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PERFORMANCE ASSESSMENT OF DISTRIBUTED INVENTORY IN PHYSICAL INTERNET

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ABSTRACT
Classical supply chain design relies on a hierarchical organization to store and distribute products over a given geographical area. Within this framework a shortage in a stock affects the whole downstream of the supply chain regardless of the inventory kept in other locations. Within the Physical Internet approach, inventories are distributed in hubs towards the market and source substitution is allowed. The Physical Internet aims to integrate logistics networks into a universal system of interconnected services through the development of protocols and standards for the routing of smart containers of various sizes. This organization enables a distributed storage of goods in hubs thanks to containerization, thus the feasibility of multi-sourcing to one ordering point. This contribution measures the impact of such an organization on stock levels and inventory costs with service level set as a constraint. The analysis focuses on the resources levels (transportation and inventory) needed by the current supply model and by the Physical Internet in order to serve a market with a (Q,R) stock policy. Starting with two supply models and with the definition of cost models as well as inventory policy, the work is based on computer simulation. The analysis tested 3 different families of criterion in order to select dynamically the source when an order is requested: Source Substitution, Minimum Ratio and Minimum Sum. The source substitution, one of the simplest, was found the more efficient and stable according to different scenarios.

Keywords: Alternative supply sources, Physical Internet, inventory, supply chain.

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1 INTRODUCTION

Generally speaking in Fast Moving Consumer Goods (FMCG) sector the cost of inventories represents up to 40% of total logistics cost Cachon et al. [1], not to mention the cost of shortages in retail shops, i.e. around 7% of references in supermarket. Therefore one of the key factors to improve the business performance of companies is to reduce the inventory level along the whole chains, while maintaining or improving product availability. At the production level, Just-in-Time principle is considered as an efficient solution to this end. However, this principle is very difficult to extend throughout sectors, for example the FMCG supply chains, because of the high variations of customers’ demand and distribution constraints. At the supply chain level, vertical and horizontal collaborations were put up front to tackle this issue and break boundaries. On top of the classic vertical way, the horizontal collaboration to optimize inventory level attracts more and more attentions, for example the inventory pooling studied in Swinney [2], and Pan et al. [3] from a transportation point a view. The previous works have already proven that the vertical or horizontal solutions, from both research results and industrial practices could be locally efficient to reduce inventories. Nevertheless, these solutions are more and more challenged by the new practices of today’s supply chains: the higher uncertainty of customer demand, the longer lead time from source points to end customers due to the delocalization of plants and also the tighter transportation constraints. Hence new distribution networks and inventory policies are appealing. Face with these new problems, we study a new modality of inventory management: distributed inventory through open logistics networks, such as the Physical Internet.

The concept of the Physical Internet (PI) is the metaphor of Digital Internet which is a good example of how to interconnect the heterogeneous and independent networks. Inspired by this example, Montreuil et al. [4] firstly stated the PI concept and gave its definition as:

"an open global logistics system leveraging interconnected supply networks through a standard set of modular containers, collaborative protocols and interfaces for increased efficiency and sustainability."

The essence of PI is to interconnect heterogeneous and independent logistics networks, and towards a common open logistics network. As an innovative concept in logistics, Montreuil [5] and Montreuil et al. [6] studied further the concept of PI network. Ballot et al. [7], Sarraj et al. [8] and Sarraj et al. [9] investigated the performance of PI network in terms of transportation in FMCG cases. The previous works have demonstrated the potential improvement on transportation through PI network.

Following the previous research works, we aim to study the inventory management issue within an open and interconnected network. As a first step, the contribution of this paper is to define and assess new strategies of replenishment within a distributed inventory system enabled by PI network, and to outline the potential of this kind of system. However, it is important to claim that our objective is not to define the best strategy of inventory management in PI network, but to study the performance of such logistics network to reduce inventory.

The paper is organised as follow. In Part 2 we illustrate the distributed inventory modality through PI network, which is different to the classic supply network. This part also defines the context of our research question: performance of replenishment policies in open logistic network such as PI network. Part 3 aims to answer the question with defining some replenishment strategies based on two factors: shipment distance and inventory level. Then the strategies will be assessed by simulation studies driven by the objective to minimize the total cost generated by transportation and inventory. In Part 4 we explore the generalization of the results obtained in Part 3 to more complex networks. Part 5 summarizes findings of this paper and highlights perspectives for future works.
2 DISTRIBUTED INVENTORY THROUGH PHYSICAL INTERNET

In Part 2 we discuss the modality of distributed inventory management within open supply networks. Figure 1 contrasts the structure of the classic dedicated distribution network of a company on the left and the open supply network on the right. A classical supply chain in FMCG sector shows a hierarchy from a source plant to a central warehouse (WH) to regional distribution centers (DC) and to point of sales (POS). The WH and DC respectively belong to suppliers and retailers (usually managed by 3PL). In this scheme each POS depends exclusively on its own regional DC. No transfer is allowed between DCs that belong to different companies and between DCs of different regions (except some special cases of intra or extra collaboration between DCs). In this case, not only the high variation of demand at the end of the chain but also the incomplete visibility on global inventory between players can generate important inventories and shortages along the distribution channel. According to this organization most of the solutions to reduce inventories are based on vertical collaboration, e.g. centralization of inventory Chopra et al. [10], Collaborative Planning, Forecasting and Replenishment (CPFR) Holweg et al. [11], vendor-managed inventory (VMI) Yao et al. [12] etc. On the right hand side, the distribution is spread over different facilities (hubs) towards the POS. The storage is also distributed among hubs. This open organization implies that a supplier is able to push products towards the market regardless of retailers. As a corollary, the retailers can be supplied by not one but several sources (hubs), i.e. multiple sourcing. There are numerous questions associated to this organization: handover of the goods property, traceability, security, etc. They are not addressed here as we focus only on replenishment policies.

Figure 1: Traditional FMCG distribution networks versus PI network based distribution network

The literature review shows a lot of papers dealing with hierarchical distribution network and product substitution. The literature about multiple sourcing in supply chain exists but with less attention on inventory management. However few papers are dealing with the very complex problem of order-up-to S and (Q,R) inventory policies with source substitution.

Despite an enormous literature about (Q,R) inventory policies and multiple suppliers inventory models Minner [13], we were not able to find one that fits with the scheme described above.

Among them, a first paper close to our problem was published by Axsäter [14]. It presents a model of multi sourcing but only in the case of unidirectional transshipment between a warehouse and another one. This is a very specific case of inventory substitution where the warehouses apply continuous review order-up-to S policies and (R,Q) policies, when replenishing from the supplier. The results are illustrated with a Poisson demand. However, no alternate sourcing is available as depicted here. A second paper Çapar et al. [15] proposes a two-stage supply chain that consists of two distribution centers and two retailers.
Each member of the supply chain uses a (Q,R) inventory policy and incurs standard inventory holding and backlog costs, as well as ordering and transportation costs. This approach is close to the one proposed here with options of alternative sourcing or lateral transhipment. At the end the paper proposes a selection rule for each retailer based on lower order cost taking into account backlog costs. Our approach is similar as we also propose selection rules and test them. However it differs by the fact that distribution network is not anymore a tree structure but a general network without backlog.

3 DEFINE AND ASSESS REPLENISHMENT POLICIES IN PHYSICAL INTERNET NETWROK

Due to the complexity of the problem, the main research objective of this paper is to define and to assess rules to chose replenishment in PI network with (Q,R) inventory policies and no backlog. In Part 2 we have discussed the modality of distributed inventory in the PI network, which enables the possibility of multi-sources replenishment and thus the unpredictability of source point for every order. Hence it is necessary to set up replenishment policies to select the best source point for each order, meanwhile determine the optimum reorder point and lot size in PI supply network.

A simulation approach is adapted to assess the performance of strategies comparing to the classic supply mode. In order to compare the results of simulation, we use the same input data for all scenarios, such as the demand (mean value and variation), lead time, etc. However, we should recall that our main objective here is to study the performance of distributed inventory through the PI network, but not to find out the best strategy of replenishment in such a system. Actually it is probably true that the best strategy will be determined specifically to actual cases with different constraints and demands. This is not the intention of this paper. Furthermore, as the first step of our research, we analyze only cycle inventory in this paper. The impact of the new strategies on safety inventory will be studied in the next step.

3.1 Classic supply network

The classic supply network is considered as reference to the PI network scenarios. As shown in Figure 1 and in Figure 4, in the classic supply model the couples of source-destination are fixed, e.g. Retailer; supplied by DC, which is supplied by WH, and so on. To simplify the problem, we make an assumption that the deterministic lead times are not subject to change during the whole period of time of the simulation and the orders are generated by a (Q,R) inventory policy.

3.2 Alternative strategies in PI supply network

Differently in the PI network every order can be satisfied by different sources points according to the real-time status of the potential sources and the criteria of selection. For example the order from retailer could be shipped from any hub, and a hub could be delivered from another hub or directly from plant according to the criteria. The two most important factors taken into account are: distance between source and ordering point; and the inventory level at source. Accordingly 3 families of criterion to select source point in PI network are defined and assessed to explore performance: Source Substitution, Minimum Ratio and Minimum Sum.

3.2.1 Source Substitution

Intuitively a logic to determine a source point is the closest one to the destination, for reasons of lead times and transportation cost. But in this case the replenishment would not be dynamic, if distance is the only criterion. Taking into account the inventory level, the Source Substitution strategy is based on a very intuitive criterion, which is as follows: the source of replenishment is the closest (in terms of km) with an available amount of inventory.
enough to satisfy the order. In other words, the criterion is Distance First - Inventory Second. This rule will be simulated in Part 3.3.

3.2.2 Criteria of the Minimum Ratio

Furthermore, we can consider the importance of transportation and inventory at the same level. The system is driven both by the minimum distance and by the local level of inventory simultaneously. The aim is to reach a more uniform distribution of inventory throughout the network, therefore decrease the overall lead time of replenishment and the total handling costs. Here we employ two common expressions to formulate the interacting: ratio or sum.

3.2.2.1 Minimum Ratio

The idea of minimum ratio criterion is to reduce the global amount of inventory through a more uniform distribution of inventory in the system. At the same time it is necessary to find a trade-off between this objective and the cost of transportation. Therefore the network flows are driven by the following criterion. Whenever a retailer or a hub needs to be replenished, the source of shipping is the one that minimizes the ratio of the distance to its current inventory of inventory:

\[
\min \left( \text{Ratio}_i = \frac{D_i}{S_i}; \forall \text{candidate sources} \right) \tag{1}
\]

Where \(D_i\) is the distance from the candidate source \(i\) to destination point; \(S_i\) is the inventory level at the candidate source \(i\); and a candidate source \(i\) should have enough inventories for the order.

Obviously the minimum ratio criterion promotes, to each order, the sourcing point which has shorter distance to destination, meanwhile has higher level of inventory. To simulate the strategy here investigated, we suppose that each ordering point of the network (either retailer or hub) calculates the minimum ratio of its sourcing points, in order to find the most convenient source (hub or manufacturing facility). For instance, every time the Retailer 1 needs to be replenished, it calculates the above-defined ratio for each hub, and sends an order to the hub where the ratio is the minimum. In like manner the replenishment of Retailer 2 takes place. In particular, the replenishment of hubs can happens between hubs, which means a hub can place his order to plant as usual, or to his adjacent hubs. This is profiting from the interconnected network through PI.

3.2.2.2 Minimum ratio modified with coefficients

The ratio above-defined is the primitive way to investigate the strategy simultaneously considering the distance and inventory level, whereas it can be extended to different forms, for example modified with the following power coefficient. The candidate source that minimizes this ratio is chosen for replenishment:

\[
\min \left( \text{Ratio}_i = \frac{D_i^\gamma}{S_i^\varphi}; \forall \text{candidate sources} \right) \tag{2}
\]

\(D_i\) and \(S_i\) having same definition as in Equation (1); \(\gamma\), \(\varphi\) are coefficients to calculate the Ratio of each candidate source \(i\).

The value of \(\gamma\) and \(\varphi\) will be determined by a simulation study, by changing their value in a range to find out the optimum one which gives the lowest total cost. This approach will be further discussed in the simulation and result parts. Obviously Equation (1) is a specific form of (2) when \(\gamma\) and \(\varphi=1\).

3.2.3 Criteria of the Minimum sum

It is also possible to implement another common mathematical expression to interpret the interacting of distance and inventory level, which is to sum up the importance of these two factors. Here we study the formulation with monomial expression or exponential expression.
3.2.3.1 Minimum sum of Monomial

The minimum sum of monomial is based on the same logic that the candidate source having the minimum value \( f(D_i, S_i) \) will be selected as a replenishing points. The value of \( f(D_i, S_i) \) will be formulated in minimal expression as follow:

\[
\text{min } ( f(D_i, S_i) = a D_i^\alpha + \beta S_i^\delta ; \text{ i\forall candidate sources }) \tag{3}
\]

\( D_i \) and \( S_i \) having same definition as in Equation (1); \( \alpha, \beta, \gamma, \delta \) are coefficients to calculate the value \( f(D_i, S_i) \) of each candidate source \( i \).

Obliviously value of the coefficients in the Equation (3) is very sensitive to the final decision. Another role of the coefficients is to differentiate the impact of transportation and inventory on decision. In a given simulation case as in Part 3.3, we can test different values of the coefficients to determine the optimum one which is hereafter considered as parameters of the simulation model.

3.2.3.2 Minimum sum of Exponential

Based on the same logic, another criterion proposed here is the minimum sum of exponential, as presented in Equation (4):

\[
\text{min } ( f(D_i, S_i) = \alpha^{D_i} + \beta^{S_i} ; \text{ i\forall candidate sources }) \tag{4}
\]

\( D_i \) and \( S_i \) having same definition as in Equation (1); \( \alpha \) and \( \beta \) are coefficients to calculate the value \( f(D_i, S_i) \) of each candidate source \( i \).

The intention of the sum of exponential expression is to strengthen the sensibility of distance and inventory level, the factors which are in exponential position, to the selection of replenishing point. The value of the coefficients will be determined in the same way presented above.

Till now we have demonstrated the four categories of strategies to select replenishing points. They are: the classic model with fixed path, the source substitution, minimum ratio of distance to inventory level, and the minimum of sum, and also their extended models in PI network. In the next part we test their performance via simulation studies.

3.3 Simulation studies to assess the strategies

To test the different strategies defined above, we developed a model in AnyLogic which is a multi-method simulation modeling software developed by XJ Technologies (www.anylogic.com). It supports all three well-known modeling approaches: system dynamics, discrete event simulation, and agent-based modeling. Moreover it allows any combination of these approaches within a single model. Without giving details of the simulation model, the following points summarize the features and assumption throughout the whole analysis:

1. Model of Discrete event simulation;
2. Time of simulation is 1000 days (all the process take place once a day);
3. Normal distribution of daily demand (whereas mean and standard deviation values are defined and presented in Table 1);
4. The pattern of network is fixed (number of plants, WHs, DCs, Retailers and hub, as well as their location, distance and lead time);
5. Simulation is driven by minimisation of total cost of the supply network (transportation cost and inventory holding cost);
6. Design of experiments to minimize the total cost in function of Inventory parameters of Lot size and reorder point (ROP), and to determine the coefficient for the equations (2)- (4), which were defined in the replenishment criteria.
7. Service rate: total shortage should be less than 100 units at the end of simulation (global service level of 99.96%).
8. Direct shipments from plant to retailers are not allowed.
In order to compare the results obtained by experiments, all scenarios have the same input data. The difference is only the replenishment strategies in different supply modes. Before introducing the simulation process, we will explain how to model the cost function of inventory holding and of transportation activity.

### 3.3.1 Cost functions

The cost elements are communicated by our research partners. Only the two most important elements are taken into account as mentioned: inventory and transportation. Basically the holding cost model is linear in function of the storing time of volume measured by a cubic meter. For instance at a Warehouse, distribution center and hub level the cost is 0.11 Euro/(day·m³), while at the Retailer the cost is given by a 3/2 ratio, which is equal to 0.165 Euro/(day·m³), as shown in Figure 2. This represents more expensive inventory in a shop.

![Figure 2: Cost function of inventory holding](image1)

The function for the transportation activity is a step function (non-continuous), whereas the transportation takes place only on road. The function chosen models full truckload operations as it is usually the case in the FMCG supply chain. As show in Figure 3, each segment stands for the cost for using a truck, which is 1.4 Euro/km. Different to the other research papers having a purely linear cost function in transportation, the piecewise linear function results in an optimized transportation scheme which minimize the total transportation cost by minimizing the number of trucks. This issue has also been discussed in Meuffels et al. [16] and Pan et al. [17].

However, knowing that the transportation is not the main concerns of this paper, the following scenario does not further seek the optimization in efficiency transportation: every time an order is processed, the lot is shipped, no matter what the size is, and the cost of transportation is proportional to the road distance. That is why all segments in Figure 3 are flat.

![Figure 3: Cost function of road transportation](image2)

### 3.3.2 Simulation models

In this part we present the input data and the setting of each scenario of simulation. The obtained results will be presented in Part 3.3.3.

#### 3.3.2.1 Scenario of classic supply network

The scenario of classic supply network is the reference to the other scenarios of PI network. A simple supply network is designed as the field of the simulation studies, as shown in Figure
4. As said, the pattern of network is fixed, as well as the distance or lead time to all scenarios.

Figure 4: Classic supply network designed for simulation

In addition to the designed network, we took a sample of flow from a database of FMCG companies. To simplify the problem, only one product is considered in the model. We analyzed the flow of this product during one year in two regions modeled as Retail 1 and Retailer 2 in Figure 4. We can further calculate the mean value and standard deviation (S.D.) of daily demand, the lead-time, lot size etc., being the essential input data to simulation. The unit of product is measured in full pallet (which is 1.73 m³). Parameters are shown in Table 1.

Table 1: Parameters of experiment simulation of classic model

<table>
<thead>
<tr>
<th>No</th>
<th>Parameters</th>
<th>Measurement Unit</th>
<th>Type</th>
<th>Value</th>
<th>Step</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Daily Demand Region 1</td>
<td>[units/day]</td>
<td>Stochastic</td>
<td>Mean=20, S.D.=4</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>Daily Demand Region 2</td>
<td>[units/day]</td>
<td>Stochastic</td>
<td>Mean=35, S.D.=7</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>Lead time DC to R</td>
<td>[Days]</td>
<td>Fixed</td>
<td>7</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>Lead time WH to DC</td>
<td>[Days]</td>
<td>Fixed</td>
<td>10</td>
<td>-</td>
</tr>
<tr>
<td>5</td>
<td>Lead time Plant to WH</td>
<td>[Days]</td>
<td>Fixed</td>
<td>14</td>
<td>-</td>
</tr>
<tr>
<td>6</td>
<td>Lot Size Retailer 1</td>
<td>[units]</td>
<td>Fixed</td>
<td>76</td>
<td>-</td>
</tr>
<tr>
<td>7</td>
<td>Lot Size Retailer 2</td>
<td>[units]</td>
<td>Fixed</td>
<td>90</td>
<td>-</td>
</tr>
<tr>
<td>8</td>
<td>ROP Retailer 1</td>
<td>[units]</td>
<td>Fixed</td>
<td>27</td>
<td>-</td>
</tr>
<tr>
<td>9</td>
<td>ROP Retailer 2</td>
<td>[units]</td>
<td>Fixed</td>
<td>47</td>
<td>-</td>
</tr>
<tr>
<td>10</td>
<td>Lot Size WH to DC 1</td>
<td>[units]</td>
<td>Discrete</td>
<td>25 (min) 800 (max)</td>
<td>25</td>
</tr>
<tr>
<td>11</td>
<td>Lot Size WH to DC 2</td>
<td>[units]</td>
<td>Discrete</td>
<td>25 (min) 800 (max)</td>
<td>25</td>
</tr>
<tr>
<td>12</td>
<td>Lot Size Plant to WH</td>
<td>[units]</td>
<td>Discrete</td>
<td>100 (min) 1000 (max)</td>
<td>50</td>
</tr>
<tr>
<td>13</td>
<td>ROP DC 1</td>
<td>[units]</td>
<td>Discrete</td>
<td>20 (min) 800 (max)</td>
<td>20</td>
</tr>
<tr>
<td>14</td>
<td>ROP DC 2</td>
<td>[units]</td>
<td>Discrete</td>
<td>20 (min) 800 (max)</td>
<td>20</td>
</tr>
<tr>
<td>15</td>
<td>ROP WH</td>
<td>[units]</td>
<td>Discrete</td>
<td>100 (min) 1000 (max)</td>
<td>20</td>
</tr>
<tr>
<td>16</td>
<td>Production Lot to plant</td>
<td>[units]</td>
<td>Fixed</td>
<td>1200</td>
<td>-</td>
</tr>
</tbody>
</table>
Parameters 1 to 9, and 16 are fixed in all scenarios of simulation, since they are known data related to the demand at retailers' level and the pattern of network. Parameters 10 to 15, which are unknown from the database, are variables to be determined by experiment simulation. We increase the value of parameter by step value from min to max value; and each combination of parameters is replicated ten times (a set of ten simulation for each combination of parameters). Then we compare the results (minimizing total cost) to find out the optimum configuration. The method is applied in the same way to the following scenarios. The optimum value of parameters 10 to 15 is presented in 3.3.3.

### 3.3.2.2 Scenarios of PI supply network

Comparing to Figure 4, the pattern of PI supply network is the same. However, since the WH, DC1 and DC2 are now considered as open HUB 1 to 3 respectively, several new replenishment paths are enabled, e.g. the shipment from plant to HUB2 or HUB3, from HUB1 to R1 or R2 and particularly the interconnection between hubs. Figure 5 shows the new paths with distance in addition to the paths presented in Figure 4.

![Figure 5: Physical Internet supply network used in simulation](image)

Parameters 1 to 9, and 16 are maintained in simulation scenarios of PI network. Note that the lead time between WH and DCs are now the lead time between Hubs; and the lead time to the new paths is proportional to the distance. Parameters 10 to 15 are no longer suitable in PI models and they are replaced by parameters 17 to 21 in Table 2. In the same way their optimum value will be determined by experiments of simulation.

### Table 2: Modification of parameters in PI network models

<table>
<thead>
<tr>
<th>No</th>
<th>Parameters</th>
<th>Measurement unit</th>
<th>Type</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Min</td>
</tr>
<tr>
<td>17</td>
<td>Lot Size HUB to HUB</td>
<td>[units]</td>
<td>Discrete</td>
<td>20</td>
</tr>
<tr>
<td>18</td>
<td>Lot Size Plant to HUB</td>
<td>[units]</td>
<td>Discrete</td>
<td>20</td>
</tr>
<tr>
<td>19</td>
<td>ROP HUB1</td>
<td>[units]</td>
<td>Discrete</td>
<td>20</td>
</tr>
<tr>
<td>20</td>
<td>ROP HUB2</td>
<td>[units]</td>
<td>Discrete</td>
<td>20</td>
</tr>
<tr>
<td>21</td>
<td>ROP HUB3</td>
<td>[units]</td>
<td>Discrete</td>
<td>20</td>
</tr>
</tbody>
</table>

Remember that another particularity in PI models is the criteria for selecting the suitable replenishing point and some of them have coefficients to be determined as well. Once fixed the value of lot size and reorder point in PI network, the next step is to determine the value of the coefficients in Equations (2) to (4). In the same way, the value of each coefficients...
ranges from -100 to +100 by step of 0.25. Every combination is tested (ten times) in a number of simulation experiments to find the optimal value in each equation. The outcome is presented in the next part.

### 3.3.3 Results and discussion

A number of simulation experiments have been executed but only the final results are presented in this part. We have overall defined 6 scenarios of simulation and enumerated as in Table 3.

**Table 3: Scenarios of simulation study**

<table>
<thead>
<tr>
<th>Sc No.</th>
<th>Model</th>
<th>Selection criteria of replenishing point</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>Classic network</td>
<td>Fixed</td>
</tr>
<tr>
<td>S2</td>
<td>PI network</td>
<td>Source Substitution</td>
</tr>
<tr>
<td>S3</td>
<td>PI network</td>
<td>Minimum Ratio</td>
</tr>
<tr>
<td>S4</td>
<td>PI network</td>
<td>Minimum Ratio modified with coefficients</td>
</tr>
<tr>
<td>S5</td>
<td>PI network</td>
<td>Minimum sum of Monomial</td>
</tr>
<tr>
<td>S6</td>
<td>PI network</td>
<td>Minimum sum of Exponential</td>
</tr>
</tbody>
</table>

The results of optimal lot size and reorder point in classic and PI network determined by simulation is presented in Table 4. The criterion to select replenishing points in PI network is Source Substitution, i.e. Scenario 2. We can see that PI network enables the replenishment between hub and the optimal lot size of each hub is the same. Remind that these values are fixed according the determinate demand at retailers’ level, in other words only for this case. Change input data certainly results in different outcomes. Knowing that the results of the other scenarios are very close to S2, the value lot size and reorder point of S2 is therefore maintained in the other scenarios in order to simplify the simulation process.

**Table 4: Optimal lot size and reorder point in classic and PI network**

<table>
<thead>
<tr>
<th>Classic network</th>
</tr>
</thead>
<tbody>
<tr>
<td>Node</td>
</tr>
<tr>
<td>WH</td>
</tr>
<tr>
<td>DC 1</td>
</tr>
<tr>
<td>DC 2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>PI network (with Source Substitution criterion)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Node</td>
</tr>
<tr>
<td>Hub 1</td>
</tr>
<tr>
<td>Hub 2</td>
</tr>
<tr>
<td>Hub 3</td>
</tr>
</tbody>
</table>

Once fixed the lot size and reorder point in PI network, the next interesting question is to analyze the impact of the criteria of replenishing point selection on total cost. By changing
the value of the coefficients in the equations (2) to (4), we have identified two main findings from the results. First, some equations are more sensitive with the coefficient value, i.e. the equation of minimum sum with monomial. The optimal values of coefficients in each equation, with which the total cost is the minimum in each scenario, are presented in Table 5. Second, different criterion has very different impact on total cost, as shown in Figure 6. In general the scenarios with minimum ratio have a better performance in this case.

Table 5: Optimal values of coefficients in criteria of replenishing point selection

<table>
<thead>
<tr>
<th>Minimum ratio</th>
<th>Minimum sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_i/S_i$</td>
<td>$D_i\gamma + \beta S_i$</td>
</tr>
<tr>
<td>$D_i\gamma$</td>
<td>$\alpha D_i + \beta S_i$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>$\varphi$</th>
<th>$\gamma$</th>
<th>$\varphi$</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\alpha$</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>15.25</td>
<td>8</td>
<td>20</td>
<td>-35.5</td>
<td>45</td>
<td>9.5</td>
</tr>
</tbody>
</table>

Figure 6 summarizes the (lowest) total cost of each scenario. Obviously S2 with source substitution criterion in PI network has the best performance in this case, which saves 9.16% of the cost in the classic model. Oppositely the S5 with Minimum Sum of Monomial criterion is not favorable. But, again, the performance of criteria can be very different according to input data and configuration of the network. In addition, their impact on safety inventory should also be studied in the next step, and considered as a part of the total cost.

Figure 6: Total cost of simulation scenarios

In this part we have defined new replenishment strategies in distributed inventory system through PI network. The strategies have been simulated in a number of experiments in a simple network having one product (one plant and one WH) and two DCs and retailers. The results show that the performance in terms of total cost of the studied strategies is very different.

4 GENERALIZATION TO EXTENDED NETWORKS

The results obtained in the previous part were based on simulation of a simplified supply network. This part aims to observe the performance of the criteria in more complex and more realistic networks and the sensitivity. To this end, we have studied several scenarios when changing the input data, for example the changing of the sites’ location so that the distance, the variation of demand, and the possibility to split orders etc. Here all scenarios
are not presented and we prefer to focus on two very interesting cases of them: one the sensitivity of coefficients’ optimal value in extended networks, and another the performance of the criteria (Source Substitution as an example) in different networks when the number of retailers increases.

4.1 Analysis of value of coefficients in extended networks

In addition to the simple network studied above (hereinafter Network A), we define two more complex networks B and C as follow to assess the criteria:

- Network A: 1 manufacturing facility, 1 warehouse, 2 distribution centres and 2 retailers (3 open hubs in PI network)
- Network B: 2 manufacturing facilities, 2 warehouses, 2 distribution centres and 2 retailers (4 open hubs in PI network)
- Network C: 3 manufacturing facilities, 2 warehouses, 3 distribution centres and 3 retailers (5 open hubs in PI network)

![Figure 7: Extended networks B and C](image)

As illustrated in Figure 7, the pattern of Network B and C are defined in a similar way as A. All details, such as site location and distance between sites etc., are not given here. Though the B and C is still far from some practical complex cases, the analysis aims to provide a better insight into how the criteria perform in function of the gradually complete networks without losing generality. For the reason of complexity we stopped the extension study at Network C.

Table 6: Sensibility of coefficients’ optimal value in extended networks

<table>
<thead>
<tr>
<th>Minimum ratio</th>
<th>Minimum sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_i / S_i$</td>
<td>$D_i^r / S_i^r$</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>$\varphi$</td>
</tr>
<tr>
<td>Network A</td>
<td>1</td>
</tr>
<tr>
<td>Network B</td>
<td>1</td>
</tr>
<tr>
<td>Network C</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 6 illustrates the optimal value of the coefficients when the networks gradually extend. As shown the value of exponential coefficients is roughly steady from Network A to C, which means the determination of their value is more or less regardless of the complexity of network. However the variation of the other linear coefficients is more significant. The outcome is very important for our next research works.
With regard to the performance of criteria, the result of Network B and C is very similar to that of A presented in Figure 6: again the Source Substitution criterion drives Network B and C to their best performance.

4.2 Variation of the number of retailers

Another scenario is to change the number of retailers in the networks. The idea is to study the performance of PI network facing to market areas with different density of retailers (we make a hypothesis that all retailers in the same market area are shipped by PI network). Since the Source Substitution criterion is the simplest but drives to the best result, only this criterion is simulated in the model as an example to compute the savings against the classic supply network. We change the number of retailers from one to six and at each time we computed the total cost of transportation cost and inventory holding cost. Figure 8 demonstrates the savings in terms of total cost in function of the quantity of retailers.

![Graph showing total savings by PI networks in terms of total number of retailers](image_url)

**Figure 8: Total saving by PI networks in terms of total number of retailers (under Source Substitution criterion)**

The trend of savings is highlighted in Figure 8: the overall savings tend to increase as the retailer number increase. Further, when comparing the curve of Network A and C, we can see that the relative increment is roughly independent of the number of suppliers (but slightly disturbed in Network B). These are important results as in the reality we have dozens of retailer of a single firm in the same market area (e.g. in the Ile-de-France), and in general they have a very different population of suppliers.

5 CONCLUSION

In this paper we proposed a first approach based on simulation to select supply source in a PI network with distributed inventories. \((Q,R)\) inventory policy without backlog was used as our inventory management system. The objective was to test different criteria to select the best source anytime a supply is requested. Within our framework and before more in-depth research the source substitution was found the more efficient rule of selection. Minimum ratio with power coefficient and Minimum sum of exponential coefficient were also found better than the classical model. More interestingly the model suggests that the benefit of the PI system increase with the size of the distribution network where it takes more advantage of economies of scale.
This paper does not aim to provide the best replenishment policy in PI network, but to give a first insight into the potential performance of different criteria in such open supply network. The obtained results are encouraging but are limited to the geometry of the network, to the simulation model and to the rules and constraints applied. The results need further investigation to demonstrate the robustness of the source substitution criterion and the performance of the PI network vs. classical supply chain organizations.

6 REFERENCES

