Design and Study of a Multiphase Axial-Flux Machine

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In this paper, a 7-phase Axial-flux Double-rotor Permanent Magnet Synchronous Machine is studied using analytical and Finite Element methods. This type of machine shows a higher sensitivity to the inductance harmonics and electromotive force (emf) compared with the 3-phase machines. So, the conventional analytical modeling method, in which only the first harmonic is taken into account, leads to significant errors in the determination of the control parameters, e.g., the frequency of Pulse Width Modulation Voltage Source Inverter. A multimachine model explains the reasons for this sensitivity and a more sophisticated analytical method is used. Results are compared with those obtained by the 3-D FEM.

Index Terms—Axial flux machines, finite 3D element method, harmonics, magnetostatic.

I. INTRODUCTION

MULTIPHASE machines have various advantages over the conventional 3-phase machines, such as higher reliability, higher torque density, lower pulsating torque, and a decomposition of the power supplied by the static converters. However, because of its specificities compared with the classical machines, the modeling and control of this type of machine must be reconsidered.

A vectorial formalism has been proposed to design a comprehensive model of this type of machine [1]. Thus, a wye-coupled 7-phase machine without reluctance and saturation effects has been proven to be equivalent to a set of three 2-phase fictitious machines [2]. Each machine is characterized by its own inductance, resistance, emf and family of harmonics. The torque of the real machine is equal to the sum of the three torques of the fictitious machines.

For a 3-phase machine a first harmonic model often gives sufficient results (first-order model) to achieve a good control of the system. The reason is that there is only one fictitious machine in this case. For a 7-phase machine, as there are three fictitious machines, three spatial harmonics should be considered in order to correctly design the machine. By acting on windings and permanent magnet shapes, it is possible to modify the harmonic spectrum of magnetomotive and electromotive forces. From this point of view, the axial machines offer a wide variety of possibilities [3], [4].

In this paper an axial double-rotor single stator NN Permanent Magnet machine with toroidal windings has been chosen. Three approaches are proposed to synthesize the model of this machine, based on the harmonic decompositions. The first two methods are based on analytical solutions. The first one is a conventional approach that takes into account only the first harmonic. The second one considers all the harmonics. Finally we use the Finite Element Method (FEM).

The results (inductances, electromotive force and time constants) obtained by the three methods will be compared. We

show also the possibilities of the 3D FEM which will be taken as a reference.

II. MACHINE MODELS

A. Multiphase Machine Model

The vectorial formalism [1] is based on the properties of the 7 by 7 inductance matrix $[L_s]$ of the stator phases. This matrix is symmetrical and circular: consequently four parameters $L_{11}, m_{12}, m_{13}, m_{14}$ are necessary to determine it. A linear application is associated with this matrix whose the eigenvalues are the inductances L_{Mk} of the fictitious machines. In the same way the electromotive forces $\vec{E}_{Mk}$ are the vectorial projections of the emf vector of the real machine onto the eigenspaces.

In the eigenspace, the electrical equation of Mk, the fictitious machine number k, can be written as follows:

$$\vec{V}_{Mk} = R_{Mk} \vec{I}_{Mk} + L_{Mk} \frac{d\vec{I}_{Mk}}{dt} + \vec{E}_{Mk}. \quad (1)$$

From this equation we can introduce an equivalent circuit for each machine as represented in Fig. 1. Moreover, a harmonic characterization of the fictitious machine is possible. From this perspective, the periodic components of any vector of the real machine are expanded using Fourier series. Then, any vector is projected onto the different eigenspaces: a harmonic repartition as summarized in Table I, [1] is obtained.

B. Analytical Model

The analytical approaches used in this paper are classical. They are based on the following assumptions: the permeability of ferromagnetic sheets is infinite and the leakage fluxes are neglected. Two kinds of spatial distributions of the axial field in the air-gaps are considered. The first one, denoted A_{11} , only takes the first harmonic into account. The second one, denoted A_T , considers a trapezoidal distribution of the field in the air-gaps. For these approaches, Ampere's Law and conservation of flux are used to determine the field in the air-gaps. Following the A_{11} approach, a design procedure has been developed for multiphase axial flux machines.

The emf, the magnetomotive forces and the four different values $L_{11}, m_{12}, m_{13}, m_{14}$ of the 7 by 7 inductance matrix of the stator phases are determined. Then, the inductances L_{Mk} and the emf of the fictitious machines can be found thanks to the vectorial formalism.

In the case of the first harmonic approach, $\vec{E}_{M2}$ and $\vec{E}_{M3}$ are equal to zero. Consequently, only one machine produces torque. The other ones can be neglected from this point of view. We find then the same result as for 3-phase machines. Nevertheless, parasitic currents appear in the circuits of the fictitious machines M2 and M3.

In the case of the A_T method, Table I gives the distribution of the harmonics in the various fictitious machines: the values of L_{Mk} and $\vec{E}_{Mk}$ are then modified. The analytical expressions that permit the determination of the values of L_{Mk} and $\vec{E}_{Mk}$ give information which can be used to modify the design of the machine. Consequently it is possible to design a machine which can be easily controlled by the Voltage Source Inverters.

C. Numerical Model

For the numerical model, we have used the FEM in the 3D magnetostatic case. In order to limit the number of unknowns, the scalar potential formulation is classically used. In these conditions Maxwells equation can be written as follows:

$$\text{div}(\mu(\mathbf{h}_s - \mathbf{grad}\varphi)) = 0 \quad (a)$$

$$\mathbf{h}_s \times \mathbf{n}|_{S_h} = 0, \varphi = 0 \text{ on } S_h \text{ and } \mathbf{b} \cdot \mathbf{n}|_{S_h} = 0 \quad (b) \quad (2)$$

where $\mathbf{h}_s$ represents the source field, $\mathbf{j}_s$ the current density and μ the magnetic permeability which depends on the magnetic field $\mathbf{h}$. S_h and S_b are the surface boundaries.

When there are permanent magnets, the constitutive relationship takes the form:

$$\mathbf{b} = \mu_n(\mathbf{h} + \mathbf{h}_c) \quad (3)$$

where μ_n is the permeability of the permanent magnets, close to that of the vacuum, and $\mathbf{h}_c$ the coercitive field.

The source field $\mathbf{h}_s$ of (2(a)) can be defined by:

$$\text{curl}\mathbf{h}_s = \mathbf{j}_s \quad (4)$$

It may be noted that there is an infinity of source fields $\mathbf{h}_s$, which verify (4). We can also note that in the magnetostatic case the current density distribution is assumed to be uniform. Consequently, $\mathbf{j}_s$ in the inductor can be written as: $\mathbf{j}_s = \mathbf{N}i$ where "i" represents the current in a winding and $\mathbf{N}$ is the turn density vector. As the current density vector $\mathbf{N}$ is divergence free, a vector $\mathbf{K}$ can be introduced so that $\text{curl}\mathbf{K} = \mathbf{N}$. Like the source field $\mathbf{h}_s$, the vector $\mathbf{K}$ is chosen so that $\mathbf{K} \times \mathbf{n} = 0$ on S_h .

In the case of the scalar potential formulation, the flux in the winding "i", a function of the excitation current in the winding "j", can be written as follows [5]:

$$\Phi_{ij} = \int_V \mu K_i \cdot (\mathbf{K}_j i_j - \mathbf{grad}\varphi) dv. \quad (5)$$

This expression will be used to determine the flux and consequently the different inductances of the studied multiphase machine.

Fig. 1. Equivalent circuit of one fictitious machine.

TABLE I
HARMONIC CHARACTERIZATION OF THE THREE FICTITIOUS MACHINES.

Fictitious 2-phase machines	Families of odd harmonics
M1	1, 13, 15, ..., $7h \pm 1$
M2	5, 9, 19, ..., $7h \pm 2$
M3	3, 11, 17, ..., $7h \pm 3$

Fig. 2. Structure of type NN.

For the rotor motion, we define a slip surface in the middle of the air gap, which requires a regular mesh [6]. The rotor displacement is modeled by a circular permutation of the unknowns, according to the mesh step. It may be noted that only rotor elements in contact with the slip surface are concerned by the permutation. At the level of the numerical algorithm, the unknown permutations are predetermined. So, the computation time and the storage memory do not increase when considering this movement.

III. STUDIED MACHINE

A. Presentation of the Studied Machine

With our method we have designed a Permanent Magnet machine of type "NN" (see Fig. 2) with eight poles. Its power is 5 kW at the nominal speed of 340 rpm. Its main design parameters are given in Table II. Fig. 3 shows one sixteenth of the studied machine.

B. Determination of Parameters

The inductances and electromotive forces have been calculated using the three proposed methods.

To model the machine with the FEM we have used a mesh with about 185 000 elements and 36 500 nodes. From the finite element method computation, at no-load, we can see in Fig. 4 the magnetic field distribution in the machine for a given position.

Still at no-load, from the FEM we have determined the flux and the emf in the different phases. The emfs of phase number one and three are shown in Fig. 5. For example, the flux and the

Fig. 3. One sixteenth of the studied machine.

TABLE II
MAIN PARAMETERS

Outer core diameter (m)	0,201
Inner core diameter (m)	0,116
Overall length of the machine (m)	0,1
Air-gap (m)	0,001

Fig. 4. Magnetic field distribution in the machine.

emf of one phase can be seen in Fig. 6. The harmonic spectrum (Fig. 7) of the emf shows the presence of harmonics until the 13th harmonic. We can note that the fifth harmonic does not appear in the spectrum.

The different inductances have also been computed with the FEM. Due to the symmetry of the machine, we have only one self inductance L_{11} and three mutual inductances (m_{12} , m_{13} , m_{14}) to calculate. The other values of inductances can be obtained by permutation.

In Table III we can compare the inductances obtained from our three methods. We denote ϵ_1 and ϵ_2 the error for Λ_{H1} and Λ_T respectively, taking FEM as a reference. We remark that the error does not exceed 26% with the Λ_{H1} method and 9% with the Λ_T method.

IV. ANALYSIS OF FICTITIOUS MACHINES

By carrying out vectorial projections and calculating eigenvalues, we have obtained the different emfs and inductances of the fictitious machines.

For the fictitious machines, we present in Table IV the RMS values of the emfs. As we could have predicted using Table I and Fig. 7, the emf value c_{M1} (resp. c_{M2} , c_{M3}) of M1 (resp. M2, M3)

Fig. 5. emfs of number 4 and 6 phases.

Fig. 6. Flux and emf of the number 6 phase.

Fig. 7. Harmonic spectrum of the emf.

TABLE III
INDUCTANCES OF THE REAL MACHINE

	FEM (mH)	Λ_{H1} (mH)	Λ_T (mH)	ϵ_1	ϵ_2
L_{11}	19,75	14,6	18	-26%	-9%
m_{12}	7,95	9,1	7,7	15%	-3%
m_{13}	-2,7	-3,25	-2,6	20%	-3,5%
m_{14}	-13,5	-13,1	-12,9	-3%	-4,5%

corresponds to the effect of the harmonic ranks 1 and 13 (resp. 5 and 9, 3, and 11). As before, we calculate, using the FEM as our reference, the errors. We denote ϵ_3 and ϵ_4 respectively the errors for Λ_{H1} and Λ_T . As we can see from the results (in Table IV),

A_{H1} and A_T underestimate slightly e_{M1} . A_{H1} is inappropriate for estimation of e_{M2} and e_{M3} since the projections are equal to zero.

In the same way we present in Table V the L_{Mk} inductances obtained by the three methods. The relative errors (using FEM as our reference) are denoted ε_5 and ε_6 for A_{H1} and A_T respectively.

It can be noted that between the A_T and FEM methods the results are close for the M1 machine (error less than -5%). As the FEM method takes into account the leakage inductance, it is normal to find higher values in this case. The results are less satisfactory for the M2 and the M3 machines. For these two machines the sensitivity of the models is effectively higher since it depends on the third and the fifth harmonics whose values are weaker compared with the first harmonic. For the A_{H1} approach, the unacceptable errors on L_{M2} and L_{M3} confirm the sensitivity of the M2 and M3 machines to the harmonics. The A_{H1} method, usual for 3-phase machine study, can not be used.

V. EFFECT ON THE PWM FREQUENCY CHOICE

From inductances values L_{M1} , L_{M2} , and L_{M3} , the time-constant of each fictitious machine is determined and given in Table VI.

In general, the PWM frequency is chosen according to the smallest electric time-constant. In our case, we have three time-constants. The PWM frequency must then respect the following equation:

$$f_{PWM} \geq \max \left(\frac{5}{\tau_{M1}}, \frac{5}{\tau_{M2}}, \frac{5}{\tau_{M3}} \right). \quad (6)$$

With the A_{H1} method, the minimum value of the PWM frequency can not be determined. In fact, only the τ_{M1} electric time-constant is then considered and the minimum frequency f_{PWM} is equal to $5/\tau_{M1} = 50$ Hz. With the FEM and the A_T methods we find a minimum PWM frequency of 625 Hz. The difference between these two values is extremely large and thus unacceptable. Undesirable parasitic currents appear in the machine if the A_{H1} method is used.

So, it is particularly important to consider the harmonics for the design of a multiphase machine. More generally, a correct predetermination of the inductances L_{M1} , L_{M2} , and L_{M3} of the fictitious machines is necessary to predict the magnitude of the currents in the machine.

VI. CONCLUSION

The determination of the necessary parameters for the control of multiphase machines is not as easy as for 3-phase ones. It has been shown that the analytical method A_{H1} based only on a first harmonic approach leads to insufficient accuracy (e.g., for the determination of the PWM frequency). A multimachine model based on a vectorial formalism has been used to explain the reasons for this phenomenon. It is consequently necessary to use

TABLE IV
RMS VALUES OF FICTITIOUS MACHINE EMF AND RELATIVE ERRORS

	FEM (V)	A_{H1} (V)	A_T (V)	ε_3	ε_4
e_{M1}	87	84	85,5	-7%	-2%
e_{M2}	8	0	7,7	-100%	-3%
e_{M3}	20	0	19,3	-100%	-3,5%

TABLE V
INDUCTANCES OF FICTITIOUS MACHINES AND RELATIVE ERRORS

	FEM (mH)	A_{H1} (mH)	A_T (mH)	ε_5	ε_6
L_{M1}	55	51	52,3	-7%	-5%
L_{M2}	4,1	0	3,2	-100%	-22%
L_{M3}	8	0	6,7	-100%	-16%

TABLE VI
VALUES OF FICTITIOUS MACHINE TIME-CONSTANTS

	FEM (ms)	A_{H1} (ms)	A_T (ms)
τ_{M1}	108,5	100,6	103,2
τ_{M2}	8	0	6,3
τ_{M3}	15,8	0	13,2

more precise methods of modeling. In the case of the relatively simple studied machine, another analytical method A_T , taking into account harmonics, has been applied. Comparisons with the FEM results show a sufficient accuracy. For more sophisticated machines (skewed slots, different windings, different magnet shapes) the analytical method is still interesting as it allows us to show how to influence the control parameters but does not give precise quantitative results. It is then necessary to use the FEM to get useful values of the control parameters.

REFERENCES

- [1] E. Semail, A. Bouscayrol, and J. P. Hautier, "Vectorial formalism for analysis and design of polyphase synchronous machines," *Eur. Phys. J.-Appl. Phys.*, vol. 22, no. 3, pp. 207-220, 2003.
- [2] E. Semail, X. Kestelyn, and A. Bouscayrol, "Right harmonic spectrum for the back-electromotive force of a n-phase synchronous motor," in *IEEE Industrial Application Society Annu. Meeting*, Seattle, WA, Oct. 3-7, 2004. CD-ROM.
- [3] A. Cavagnino, M. Lazzari, F. Profumo, and A. Tenconi, "A comparison between the axial flux and the radial flux structures for PM synchronous motors," *IEEE Trans. Ind. Appl.*, vol. 38, no. 6, Nov./Dec. 2002.
- [4] S. Huang, J. Luo, F. Leonardi, and T. A. Lipo, "A comparison of power density for axial machines based on general purpose sizing equations," *IEEE Trans. Energy Conversion*, vol. 14, no. 2, pp. 185-192, Jun. 1999.
- [5] Y. Le Menach, S. Clenet, and F. Piriou, "Numerical model to discretize source fields in the 3D finite element method," *IEEE Trans. Mag.*, vol. 36, pp. 676-679, 2000.
- [6] Y. Kawase, T. Yamaguchi, and Y. Hayashi, "Analysis of cogging torque of permanent magnet motor by 3D finite element method," *IEEE Trans. Magn.*, vol. 31, no. 3, pp. 2044-