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To cite this version:

Suat Topsu, Luc Chassagne, Darine Haddad, Yasser Alayli, Patrick Juncar. High accuracy velocity control method for the french moving-coil watt balance. Review of Scientific Instruments, 2004, 75 (11), pp.4824-4827. 10.1063/1.1809302 . hal-00870592

HAL Id: hal-00870592
https://hal.science/hal-00870592
Submitted on 7 Oct 2013

REVIEW OF SCIENTIFIC INSTRUMENTS VOLUME 75, NUMBER 11 NOVEMBER 2004

High accuracy velocity control method for the french moving-coil watt balance

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(Received 9 February 2004; accepted 20 August 2004; published 1 November 2004)

We describe a novel method of velocity control dedicated to the French moving-coil watt balance. In this project, a coil has to move in a magnetic field at a velocity of 2 mm s^{-1} with a relative uncertainty of 10^{-9} over 60 mm. Our method is based on the use of both a heterodyne Michelson's interferometer, a two-level translation stage, and a homemade high frequency phase-shifting electronic circuit. To quantify the stability of the velocity, the output of the interferometer is sent into a frequency counter and the Doppler frequency shift is recorded. The Allan standard deviation has been used to calculate the stability and a $\sigma_y(\tau)$ of about 2.2×10^{-9} over 400 s has been obtained. © *2004 American Institute of Physics.* [DOI: 10.1063/1.1809302]

I. INTRODUCTION

At present, the base units of the SI are related to atomic properties or fundamental constants, except for the unit of mass. Actually, the kilogram is still defined in relation to an artifact, a platinium–iridium cylinder kept at the Bureau International des Poids et Mesures (BIPM). The results of periodic comparisons between national kilogram prototypes and the standard unit of mass have called the definition of this unit into question as a mean drift of 5×10^{-8} kg has been observed over the last century.[1] One strategy to improve the status of the kilogram is the watt balance project which relates the unit of mass to Planck's constant via electrical energy.[2–5] One can show[6] that if the electrical quantities are measured using the Josephson effect[7] and the quantum Hall effect,[8] the test mass m can be expressed in terms of the meter, the second and the Planck's constant h as given by Eq. (1),

$$m = K\frac{f_J^2}{gV}\times h, \tag{1}$$

where f_J is the Josephson frequency, g the local acceleration of gravity, V the velocity of the moving-coil, and K a constant. To be able to measure a mass with a relative standard uncertainty of 10^{-8}, it is necessary to measure each physical parameter with a better one. In this article, we present a method that will permit us to control the velocity of the moving-coil with a relative uncertainty of 10^{-9}. In the French setup, the weight of the 500 g gold platinum alloy test mass will be balanced by a 5 mA current flowing in a circular coil of 27 mm diameter immersed in a 1 T radial magnetic field generated by a permanent magnet (SmCo). The coil and the test mass will be moved at a constant velocity of 2 mm s^{-1} producing a 1 V induced voltage at the coil terminals. The vertical displacement will be assured thanks to a set of six flexure strips monitored by a magnetic linear motor and a piezoelectric actuator.

II. HIGH ACCURACY INTERFEROMETRIC TECHNIQUE FOR VELOCITY CONTROL

The velocity must be measured with a traceable method with regard to the definitions of the meter and the second in SI. For this purpose, we decided to use heterodyne optical interferometry with a laser which frequency is one of those recommended by the Comité International des Poids et Mesures to define the meter. Time is measured using a high stability quartz oscillator calibrated in regard to a cesium atomic clock.

Consider a heterodyne Michelson's interferometer with a moving mirror mounted on a translation stage. Even with the better high precision translation stages, it is impossible to obtain a stability of the velocity below to 10^{-6} in relative value over centimetric displacement. This can be achieved only with a two-levels translation stage. The first level (TS1) has a long travel range but a low positioning accuracy rather than the second level (TS2) has a short travel range but a high sensitivity (generally a piezoelectric actuator). The aim of TS2 is to compensate the translation defects of TS1. The principle of our method is depicted in Fig. 1. A heterodyne laser source emits two orthogonally polarized beams separated in frequency by $\delta\nu=|\nu_2-\nu_1|$. The optical beams pass through the Michelson's interferometer. They are separated thanks to a polarization beamsplitter. One of the beams is retroreflected using a fixed corner cube and the other using a movable corner cube supported by a two-levels translation stage. The two beams are recombined at the output of the interferometer and mixed thanks to a polarizer resulting in a signal s_3. This signal contains the information about the velocity.

TS1 is a commercial high accuracy translation stage with its servo control system. This servo control system is independent and different compared to the control system of TS2. The velocity control of TS2 is based on the use of a high frequency phase-shifting electronic circuit. This circuit gen-

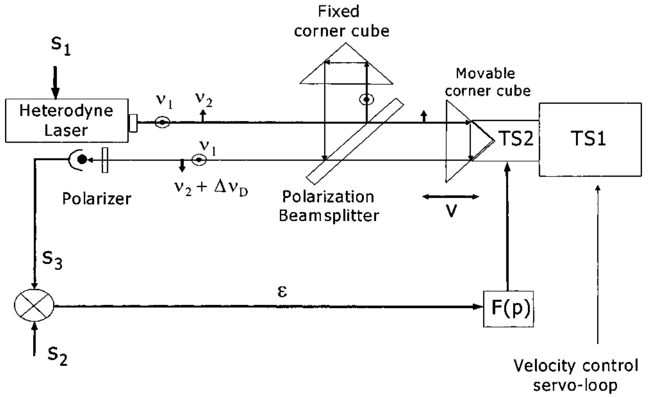

FIG. 1. Principle of the high accuracy velocity control method dedicated to the BNM watt balance experiment. A moving mirror of a Michelson's interferometer is mounted on a two-levels translation stage, one rough (TS1) and one fine (TS2). A motion of TS1 leads to a Doppler phase shift of the signal s_3 equal to $\phi_D \pm \delta\phi_1$, where $\delta\phi_1$ is the phase noise due to the fluctuation of V. Simultaneously phase shifts equal to $\phi_D \pm \delta\phi_2$ are made onto s_2 leading also to a velocity V for the movable mirror but with TS2 and with an uncertainty $\delta\phi_2 \ll \delta\phi_1$. The result is that the stability of the velocity of the mirror is as fine as permitted by TS2 over the entire travel range of TS1.

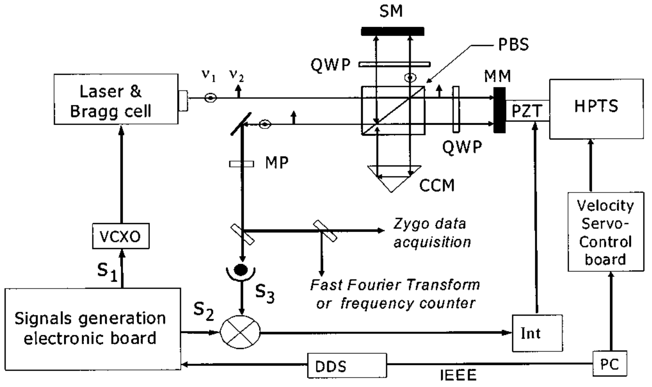

FIG. 2. Experimental setup. SM, Stationary Mirror; MM, movable mirror; PBS, Polarization Beamsplitter; QWP, Quarter Wave Plate; HPTS, High Precision Translation Stage; VCO, Voltage Control Oscillator; CCM, Corner Cube Mirror; PZT, Piezoelectric Actuator; Int, Integrator; MP, Mixing Polarizer; DDS, Digital Direct Signal generator. The part of the optical beam sent to the Fast Fourier Transform apparatus or to the frequency counter is first demodulated with a reference signal at a frequency of 20 MHz. The signal generator is controlled by a computer (PC) via an IEEE communication port.

erates two synchronized signals s_1 and s_2 both at a frequency $\delta\nu$. It allows us to also produce phase shifts on these signals. Signal s_2 is sent to a mixer to be phase-compared with s_3. Signal s_1 feeds a Bragg cell (not seen in Fig. 1) placed in the laser head to perform the two optical beams separated in frequency by $\delta\nu$ and orthogonally polarized thanks to a birefringent plate. A motion of the mirror with a velocity $V \pm \delta V_1$ is generated via TS1. The motion of TS1 leads to a Doppler phase shift per unit of time of the signal s_3, equal to $\phi_D \pm \delta\phi_1$, where $\delta\phi_1$ is the phase noise due to δV_1. Simultaneously, the phase shift equal to $\phi_D \pm \delta\phi_2$ is made onto s_2 leading also to a velocity $V \pm \delta V_2$ for the movable mirror but with an uncertainty $\delta V_2 \ll \delta V_1$. As $\delta\phi_2 \ll \delta\phi_1$, signal error ε is given by

$$\varepsilon = \pm K(\delta\phi_1 - \delta\phi_2) \approx \pm K \times \delta\phi_1, \tag{2}$$

where K is a constant factor. Hence, the servo-loop control of TS2 will improve the defect $\delta\phi_1$ of TS1. The result is that ε is always around zero and the stability of the velocity of the mirror is as fine as permitted by TS2 over the entire travel range of TS1. Notice that it is possible to control the direction of the displacement of the mirror by making phase shifts either on the signal s_2 either on the signal s_1. The limit of this method is due to the fact that it is not possible to equalize experimentally the two velocities perfectly. Hence the maximum travel of the movable mirror is limited by the maximum travel range of the piezoelectric actuator. However, with a 3 μm travel range piezoelectric actuator and our electronics, the movable mirror has been displaced over 80 mm without breaking the loop lock of TS2 which is enough in the case of the French watt balance.

III. EXPERIMENTAL SETUP

The experimental setup is shown in Fig. 2. We use a commercial interferometric system (ZMI2001, Zygo) with a double-pass interferometer. Its resolution is equal to 0.31 nm. The difference between the two optical components $\delta\nu$ is equal to 20 MHz. The translation stage is a two-levels one. The first level TS1 is a high precision translation stage from Aerotech (ALS20010) monitored by a magnetic linear motor. It is designed to have a velocity control at the 10^{-5} accuracy level in relative value. The drive system is composed of a linear brushless servo motor. The total travel is about 100 mm and the maximum velocity is 2 m s^{-1}. The second level TS2 is a piezoelectric translator (PZT). The maximum travel range is 3 μm for an applied voltage of 40 V. The velocity control of this stage is performed thanks to the method we described above. The signal generation board is made from high speed and low phase noise logic components and a high frequency clock (640 MHz). Minimum phase shift achievable on s_1 or s_2 is equal to $2\pi/32$. The beam at the output of the Michelson's interferometer is divided in three parts. One part is used to generate the signal s_3 at the mixer level. Another part is sent to the Zygo data acquisition board and then transferred to a PC using a PCI-VME connection to visualize the displacement of the movable mirror. The third part of the beam is used to measure the Doppler frequency shift using either a fast Fourier transform (FFT) apparatus either a frequency counter. For this purpose, the useful signal is first demodulated with a reference signal at a frequency $\delta\nu$ (not represented in Fig. 2).

IV. RESULTS AND DISCUSSIONS

A. Spectral analysis of the servo loop control efficiency

We measure the Doppler frequency shift during the displacement of the mirror by sending the output of the Michelson's interferometer into a FFT apparatus (SR785, Stanford) as seen in Fig. 2. Figure 3 shows the Doppler frequency shift of the optical beams during the displacement of the mirror with a velocity of 0.2 mm s^{-1} using only TS1. Such a velocity corresponds to a mean frequency of about 1.264 kHz with a dispersion due to the instability of the velocity during the displacement. Once the second level translation

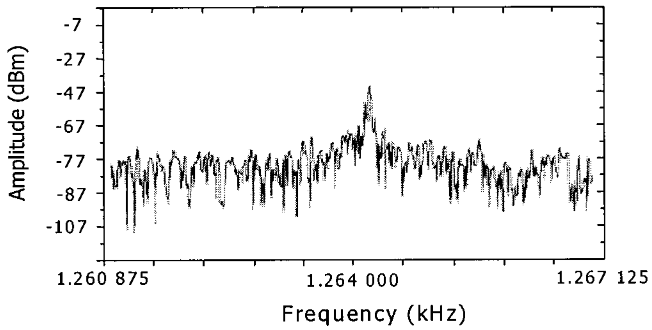

FIG. 3. Spectral measurement of the Doppler frequency shift during the displacement of the mirror. The velocity of the mirror is equal to 0.2 mm s^{-1}. Only the rough level of the two-levels translation stage is used.

stage TS2 and its velocity control loop get started the peak becomes narrow (Fig. 4) showing hence the efficiency of our velocity control method.

B. Characterization of the performance of the velocity control servo-loop using the Allan variance

To quantify the stability of the velocity when the two control servo-loops are running, we use the Allan standard deviation. It is currently used in time frequency metrology for an estimation of the stability of primary frequency standards.[9] For this purpose, the signal coming from one part of the beam at the output of the Michelson's interferometer at a frequency $\delta\nu \pm \nu_D$ (where $\delta\nu = 20$ MHz and ν_D is the Doppler frequency shift) is demodulated with a reference signal at a frequency 20 MHz coming from our high frequency electronic circuit. The frequency of the resulting signal is measured thanks to a high stability frequency counter HP53132A with a relative accuracy of 10^{-10}. The mirror is moved over 80 mm with a velocity of 2 mm s^{-1} leading, respectively, to an acquisition time of 40 s. During this time, the Doppler frequency shift is measured over 10 s giving one measurement point. During the 40 s, 3 points are recorded. This step is repeated 34 times leading to 102 points of 10 s. We use the overlapped Allan variance $\sigma_y^2(\tau)$ calculus. The maximum estimation time is then about 400 s. In the same way, the overlapped Allan standard deviation for a velocity of 0.2 mm s^{-1} has been calculated with a set of 200 data. Each data is measured over 30 s. The maximum estimation time with the overlapped Allan standard deviation is then 2700 s. Figures 5 and 6 represent the standard deviation of the Doppler frequency shift corresponding, respectively, to a velocity of the moving mirror of 0.2 mm s^{-1} (i.e., 1.264 kHz) and 2 mm s^{-1} (i.e., 12.641 kHz). One can show that $\sigma_y(\tau)$ is about 3.5×10^{-9} over 2700 s for a velocity of 0.2 mm s^{-1} (Fig. 5) and 2.2×10^{-9} over 400 s for a velocity of 2 mm s^{-1} (Fig. 6). This result shows that in terms of stability, our method reaches the specifications imposed by the moving-coil watt balance project.

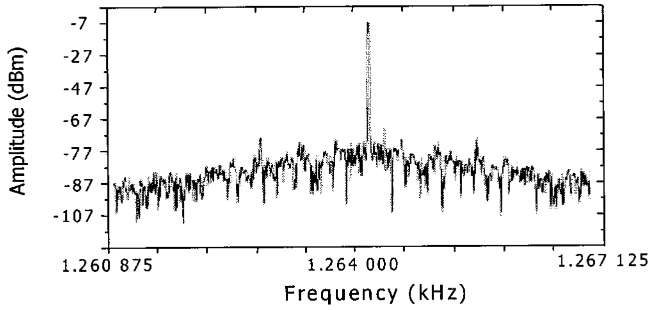

FIG. 4. Spectral measurement of the Doppler frequency shift using the two levels of control. The velocity of the mirror is equal to 0.2 mm s^{-1}. The two levels of the translation stage are used. Once the second level translation stage and its velocity control loop get started the peak becomes narrow (by comparison to Fig. 3).

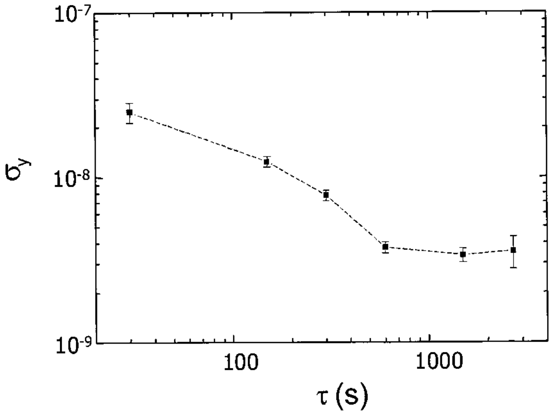

FIG. 5. Allan standard deviation of the Doppler frequency shift for a velocity of **0.2 mm s^{-1}**. $\sigma_y(\tau)$ has been calculated with 200 data. Each point is averaged over 30 s.

C. Absolute value of the velocity

1. Measurement of the velocity from the Doppler frequency shift

As we use a double-pass interferometer, the Doppler frequency shift for a velocity V is equal to

$$\nu_D = \nu_0 \times \frac{4V}{nc}, \qquad (3)$$

where n is the refractive index of air. In our case, the laser frequency has been calibrated with regard to a national reference of the Institut National de Métrologie (Paris, France) and is equal to $\nu_0 = 473.612\,117\,6 \times 10^{12}$ Hz. With V

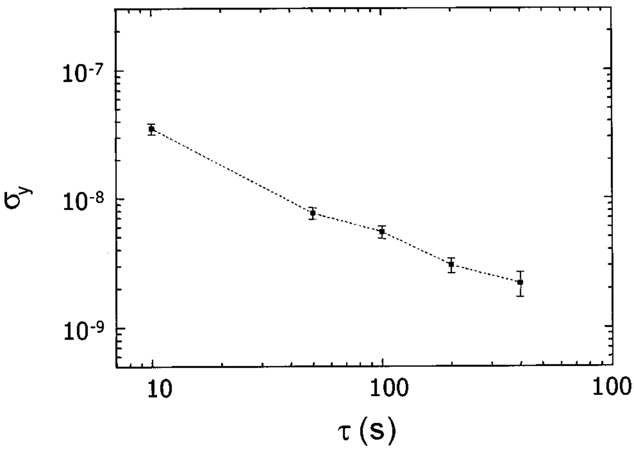

FIG. 6. Allan standard deviation of the Doppler frequency shift for a velocity of **2 mm s^{-1}**. $\sigma_y(\tau)$ has been calculated with 100 data. Each point is averaged over 10 s.

$=2\ \mathrm{mm\,s^{-1}}$, one obtains a theoretical value (taking $n=1$) of the Doppler frequency shift of 12 638.399 80 Hz. To compare with experimentation, we measure the Doppler frequency shift of the optical beam at the output of the Michelson's interferometer. As previously, one part of the beam stemmed from the interferometer is sent onto a detector and mixed with a reference signal at a frequency of 20 MHz. The resulting signal is sent to the frequency counter HP 53132A and the value is recorded on a computer. For this purpose, the mirror moves over 80 mm and the Doppler frequency shift is recorded over 40 s. This step is repeated until we obtain 100 points. We measure an arithmetical mean value of 12 641.873 76 Hz corresponding to a velocity of $2.000\,549\,747\ \mathrm{mm\,s^{-1}}$. The discrepancy is first due to the refractive index of air. Actually, this experiment is not made in an air-conditioned room and the optical path difference between both arms of the Michelson's interferometer is about 1 m. Nevertheless, as the watt balance experiment is planned to be made under vacuum this error is not significant. Second, it is impossible to impose exactly a velocity of $2\ \mathrm{mm\,s^{-1}}$ on TS1 because of its limited accuracy. Furthermore, what is interesting for us is not to have exactly a velocity of $2\ \mathrm{mm\,s^{-1}}$ but a relative uncertainty of 10^{-9} on V. The relative uncertainty on V by this method could reach 10^{-10} if the experiment is made under vacuum.

2. Measurement of the velocity using the number of steps

It is also possible to know the value of V using the repetition rate of the phase shifts and the value of the corresponding displacement Δx. Actually, when a phase shift $\Delta\phi$ is generated on s_2 the movable mirror will move until both phases of s_2 and s_3 become equal thanks to the lock-in electronic system as seen in Fig. 2. If these phase shifts are quantified, it becomes possible to move the mirror step by step.[10] The displacement value Δx of the mirror corresponding to the phase shift $\Delta\phi$ is

$$\Delta x = \frac{\Delta\phi}{2\pi}\frac{\lambda_0}{4n}, \tag{4}$$

where λ_0 is the laser wavelength in vacuum and n the refractive index of air. The period of repetition of the phase shifts can be controlled thanks to a high-stability quartz oscillator (DS345 - Stanford), hence we can impose an uniform speed to the moving mirror given by

$$V = \nu' \times \Delta x, \tag{5}$$

where ν' is the repetition rate of the phase shifts. With $V=2\ \mathrm{mm\,s^{-1}}$ and $\Delta\phi=2\pi/32$ corresponding to a step value of $\Delta x=4.945\,246\,313$ nm, we obtain a theoretical value $(n=1)$ of $\nu'=404\,428$ Hz. The experimental value of ν' is determined so as to minimize the error signal ε at the mixer output (see Fig. 2). We found a value of 403 209 Hz corresponding to a velocity of 1.993 971 733 nm. This experimental value is different from the value found with the previous method because these two experiments have been made at different times. The tolerance on ν' depends on the travel range of the piezoelectric actuator and the total displacement range of the movable mirror. With our 3 μm travel range PZT and for a total displacement of 80 mm, a tolerance of 1 Hz on the value of ν' is enough. This could be easily achievable with the digital signal generator DS345 from Stanford. The discrepancy between the theoretical and experimental values is essentially due to the refractive index of air and to the limited accuracy of the velocity control servo-loop TS1. One can show that the relative uncertainty on V is given

$$\frac{\sigma_{\langle V\rangle}}{\langle V\rangle} = \frac{2\sqrt{3}\langle\Delta x\rangle^{1/2}\sigma_{\langle\Delta x\rangle}}{D^{3/2}}, \tag{6}$$

where $\langle\Delta x\rangle$ is the mean step value, $\sigma_{\langle\Delta x\rangle}$ the noise level on each step, and D the total displacement value. Neglecting the refractive index of air, the relative uncertainty on $\langle V\rangle$ could reach 10^{-12} (using $\langle\Delta x\rangle=4.945\,297\,313$ nm, $\sigma_{\langle\Delta x\rangle}=0.22$ nm and $D=80$ mm).

ACKNOWLEDGMENT

This project is funded by the Bureau National de Métrologie of France and overseen by G. Genevès of the BNM-LAMA/LNE Laboratory.

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