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High accuracy velocity control method for the french moving-coil watt balance

Suat Topcu, Luc Chassagne, Darine Haddad, and Yasser Alayli
LIRIS, Université de Versailles, 45 Avenue des États-Unis, 78035 Versailles, France

Patrick Juncar
BNM-INM/CNAM, 292 Rue Saint Martin, 75141 Paris, France

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We describe a novel method of velocity control dedicated to the French moving-coil watt balance. In this project, a coil has to move in a magnetic field at a velocity of 2 mm s\(^{-1}\) with a relative uncertainty of 10\(^{-9}\) over 60 mm. Our method is based on the use of both a heterodyne Michelson’s interferometer, a two-level translation stage, and a homemade high frequency phase-shifting electronic circuit. To quantify the stability of the velocity, the output of the interferometer is sent into a frequency counter and the Doppler frequency shift is recorded. The Allan standard deviation has been used to calculate the stability and a \(\sigma_\text{V}(\tau)\) of about 2.2 \(\times 10^{-9}\) over 400 s has been obtained. © 2004 American Institute of Physics. [DOI: 10.1063/1.1809302]

I. INTRODUCTION

At present, the base units of the SI are related to atomic properties or fundamental constants, except for the unit of mass. Actually, the kilogram is still defined in relation to an artifact, a platinum–iridium cylinder kept at the Bureau International des Poids et Mesures (BIPM). The results of periodic comparisons between national kilogram prototypes and the standard unit of mass have called the definition of this unit into question as a mean drift of 5 \(\times 10^{-8}\) kg has been observed over the last century.\(^1\) One strategy to improve the status of the kilogram is the watt balance project which relates the unit of mass to Planck’s constant via electrical standards. The kilogram is still defined in relation to an artifact, a platinium–iridium cylinder kept at the Bureau International des Poids et Mesures. To be able to measure a mass with a relative standard uncertainty of 10\(^{-9}\), it is necessary to measure each physical parameter with a better one. In this article, we present a method that will permit us to control the velocity of the moving-coil with a relative uncertainty of 10\(^{-9}\). In the French setup, the weight of the 500 g gold platinum alloy test mass will be balanced by a 5 mA current flowing in a circular coil of 27 mm diameter immersed in a 1 T radial magnetic field generated by a permanent magnet (SmCo). The coil and the test mass will be moved at a constant velocity of 2 mm s\(^{-1}\) producing a 1 V induced voltage at the coil terminals. The vertical displacement will be assured thanks to a set of six flexure strips monitored by a magnetic linear motor and a piezoelectric actuator.

II. HIGH ACCURACY INTERFEROMETRIC TECHNIQUE FOR VELOCITY CONTROL

The velocity must be measured with a traceable method with regard to the definitions of the meter and the second in SI. For this purpose, we decided to use heterodyne optical interferometry with a laser which frequency is one of those recommended by the Comité International des Poids et Mesures to define the meter. Time is measured using a high stability quartz oscillator calibrated in regard to a cesium atomic clock.

Consider a heterodyne Michelson’s interferometer with a moving mirror mounted on a translation stage. Even with the better high precision translation stages, it is impossible to obtain a stability of the velocity below to 10\(^{-6}\) in relative value over centimetric displacement. This can be achieved only with a two-levels translation stage. The first level (TS1) has a long travel range but a low positioning accuracy rather than the second level (TS2) has a short travel range but a high sensitivity (generally a piezoelectric actuator). The aim of TS2 is to compensate the translation defects of TS1. The principle of our method is depicted in Fig. 1. A heterodyne laser source emits two orthogonally polarized beams separated in frequency by \(\delta \nu = |\nu_1 - \nu_2|\). The optical beams pass through the Michelson’s interferometer. They are separated thanks to a polarization beamsplitter. One of the beams is retroreflected using a fixed corner cube and the other using a movable corner cube supported by a two-levels translation stage. The two beams are recombined at the output of the interferometer and mixed thanks to a polarizer resulting in a signal \(s_3\). This signal contains the information about the velocity.

TS1 is a commercial high accuracy translation stage with its servo control system. This servo control system is independent and different compared to the control system of TS2. The velocity control of TS2 is based on the use of a high frequency phase-shifting electronic circuit. This circuit gen-
erates two synchronized signals $s_1$ and $s_2$ both at a frequency $\delta v$. It allows us to also produce phase shifts on these signals. Signal $s_2$ is sent to a mixer to be phase-compared with $s_3$. Signal $s_1$ feeds a Bragg cell (not seen in Fig. 1) placed in the laser head to perform the two optical beams separated in frequency by $\delta v$ and orthogonally polarized thanks to a birefringent plate. A motion of the mirror with a velocity $V \pm \delta V_1$ is generated via TS1. The motion of TS1 leads to a Doppler phase shift per unit of time of the signal $s_3$, equal to $\phi_2 \pm \delta \phi_1$, where $\delta \phi_1$ is the phase noise due to the fluctuation of $V$. Simultaneously phase shifts equal to $\phi_2 \pm \delta \phi_2$ are made onto $s_2$ leading also to a velocity $V$ for the movable mirror but with TS2 and with an uncertainty $\delta \phi_2 \ll \delta \phi_1$. The result is that the stability of the velocity of the mirror is as fine as permitted by TS2 over the entire travel range of TS1.

where $K$ is a constant factor. Hence, the servo-loop control of TS2 will improve the defect $\delta \phi_1$ of TS1. The result is that $\epsilon$ is always around zero and the stability of the velocity of the mirror is as fine as permitted by TS2 over the entire travel range of TS1. Notice that it is possible to control the direction of the displacement of the mirror by making phase shifts either on the signal $s_2$ either on the signal $s_1$. The limit of this method is due to the fact that it is not possible to equalize experimentally the two velocities perfectly. Hence the maximum travel of the movable mirror is limited by the maximum travel range of the piezoelectric actuator. However, with a 3 µm travel range piezoelectric actuator and our electronics, the movable mirror has been displaced over 80 mm without breaking the loop lock of TS2 which is enough in the case of the French watt balance.

**III. EXPERIMENTAL SETUP**

The experimental setup is shown in Fig. 2. We use a commercial interferometric system (ZMI2001, Zygo) with a double-pass interferometer. Its resolution is equal to 0.31 nm. The difference between the two optical components $\delta v$ is equal to 20 MHz. The translation stage is a two-levels one. The first level TS1 is a high precision translation stage from Aerotech (ALS20010) monitored by a magnetic linear motor. It is designed to have a velocity control at the $10^{-5}$ accuracy level in relative value. The drive system is composed of a linear brushless servo motor. The total travel is about 100 mm and the maximum velocity is 2 m s$^{-1}$. The second level TS2 is a piezoelectric translator (PZT). The maximum travel range is 3 µm for an applied voltage of 40 V. The velocity control of this stage is performed thanks to the method we described above. The signal generation board is made from high speed and low phase noise logic components and a high frequency clock (640 MHz). Minimum phase shift achievable on $s_1$ or $s_2$ is equal to $2\pi/32$. The beam at the output of the Michelson’s interferometer is divided in three parts. One part is used to generate the signal $s_1$ at the mixer level. Another part is sent to the Zygo data acquisition board and then transferred to a PC using a PCI-VME connection to visualize the displacement of the movable mirror. The third part of the beam is used to measure the Doppler frequency shift using either a fast Fourier transform (FFT) apparatus either a frequency counter. For this purpose, the useful signal is first demodulated with a reference signal at a frequency $\delta v$ (not represented in Fig. 2).

**IV. RESULTS AND DISCUSSIONS**

**A. Spectral analysis of the servo loop control efficiency**

We measure the Doppler frequency shift during the displacement of the mirror by sending the output of the Michelson’s interferometer into a FFT apparatus (SR785, Stanford) as seen in Fig. 2. Figure 3 shows the Doppler frequency shift of the optical beams during the displacement of the mirror with a velocity of 0.2 mm s$^{-1}$ using only TS1. Such a velocity corresponds to a mean frequency of about 1.264 kHz with a dispersion due to the instability of the velocity during the displacement. Once the second level translation

**FIG. 1. Principle of the high accuracy velocity control method dedicated to the BNM watt balance experiment. A moving mirror of a Michelson’s interferometer is mounted on a two-levels translation stage, one rough (TS1) and one fine (TS2). A motion of TS1 leads to a Doppler phase shift of the signal $s_1$ equal to $\phi_2 \pm \delta \phi_1$, where $\delta \phi_1$ is the phase noise due to the fluctuation of $V$. Simultaneously phase shifts equal to $\phi_2 \pm \delta \phi_2$ are made onto $s_2$ leading also to a velocity $V$ for the movable mirror but with TS2 and with an uncertainty $\delta \phi_2 \ll \delta \phi_1$. The result is that the stability of the velocity of the mirror is as fine as permitted by TS2 over the entire travel range of TS1.**

**FIG. 2. Experimental setup. SM, Stationary Mirror; MM, movable mirror; PBS, Polarization Beamsplitter; QWP, Quarter Wave Plate; HPTS, High Precision Translation Stage; VCO, Voltage Control Oscillator; CCM, Corner Cube Mirror; PZT, Piezoelectric Actuator; Int, Integrator; MP, Mixing Polarizer; DDS, Digital Direct Signal generator. The part of the optical beam sent to the Fast Fourier Transform apparatus or to the frequency counter is first demodulated with a reference signal at a frequency of 20 MHz. The signal generator is controlled by a computer (PC) via an IEEE communication port.**
stage TS2 and its velocity control loop get started the peak becomes narrow (Fig. 4) showing hence the efficiency of our velocity control method.

B. Characterization of the performance of the velocity control servo-loop using the Allan variance

To quantify the stability of the velocity when the two control servo-loops are running, we use the Allan standard deviation. It is currently used in time frequency metrology for an estimation of the stability of primary frequency standards.9 For this purpose, the signal coming from one part of the beam at the output of the Michelson’s interferometer at a frequency \( \delta v \pm v_D \) (where \( \delta v \approx 20 \text{ MHz} \) and \( v_D \) is the Doppler frequency shift) is demodulated with a reference signal at a frequency 20 MHz coming from our high frequency electronic circuit. The frequency of the resulting signal is measured thanks to a high stability frequency counter HP53132A with a relative accuracy of \( 10^{-10} \). The mirror is moved over 80 mm with a velocity of 2 mm s\(^{-1}\) leading, respectively, to an acquisition time of 40 s. During this time, the Doppler frequency shift is measured over 10 s giving one measurement point. During the 40 s, 3 points are recorded. This step is repeated 34 times leading to 102 points of 10 s. We use the overlapped Allan variance \( \sigma_v^2(\tau) \) calculus. The maximum estimation time is then about 400 s. In the same way, the overlapped Allan standard deviation for a velocity of 0.2 mm s\(^{-1}\) has been calculated with a set of 200 data. Each point is averaged over 30 s. The maximum estimation time with the overlapped Allan standard deviation is then 2700 s. Figures 5 and 6 represent the standard deviation of the Doppler frequency shift corresponding, respectively, to a velocity of the moving mirror of 0.2 mm s\(^{-1}\) (i.e., 1.264 kHz) and 2 mm s\(^{-1}\) (i.e., 12.641 kHz). One can show that \( \sigma_v(\tau) \) is about \( 3.5 \times 10^{-9} \) over 2700 s for a velocity of 0.2 mm s\(^{-1}\) (Fig. 5) and \( 2.2 \times 10^{-9} \) over 400 s for a velocity of 2 mm s\(^{-1}\) (Fig. 6). This result shows that in terms of stability, our method reaches the specifications imposed by the moving-coil watt balance project.

C. Absolute value of the velocity

1. Measurement of the velocity from the Doppler frequency shift

As we use a double-pass interferometer, the Doppler frequency shift for a velocity \( V \) is equal to

\[
\nu_D = \nu_0 \times \frac{4V}{nc},
\]

where \( n \) is the refractive index of air. In our case, the laser frequency has been calibrated with regard to a national reference of the Institut National de Métrologie (Paris, France) and is equal to \( \nu_0 = 473.612\ 117\ 6 \times 10^{12} \text{ Hz} \). With \( V \)
= 2 mm s\(^{-1}\), one obtains a theoretical value (taking \(n = 1\)) of the Doppler frequency shift of 12 638 399 80 Hz. To compare with experimentation, we measure the Doppler frequency shift of the optical beam at the output of the Michelson’s interferometer. As previously, one part of the beam stemmed from the interferometer is sent onto a detector and mixed with a reference signal at a frequency of 20 MHz. The resulting signal is sent to the frequency counter HP 53132A and the value is recorded on a computer. For this purpose, the mirror moves over 80 mm and the Doppler frequency shift is recorded over 40 s. This step is repeated until we obtain 100 points. We measure an arithmetical mean value of 12 641 873 76 Hz corresponding to a velocity of 2 000 549 747 mm s\(^{-1}\). The discrepancy is first due to the refractive index of air. The period of repetition of the phase shifts and the value of the corresponding displacement \(\Delta x\) is generated on steps 1–10. The displacement value \(\Delta x\) can be controlled thanks to a high-stability quartz oscillator (Michelson’s interferometer) and the optical path difference between both arms of the Michelson’s interferometer is about 1 m. Nevertheless, as the watt balance experiment is planned to be made under vacuum this error is not significant. Second, it is impossible to impose exactly a velocity of 2 mm s\(^{-1}\) on TS1 because of its limited accuracy. Furthermore, what is interesting for us is not to have exactly a Doppler frequency shift of 12 638 399 80 Hz. To compare with experimentation, we measure the experimental values of the Doppler frequency shift of 12 638 399 80 Hz. The discrepancy between the theoretical and experimental values is essentially due to the refractive index of air and to the limited accuracy of the velocity control servo-loop TS1. One can show that the relative uncertainty on \(V\) is given by

\[
\sigma_{(V)} = \frac{2\sqrt{3(\Delta x)^{1/2}}\sigma_{(\Delta x)}}{D^{3/2}},
\]

where \(\langle V \rangle\) is the mean step value, \(\sigma_{(\Delta x)}\) the noise level on each step, and \(D\) the total displacement value. Neglecting the refractive index of air, the relative uncertainty on \(\langle V \rangle\) could reach \(10^{-12}\) (using \(\langle \Delta x \rangle = 4.945 297 313\) nm, \(\sigma_{(\Delta x)} = 0.22\) nm and \(D = 80\) mm).

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