ADMM algorithm for demosaicking deblurring denoising

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Abstract

The paper is concerned with the problem of demosaicking, deblurring and denoising a color image in the same time. The global model of the acquisition chain for a color image contains these three effects, then doing restoration in the same time as demosaicking makes sense. We propose to take into account for correlation of spectral bands (R,G,B colors) by minimizing a criterion written in a nearly decorrelated basis. Then we adapt the Alternating Direction Multipliers Minimization (ADMM) method to get the solution.

1 Introduction

In this paper we address a classical and challenging problem in image processing: the so called demosaicking problem which arises in digital cameras for reconstructing color images. Let us give a short description.

Most digital cameras use a single sensor which is placed in front of a color filter array: the Bayer Matrix. The sensor therefore sampled only one color per spatial position and the observed image is degraded by the effect of mosaic generation. It is therefore necessary to implement, possibly fast, algorithms to define an image with three color components by spatial position. The set of techniques used in the literature, to solve this problem, is huge. Without claiming of being exhaustive we refer the reader to [10] for a broad and recent survey (see also [1, 2, 4]).

The originality of our research, in this context, is to define a variational method well suited to take into account all possible degradation effects due to: mosaic effect, blur and noise. Looking at the literature in this direction it is worth mentioning the work by Condat (see [3] and references therein) where a demosaicking-denoising method is proposed, but

without taking into account blur effects. In [7, 11] all the degradation effects are considered but with regularization energies which does not allow for fast convex optimization technique. Indeed in these works, in order to take into account the correlation between the RGB components, the prior regularization term has a complicated expression.

Here we analyze and test a new method to solve, in a more direct and possibly faster way, the demosaicking-deblurring-denoising problem. Our approach is based on two steps. The first one, as in [3], is working in a suitable basis where the three color components are statistically decorrelated. Then we are able to write our problem as a convex minimization problem. Finally to solve such a problem and to restore the image, we adapt to our framework the, well known, ADMM (Alternating Direction Multipliers Minimization) convex optimization technique. The ADMM method and its variants are largely used to solve convex minimization problems in image processing. We refer the reader to [6], for a general dissertation on convex optimization techniques, such as ADMM methods or others, and their applications to image processing.

Organization of the paper

The paper is organized as follows. Section 2 is devoted to notation in a discrete setting. In section 3 we give a short description of the general ADDM method. In section 4 we define the new basis for which the channels of the color image are decorrelated. Then we introduce the Bayer matrix and the blur operator. Section 5 is concerned with the definition of our variational model. We also show how to adapt the classical ADMM algorithm to our case.

Finally in the last section we give some applications of our algorithm on color images of big sizes.

2 Discrete setting

We define the discrete rectangular domain Ω of step size $\delta x = 1$ and dimension $d_1 d_2$. $\Omega = \{1, ..., d_1\} \times \{1, ..., d_2\} \subset \mathbb{Z}^2$. In order to simplify the notations we set $X = R^{d_1 \times d_2}$ and $Y = X \times X$. $u \in X$ denotes a matrix of size $d_1 \times d_2$. For $u \in X$, $u_{i,j}$ denotes its (i,j)-th component, with $(i,j) \in \{1, ..., d_1\} \times \{1, ..., d_2\}$. For $g \in Y$, $g_{i,j}$ denotes the (i,j)-th component of with $g_{i,j} = (g^1_{i,j}, g^2_{i,j})$ and $(i,j) \in \{1, ..., d_1\} \times \{1, ..., d_2\}$. We endowed the space X and Y with standard scalar product and standard norm. For $u, v \in X$:

$$\langle u, v \rangle_X = \sum_{i=1}^{d_1} \sum_{j=1}^{d_2} u_{i,j} v_{i,j}.$$

For $g, h \in Y$:

$$\langle g, h \rangle_Y = \sum_{i=1}^{d_1} \sum_{j=1}^{d_2} \sum_{l=1}^{2} g^l_{i,j} h^l_{i,j}.$$

For $u \in X$ and $p \in [1, +\infty)$ we set:

$$\|u\|_p := (\sum_{i=1}^{d_1} \sum_{j=1}^{d_2} |u_{i,j}|^p)^{\frac{1}{p}}.$$

For $g \in Y$ and $p \in [1, +\infty)$:

$$\|g\|_p := \left(\sum_{i=1}^{d_1} \sum_{j=1}^{d_2} \left(\sqrt{(g^1_{i,j})^2 + (g^2_{i,j})^2} \right)^p \right)^{\frac{1}{p}}.$$

If G, F are two vector spaces and $H : G \to F$ is a linear operator the norm of H is defined by

$$\|H\| := \max_{\|\|u\|_G \leq 1} (\|Hu\|_F).$$

3 ADMM algorithm for constrained minimization problem

In this paper we will describe a new algorithm to perform, in the same time, demosaicking-deblurring-denoising. To this purpose we will adapt to our context an ADMM type algorithm. We recall the relevant features necessary to illustrate the application of such a method to our setting. We refer the reader to [6] and references therein, for a general dissertation on convex optimization. The Alternating Direction Minimization Multipliers method (ADMM) is a particular optimization technique well suited for constrained minimization problem of the following form:

$$\min_{u,z} J(z) + G(u) \quad \text{subject to } Ez + Fu = b \tag{1}$$

where $J, G : \mathbb{R}^d \to \mathbb{R}$ are closed, convex, proper functions and where E and F are matrices.

To solve problem (1) one considers the augmented Lagrangian and seeks its stationary points.

$$L_\alpha(z, u, \lambda) = J(z) + G(u) + \langle \lambda, Fu + Ez - b \rangle + \frac{\alpha}{2} \|Fu + Ez - b\|^2. \tag{2}$$

Then one iterate as follows:

$$\begin{cases} (z^{k+1}, u^{k+1}) = \operatorname{argmin}_{z,u} L_\alpha(z, u, \lambda^k) \\ \lambda^{k+1} = \lambda^k + \alpha(Fu^{k+1} + Ez^{k+1} - b), \qquad \lambda^0 = 0 \end{cases} \tag{3}$$

The following result has been proven in [5].

Theorem 3.1 (Eckstein, Bertsekas) *Suppose E has full column rank and $G(u) + \|F(u)\|^2$ is strictly convex. Let λ_0 and u_0 arbitrary and let $\alpha > 0$. Suppose we are also given sequences $\{\mu_k\}$ and $\{\nu_k\}$ with $\sum_k^\infty \mu_k < \infty$ and $\sum_k^\infty \nu_k < \infty$. Assume that*

1. $\|z^{k+1} - \operatorname{argmin}_{z \in \mathbb{R}^N} J(z) + \langle \lambda^k, Ez \rangle + \frac{\alpha}{2} \|Fu^k + Ez - b\|^2\| \leq \mu_k$
2. $\|u^{k+1} - \operatorname{argmin}_{z \in \mathbb{R}^M} G(u) + \langle \lambda^k, Fu \rangle + \frac{\alpha}{2} \|Fu + Ez^{k+1} - b\|^2\| \leq \nu_k$
3. $\lambda^{k+1} = \lambda^k + \alpha(Fu^{k+1} + Ez^{k+1} - b)$.

If there exists a saddle point of $L_\alpha(z, u, \lambda)$ then $(z^k, u^k, \lambda^k) \to (z^*, u^*, \lambda^*)$ which is such a saddle points. If no such saddle point exists, then at least one of the sequences $\{u^k\}$ or $\{\lambda_k\}$ is unbounded.

4 Bayer filter, acquisition operators and decorrelated basis

4.1 Bayer filter

As said in the introduction most digital cameras use a single CCD sensor with a color filter array (CFA) placed in front of it. The resulting CFA image u^{CFA} is a scalar image having one color component per spatial location. In a discrete setting, if $u^c = (u^R, u^G, u^B)^T$ denotes the initial color image in the RGB basis, the CFA image is of the form (Alleyson et al. [2]):

$$u^{CFA}(i,j) = u^R(i,j)m^R(i,j) + u^G(i,j)m^G(i,j) + u^B(i,j)m^B(i,j) \tag{4}$$

where m^R, m^G and m^G are subsampling functions taking only the values 0 or 1. For the Bayer CFA these functions are expressed as (see [4]):

$$\begin{aligned} m^R(i,j) &= 1/4(1-(-1)^i)(1+(-1)^j) \\ m^G(i,j) &= 1/2(1+(-1)^{i+j}) \\ m^B(i,j) &= 1/4(1+(-1)^i)(1-(-1)^j) \end{aligned}$$

Indeed the fact that the subsampling functions take the value 0 or 1 at pixel (i,j) depends only of the parity of i and j. We denote by A the operator which transforms the initial RGB image u^c into the CFA image u^{CFA}, i.e. $A(u^c) = u^{CFA}$.

4.2 Acquisition operators

Beside of mosaicking effect the initial color image u^c is also degraded during the acquisition process by the presence of blur and noise.

Concerning the blur operator we assume that it is the same for every components. In particular we suppose the following form (with abuse of notation):

$$H = \begin{bmatrix} H & 0 & 0 \\ 0 & H & 0 \\ 0 & 0 & H \end{bmatrix}$$

where H is a matrix representing the standard convolution with some Gaussian kernel. The noise b is supposed to be additive, white and Gaussian, independent of u. So the total acquisition chain can be modeled as

$$u^c = [u^R(i,j), u^G(i,j), u^B(i,j)]^T \to H(u^c) \to AH(u^c) \to AH(u^c) + b = u^{CFA} \tag{5}$$

and the problem consists in reconstructing u^c from u^{CFA} knowing that

$$AH(u^c) + b = u^{CFA}.$$

In fact it is well known that the RGB components are strongly statistically correlated [8, 9]. So it will be better to find another representation $u^d = (\phi, \psi_1, \psi_2)$ in which the components carry independent information.

4.3 A decorrelated basis

The problem is: starting from the RGB system, we want to find another basis in the 3D color space in which components carry non correlated information. Of course it depends on what we mean by independent information. To answer this question we draw our inspiration from the work by Alleyson et al. [2]. From their approach we derive direct calculus to find the new basis. Recall that the CFA image has the form

$$u^{CFA}(i,j) = u^R(i,j)m^R(i,j) + u^G(i,j)m^G(i,j) + u^B(i,j)m^B(i,j). \tag{6}$$

What we want to do is to rewrite u^{CFA} as a linear combination of three functions ψ_0, ψ_1, ψ_2 representing a new "decorrelated" basis in the 3D color space. To do that the idea is first to replace functions m^R, m^G, m^B by their expression:

$$u^{CFA}(i,j) = u^R(i,j)\frac{(1-(-1)^i)(1+(-1)^j)}{4} + u^G(i,j)\frac{(1+(-1)^{i+j})}{2} + u^B(i,j)\frac{(1+(-1)^i)(1-(-1)^j)}{4} \tag{7}$$

Then developing (7) and reorganizing the terms we get:

$$u^{CFA}(i,j) = 1/4u^R(i,j) + 1/2u^G(i,j) + 1/4u^B(i,j) + u^R(i,i)\tilde{m}^R(i,j) + u^G(i,j)\tilde{m}^G(i,j) + u^B(i,j)\tilde{m}^B(i,j) \tag{8}$$

where

$$\tilde{m}^R(i,j) = 1/4((-1)^j - (-1)^i - (-1)^{i+j})$$
$$\tilde{m}^G(i,j) = 1/2(-1)^{i+j}$$
$$\tilde{m}^B(i,j) = 1/4(-(-1)^+(-1)^i - (-1)^{i+j})$$

then by denoting

$$\psi_0(i,j) = 1/4u^R(i,j) + 1/2u^G(i,j) + 1/4u^B(i,j)$$

$$\psi(i,j) = u^R(i,j)\tilde{m}^R(i,j) + u^G(i,j)\tilde{m}^G(i,j) + u^B(i,j)\tilde{m}^B(i,j)$$

we obtain

$$u^{CFA}(i,j) = \psi_0(i,j) + \psi(i,j) \tag{9}$$

Alleyson et al. [2] named ψ_0 and ψ respectively the luminance and the scalar chrominance.

Indeed at each pixel position $\psi_0(i,j)$ can be viewed as the barycentre of the component color values. The coefficients $p^R = 1/4, p^G = 1/2$ and $p^B = 1/4$ associated respectively to u^R, u^G and u^B are constant in space. Therefore the percentage of red, green and blue pixel values remains constant over all spatial positions. Thus ψ_0 does not carry spectral information but only luminance information. Spectral information is carried out by $\psi = u^{CFA} - \psi_0$.

Before coming back to the choice of a basis in the $3D$ color space in which components carry non redundant information, let us rewrite differently ψ by making some algebraic manipulation. With (6), (9) and the definition of ψ we have (to simplify the notation we drop the pixel position (i,j)):

$$u^{CFA} = \sum_{l\in(R,G,B)} u^l m^l = \psi_0 + \sum_{l\in(R,G,B)} u^l \tilde{m}^l$$

but since $\sum_{l\in(R,G,B)} m^l = 1$, from the above equality, we deduce:

$$\sum_{l\in(R,G,B)} (u^l - \psi_0) m^l = \sum_{l\in(R,G,B)} u^l \tilde{m}^l = \psi.$$

Let us denote $\psi^l = u^l - \psi_0$ for $l \in (R, G, B)$, we get $\psi = \sum_{l\in(R,G,B)} \psi^l m^l$ with

$$\begin{aligned}
\psi^R &= 3/4u^R - 1/2u^G - 1/4u^B \\
\psi^G &= -1/4u^R + 1/2u^G - 1/4u^B \\
\psi^B &= -1/4u^R - 1/2u^G + 3/4u^B.
\end{aligned}$$

Finally we have obtained that $u^{CFA} = \psi_0 + \sum_{l\in(R,G,B)} \psi^l m^l$. Now let us recall that what we want to do is to write u^{CFA} as $u^{CFA} = \sum_{l\in(0,1,2)} a_l \psi_l$ where ψ_0, ψ_1, ψ_2 represent a new "decorrelated" basis in the 3D color space. Thanks to the previous discussion it seems natural to choose as a first component the luminance

$$\psi_0 = 1/4u^R + 1/2u^G + 1/4u^B$$

and then for completing the basis to choose ψ_1 and ψ_2 among the three spectral informations ψ^l with $l \in (R, G, B)$. But these three components are dependent since it is easy to verify that $1/4\psi^R + 1/2\psi^G + 1/4\psi^B = 0$. So, we can choose any two components among ψ^R, ψ^G and ψ^B or any combination of it. In the literature most of the authors choose $\psi_1 = \psi^G = -1/4u^R + 1/2u^G - 1/4u^B$ and $\psi_2 = 1/2(\psi^G + \psi^B) = -1/4u^R + 1/4u^B$. With this choice we can verify that

$$u^{CFA} = \psi_0 + (m^R + m^G - m^B)\psi_1 + 2(m^B - m^R)\psi_2.$$

By using now on the classical notations $u^L = \psi_0$ (the luminance), $u^{G/M} = \psi_2$ (the green/magenta chrominance) and $u^{R/B} = \psi_1$ (the red/blue chrominance) the change of basis have the following expression:

$$u^d = \begin{bmatrix} u^L \\ u^{G/M} \\ u^{R/B} \end{bmatrix} = \begin{bmatrix} \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ -\frac{1}{4} & \frac{1}{2} & -\frac{1}{4} \\ -\frac{1}{4} & 0 & \frac{1}{4} \end{bmatrix} \begin{bmatrix} u^R \\ u^G \\ u^B \end{bmatrix} = T(u^c) \tag{10}$$

and

$$u^c = \begin{bmatrix} u^R \\ u^G \\ u^B \end{bmatrix} = \begin{bmatrix} 1 & -1 & -2 \\ 1 & 1 & 0 \\ 1 & -1 & 2 \end{bmatrix} \begin{bmatrix} u^L \\ u^{G/M} \\ u^{R/B} \end{bmatrix} = T^{-1}(u^d). \tag{11}$$

Hereafter u^c denotes the image in the canonical basis R, G, B, while u^d is the image in the basis $L, C^{G/M}, C^{R/B}$.

Let us remark:

- Alleyson in [2] showed that the Fourier transform of u^L is concentrated in the low frequency domain while the Fourier transform of the chrominance $u^{G/M}$ and $u^{R/B}$ are concentrated in the high frequency domain. Moreover the support of the three components in the Fourier domain are disjoint.
- In [8, 9] it is shown that, from a statistical point of view, the coefficients in the new basis are approximatively decorrelated.

5 The variational model

Let us start by recalling the acquisition camera sequence. We have (see (5)):

$$u^c \to H(u^c) \to AH(u^c) \to AH(u^c) + b = u^{CFA}$$

On the other hand from (11) we have $u^c = T^{-1}(u^d)$. By a change of variable, the observation equation (5) becomes

$$AHT^{-1}(u^d) + b = u_0. \tag{12}$$

where we have set $u_0 = u^{CFA}$.

The idea is then to restore u^d by working with the, much more convenient, decorrelated basis $L, C^{G/M}, C^{R/B}$. Finally, once u^d is restored, we simply set $u^c = T(u^d)$.

In order to retrieve u^d, we have to solve an ill posed inverse problem. So that as usual we seek for minimizer of an energy given by an L^2-discrepancy term plus a regularization penalty.

Now the key point is that, since we are working in the decorrelated basis, it makes sense to consider the following minimization problem:

$$\arg\min_{u^d} \|\nabla u^L\|_1 + \|\nabla u^{G/M}\|_1 + \|\nabla u^{R/B}\|_1 + \mu\|AHT^{-1}(u^d) - u_0\|_2^2.$$

(the norms are the ones defined in Section 2). Notice that the regularizing terms are uncoupled.

5.1 Application of ADMM method to our problem

In order to apply the ADMM method, we must rewrite the problem

$$\arg\min_{u^d} \|\nabla u^L\|_1 + \|\nabla u^{G/M}\|_1 + \|\nabla u^{R/B}\|_1 + \mu\|AHT^{-1}(u^d) - u_0\|_2^2, \tag{13}$$

in the form (1):

$$\min_{u^d, z} J(z) + G(u^d) \quad \text{subject to } Ez + Fu^d = b.$$

For our problem, we set

$$G(u^d) = 0$$

and

$$z = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ v \end{bmatrix} = \begin{bmatrix} \nabla u^L \\ \nabla u^{G/M} \\ \nabla u^{R/B} \\ AHT^{-1}(u^d) - u_0 \end{bmatrix}, \tag{14}$$

(15)

$$E = -I, \quad F = \begin{bmatrix} \nabla \\ AHT^{-1} \end{bmatrix} \quad \text{where} \quad \nabla = \begin{bmatrix} \nabla^L & 0 & 0 \\ 0 & \nabla^{G/M} & 0 \\ 0 & 0 & \nabla^{R/B} \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ u_0 \end{bmatrix}$$

We also need the dual variable

$$\lambda = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ q \end{bmatrix}.$$

To simplify the notation we write

$$z = \begin{bmatrix} w \\ v \end{bmatrix} = \begin{bmatrix} \nabla u^d \\ AHT^{-1}(u^d) - u_0 \end{bmatrix}, \quad F = \begin{bmatrix} \nabla \\ K \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ u_0 \end{bmatrix} \tag{16}$$

and finally

$$\lambda = \begin{bmatrix} p \\ q \end{bmatrix}$$

We can now write down the corresponding augmented lagrangian as:

$$\begin{aligned} & L_\alpha(z, u^d, \lambda) = \|w\|_1 + \mu\|v\|_2^2 + \langle p, \nabla u^d - w\rangle + \langle q, Ku^d - u_0 - v\rangle \\ + \quad & \frac{\alpha}{2}\|v - Ku^d + u_0\|^2 + \frac{\alpha}{2}\|\nabla u^d - w\|^2. \end{aligned} \tag{17}$$

The ADMM iterations are then given by:

$$\begin{aligned} w^{k+1} &= \operatorname*{argmin}_w \|w\|_1 + \frac{\alpha}{2}\|w - \nabla(u^d)^k - \frac{p^k}{\alpha}\|_2^2 \\ v^{k+1} &= \operatorname*{argmin}_v \mu\|v\|_1 + \frac{\alpha}{2}\|v - K(u^d)^k + u_0 - \frac{q^k}{\alpha}\|_2^2 \\ (u^d)^{k+1} &= \operatorname*{argmin}_{u^d} \frac{\alpha}{2}\|\nabla u^d - w^{k+1} + \frac{p^k}{\alpha}\|_2^2 + \frac{\alpha}{2}\|Ku^d - v^{k+1} - u_0 + \frac{q^k}{\alpha}\|_2^2 \\ p^{k+1} &= p^k + \alpha(\nabla(u^d)^{k+1} - w^{k+1}) \\ q^{k+1} &= q^k + \alpha(K(u^d)^{k+1} - u_0 - v^{k+1}), \end{aligned}$$

with $p^0 = q^0 = 0$ and $\alpha > 0$.

The standard explicit formulas for w^{k+1}, v^{k+1} and $(u^d)^{k+1}$ are:

$$\begin{aligned} w^{k+1} &= S_{\frac{1}{\alpha}}(\nabla(u^d)^k + \frac{p^k}{\alpha}) \\ v^{k+1} &= S_{\frac{\mu}{\alpha}}(K(u^d)^k - u_0 + \frac{q^k}{\alpha}) \\ (u^d)^{k+1} &= (-\Delta + K^*K)^{-1}\big(\nabla^*(w^{k+1} - \frac{p^k}{\alpha}) + K^*(v^{k+1} + u_0 - \frac{q^k}{\alpha})\big) \end{aligned} \tag{18}$$

where $S_{\frac{1}{\alpha}}(t)$ is the standard soft thresholding, that is

$$S_{\frac{1}{\alpha}}(t) = \begin{cases} t - \frac{1}{\alpha} sign(t) & |t| > \frac{1}{\alpha} \\ 0 & \text{otherwise.} \end{cases}$$

$S_{\frac{\mu}{\alpha}}$ is defined in the same way, up to the obvious replacement of $\frac{1}{\alpha}$ with $\frac{\mu}{\alpha}$. K^* denotes the adjoint matrix of the matrix $K = AHT^{-1}$ given by $K^* = (T^{-1})^* H^* A^*$. Note that one can compute all of these adjoint operators. Δ denotes the usual Laplace's operator.

Concerning the last iteration of system (18), we can verify by induction that in fact the sum $\nabla^* \frac{p^k}{\alpha} - K^* \frac{q^k}{\alpha} = 0$, so (18) simplifies:

$$(u^d)^{k+1} = (-\Delta + K^* K)^{-1} \big(\nabla^* w^{k+1} + K^*(v^{k+1} + u_0)\big) \tag{19}$$

To solve it numerically we used a classical conjugate gradient method.

6 Numerics

We test our method on images of big size (number of pixels $d_1 d_2 = P$: $500 \times 700 \leq P \leq 2200 \times 4000$).

In order to have a blurred mosaicked and noisy image to test, we follow the following standard procedure:

1 we pick a color image as a reference u^c, which is a good approximation of a color image without mosaicking effect.;

2 we apply in the right order the acquisition operator to get the observed degraded image u_0:

$$u_0 = AHu^c + b;$$

3 we formally write $u^c = T^{-1} u^d$ and we work with the new basis $(u^L, u^{\frac{G}{M}}, u^{\frac{R}{B}})$. So we have

$$u_0 = AHT^{-1}(u^d) + b;$$

4 We apply the ADMM algorithm to restore u^d;

5 We set $u^c = T(u^d)$.

As blur operator we always have considered a standard Gaussian low pass filter standard deviation $\epsilon = 1$. In figures 1, 2, 3, we restore an image of size 2200×2000 with a low level of noise. When the level noise is high, μ cannot be too small otherwise, the algorithm does not perform a good demosaicking. In this case the parameter μ is chosen in order to have a good balancing between denoising and demosaicking. In figure 4 we show the restoration results of an image reference detail with different values of the parameter μ. Then in figures 6 and 7 we show the restoration result obtained on the whole image.

We deal with rescaled images in $[0, 1]$. We made run the Matlab code on an Intel(R) Xeon(R) CPU 5120 @ 1.86GHz.

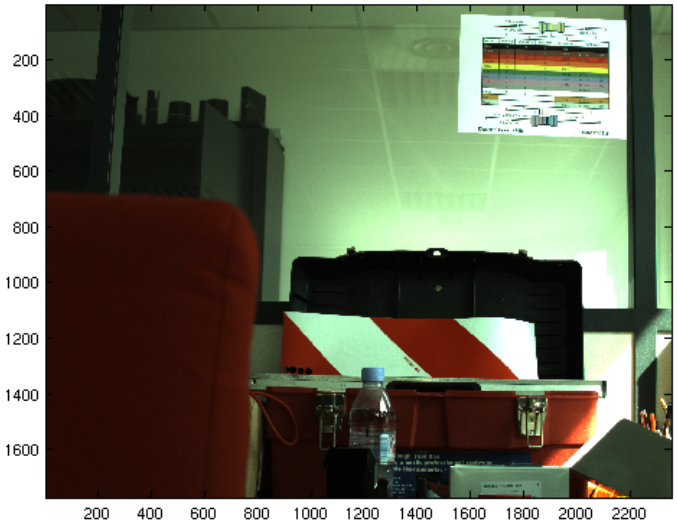


Figure 1: Original image $u^c = T^{-1}(u^d)$. Size image 2200x2000.

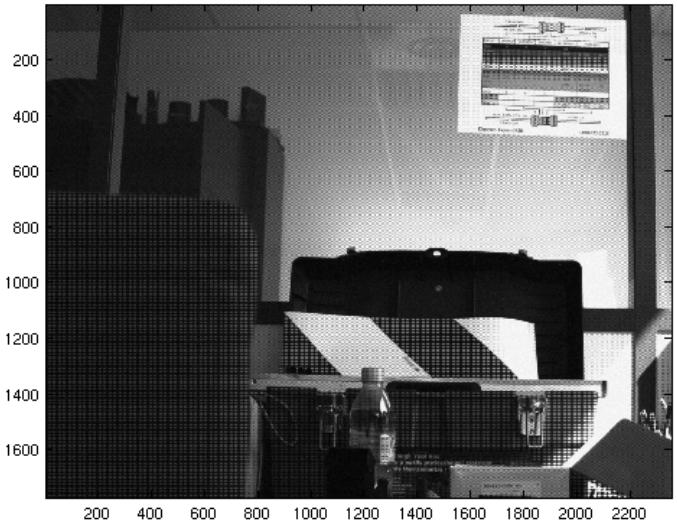


Figure 2: Observed mosaicked blurred and noisy image $u_0 = AHT^{-1}(u^d) + b$. $\sigma = 0.01$.

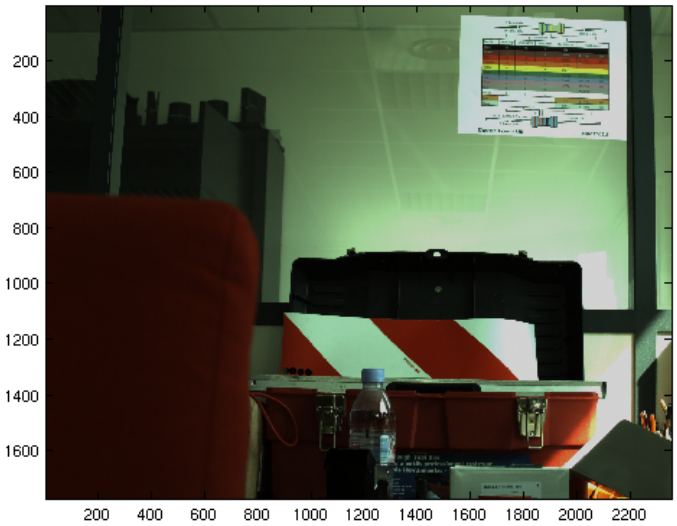


Figure 3: Restored image $u^c = T(u^d)$. CPU time about 30mn, number of iterations 30, $\mu = 30$.

The results presented here show good performance of the algorithm in terms of visual quality. The restored images show no false color for image of Figure 1and very few for image in Figures 4 and 5.

7 Conclusion

In this paper we have introduced in an intuitive way a basis for color image where the components are nearly statistically decorrelated. Then this basis is useful to write a criterion where regularization terms are independent so that standard minimization algorithms can be used. We show that ADMM algorithm can be applied in our case and that the method give good results, avoiding most of false color artifacts. In order to bring an automatic algorithm, estimation of regularizing parameter must now be performed.

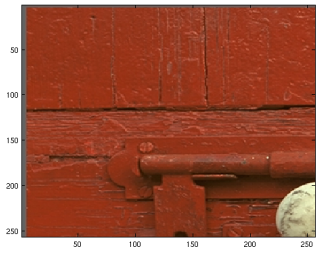
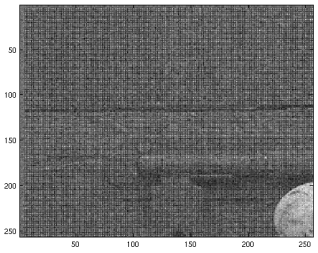
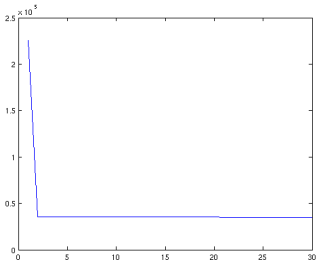
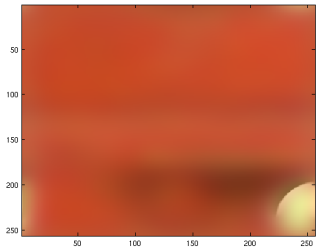
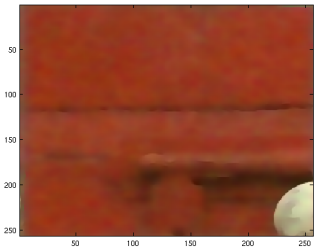
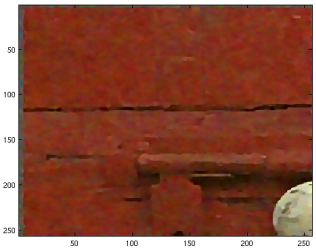
Detail of original image	Observed image	convergence of the algorithm
		
restored image with $\mu = 0.5$	restored image with $\mu = 5$	restored image with $\mu = 50$
		

Figure 4: Top left: crop of the original image. Crop size 256x256. Top center: blurred mosaicked noisy image. Top right: convergence of the algorithm. Down left: restored image with a small μ to promote the denoising against the demosaicking. Number of iterations 30. Down center and down right : restored image with a greater value of μ to promote demosaicking against the denoising. Number of iterations 30. CPU time about 2mn.

Figure 5: Original image of size 768x512.

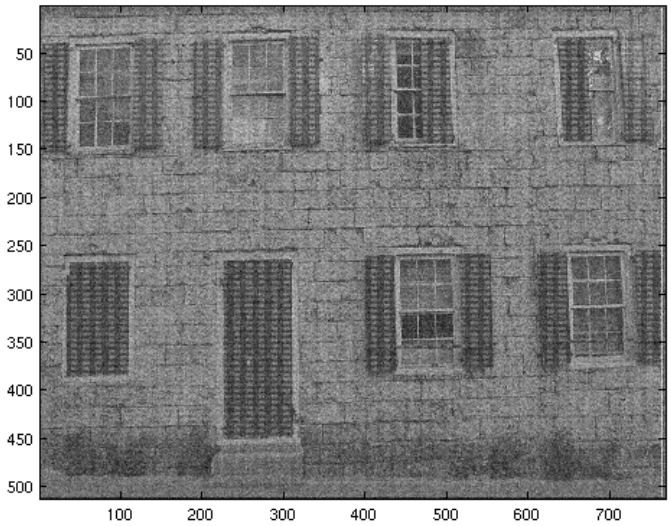


Figure 6: Observed mosaicked blurred and noisy image. $\sigma = 0.5$.

Figure 7: Restored image $u^c = T(u^d)$. CPU time about 15 mn, number of iterations 30 $\mu = 20$.

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