow [\{j \in N \mid x_j \in \mathcal{V}_j\} \in \mathcal{F} \text{ and } \{j \in N \mid x_j \in \mathcal{V}_j\} = \emptyset].$$

Indeed, Bouyssou and Marchant (Bouyssou and Marchant 2007a) proposed the following result.

Theorem 7 *(Bouyssou and Marchant 2007a) A partition* $\langle \mathcal{A}, \mathcal{U} \rangle$ *has a representation in the non-compensatory sorting model with veto if only if for each* $x_j, y_j \in X_j$ *and all* $a_{-j}, b_{-j} \in X_{-j}, j = 1, \ldots, n$,

$$(3v - \textit{graded}) \ [(x_j, a_{-j}) \in \mathcal{A} \textit{ and } (y_j, a_{-j}) \in \mathcal{A} \textit{ and } (y_j, b_{-j}) \in \mathcal{A} \textit{ and } (z_j, c_{-j}) \in \mathcal{A}]$$

$$\Rightarrow$$

$$[(x_j, b_{-j}) \in \mathcal{A} \textit{ and } (z_j, a_{-j}) \in \mathcal{A}],$$

*and above condition (***Linear***) holds.*

In (Bouyssou and Marchant 2007b), this approach has been extended to give an axiomatic basis to the non-compensatory sorting in the case of more than two classes.

3.3.4 The Meaningfulness of Electre Methods

In (Martel and Roy 2006), the authors analyze the meaningfulness of the assertions of the type "a outranks a' for such and such method", in particular, for Electre methods.

The notion of meaningfulness (Suppes 1959) comes from the measurement theory. This theory (Luce, Krantz, Suppes, and Tversky 1990) deals with the way one can represent certain information (in particular, information of qualitative nature) coming from a given category of phenomena through a set of numerical values, in such a way that this representation must adequately reflect certain properties of the considered category of phenomena.

In order to obtain a meaningful assertion (with respect to a considered category of phenomena) based on the computations that make use of the numerical representation, it is necessary that its validity or non-validity will not be affected when one uses another adequate measure or way of representing the phenomena. Indeed, meaningfulness in MCDA is related to invariance of results with respect to some admissible transformation of the criterion scales.

In Electre methods, when there are no ambiguity zones (all the preference thresholds are equal to the indifference thresholds), the meaningfulness is ensured, even for purely ordinal scales. If, for some criteria, the indifference thresholds are strictly lower than the preference thresholds, the loss of meaningfulness is locally restricted to the ambiguity zones between these thresholds. Consequently, Electre methods are meaningful without any restriction for interval criteria scales (Martel and Roy 2006).

3.4 Other Aspects

This section is devoted to other aspects related to ELECTRE methods, that do not fit the previous sections, but, nevertheless are important for several reasons.

3.4.1 The Relative Importance of Criteria

The metaphor of weight is very often a source of misunderstanding (Roy 2010). Knowing the weight of different objects allows to line them up from the heaviest to the lightest one. Similarly, the talk about the (relative) weight of two criteria assumes implicitly that the assertion "this criterion is more important than the other one" makes a sense. It leads to suppose that the weight of a criterion has an intrinsic character, that is to say that it depends only on the point of view reflected by it, and does not depend on the manner in which it is modeled (the nature of the scales, the range of the scales, the possible unit, ...). Very often researchers and practitioners had the opportunity to notice that it is in such a way that a decision maker uses (even before talking to him/her) the expression "weight of a criterion". This parameter holds different names, according to the type of model in which it intervenes. It is, nevertheless, the term *weight* which is the most often used.

It is, in general, the notion of more or less big importance between two criteria that makes naturally sense in the head of the decision makers. Simos (Simos 1990) proposed a procedure that was further revised by Figueira and Roy (Figueira and Roy 2002). These authors proposed a method, called SRF, for assessing the importance coefficients of criteria having exactly the above meaning. They also stressed the fact that SRF must not be used for the coefficients (called inappropriately weights) of a weighted-sum, and that it must be reserved for intrinsic weights (independent on the very nature of the scales) corresponding to the number of voices which could be allocated to every criterion in a voting process. It should be noted that SRF first exploits the ordinal character of the criteria scales, which means that the units and the range of the scales play no role in the assessment of the importance coefficients (to be more rigorous, a very local and minimal role). As mentioned before, the decision makers, who express themselves spontaneously about the notion of importance of criteria, make, in general, no link between this notion and the nature of the scales. The MCAP used to aggregate this information must reflect such a fact adequately.

3.4.2 Concordant Outranking with Criteria of Ordinal Significance

In (Bisdorff 2004), a new contribution to robustness concerns in MCDA was proposed. More precisely, a complete preorder π on the family of criteria F is considered, which is a ranking of significance of criteria, to be taken into account in the construction of the comprehensive outranking relation S. The weights are π-compatible if for all $g_j, g_{j'} \in F$, $w_j = w_{j'}$ if g_j and $g_{j'}$ have the same rank of significance in π, and $w_j > w_{j'}$ if g_j has a higher rank of significance than $g_{j'}$ in π. If for $(a, a') \in A \times A$ the concordance index $c(a, a') > 0.5$, for every π-compatible set of weights, there is an ordinal concordance of a over a', which is denoted by $aC_\pi a'$.

3.4.3 Evolutionary Approaches

Evolutionary algorithms are starting to be used in order to deal with large scale problems, as well as, to mitigate the complexity of some computations in ELECTRE methods, mainly due to some non-linearities existing in the formulas used in these methods.

In (Doumpos, Marinakis, Marinaki, and Zopounidis 2009), an evolutionary approach was proposed to deal with construction of outranking relations in the context of ELECTRE TRI-B.

In (Leyva-López, Sánchez, and Contreras 2008), a new MCDA method was proposed for ranking problems. It makes use of the ELECTRE III method to build a fuzzy outranking relation and exploit it through the application of a multi-objective genetic algorithm.

3.4.4 The Epissure Method for the Assessment of Non-financial Performance

A new approach making use of the Electre Tri-B method is presented in Chapter 13 of (Zopounidis and Pardalos 2010). This new approach, called Epissure (splice, which is a nautical term meaning a joint made by splicing) has been designed by André (André 2009) for evaluating non-financial performances of companies.

Because of the fierce competition in markets among companies and institutions, and because of a strong pressure by international entities to take into account other kinds of performance criteria than financial ones, there was a need of a new approach to the evaluation of non-financial performance of the companies. Epissure responds to this need.

Two normative principles were laid down *ex-ante* to ground the approach:

Principle 1: The approach must be hierarchical, i.e., classified into successive levels, wherein the levels match a hierarchy of responsibilities *vis-à-vis* the successive aggregates of performance that contribute to the performance summary.

Principle 2: At each hierarchical level (except perhaps for some at the lowest levels), the evaluations rely on ordinal verbal scales. The number of degrees on the scales must be adjusted to its matching levels; the number of degrees must be high enough to mirror evolutions and be understandable by the stakeholders operating at the said level.

A consultation process, called a framed consultation process, is an integral part of the Epissure approach. As any other consultation approach, the objective is that the different stakeholders involved in the evaluation reach a common outlook.

The Epissure approach was tested and set up within several companies for the purpose of evaluating sponsorship projects and deciding on their follow-up. The results seem to indicate that this approach is decidedly appropriate for evaluating non-financial performance. Another application concerning evaluation of the environmental performance of the Company *Total* is described in (André and Roy 2007).

3.4.5 Group Decision Aiding

In (Damart, Dias, and Mousseau 2007), the authors proposed a framework for group decision aiding, when groups are willing to cooperate. It is based on an inference based approach (see sub-section 3.1) to the Electre Tri-B method. The implemented procedure is of an interactive nature, and it is based on a "rule" that preserves the coherence of judgements about the sorting examples at both the individual and the group level. As mentioned in point 3.2.5, another inference based approach to group decision with Electre methods, has been proposed as Electre^{GKMS}-Group method (Greco, Mousseau, and Słowiński 2009; Greco, Kadziński, Mousseau, and Słowiński 2010). It employs robust ordinal regression to work with all outranking models compatible with holistic preference information.

3.4.6 Recent applications

In what follows we present some recent applications of Electre methods.

1) Sorting cropping systems (Arondel and Girardin 2000).

2) Land-use suitability assessment (Joerin, Thériault, and Musy 2001).

3) Greenhouse gases emission reduction (Georgopoulou, Sarafidis, Mirasgedis, Zaimi, and Lalas 2003).

4) Risk zoning of an area subjected to mining-induced hazards (Merad, Verdel, Roy, and Kouniali 2004).

5) Participatory decision-making on the localization of waste-treatment plants (Norese 2006).

6) Material selection of bipolar plates for polymer electrolyte membrane fuel cell (Shanian and Savadogo 2006).

7) Assisted reproductive technology (Matias 2008).

8) Promotion of social and economic development (Autran-Gomes, Rangel, and Moreira 2009).

9) Sustainable demolition waste management strategy (Roussat, Dujet, and Méhui 2009).

10) Assessing the risk of nano-materials (Tervonen, Linkov, Figueira, Steevens, Chappell, and Merad 2009).

4 Concluding Remarks

Electre methods have a long history of successful real-world applications with considerable impact on human decisions. Several application areas can be pointed out (see (Figueira, Roy, and Mousseau 2005)): agriculture and forest management, energy, environment and water management, finance, military, project selection (call for tenders), transportation, medicine, nano-technologies, ... As every MCDA method, also Electre methods have their theoretical limitations. This is why, when applying these methods, analysts should first check if their theoretical characteristics respond to the characteristics of the context in which they will be used.

In this paper, we tried to show that research on Electre methods is not a dead field. Rather the opposite, it is still evolving and gains acceptance thanks to new application areas, new methodological and theoretical developments, as well as user-friendly software implementations.

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