M-nets: A survey

Hanna Klaudel, Franck Pommereau

Noname manuscript No.
(will be inserted by the editor)

Hanna Klaudel · Franck Pommereau

M-nets: a survey.

the date of receipt and acceptance should be inserted later

Abstract This paper surveys the research related to the model of M-nets since it was introduced in 1995. M-nets are high-level labelled Petri nets which can be composed, like process algebra terms, using various operators. We present the core model, several of its extensions and the main applications.

Keywords High-level Petri nets, compositions, action refinement, synchronisation.

1 Introduction

The model of *Petri nets* [68] is based on concepts coming from automata theory, linear algebra and graph theory. Besides the general advantages of a formal model and the verification methods based on linear algebra, Petri nets are additionally attractive since they have a simple graphical representation. This characteristics is important already in the design process showing how a concurrent system is built and distributed in space. It gives a clear image of concurrency, sequentiality and conflict, both on the concrete visual level and on the graph-theoretic level. In particular, such an explicit representation of concurrency is suitable when studying non sequential ("true concurrency" or partial order) semantics of concurrent systems.

On the other hand, the *modular design* of large systems allows to reduce and manage their complexity. This is possible either in a bottom-up way by composing smaller subsystems, or going top-down by refining parts of a rough model by more detailed system descriptions. In both cases, systems

H. Klaudel
IBISC, Université d'Evry, 523 place des Terrasses, 91000 Evry, France, E-mail: klaudel@ibisc.univ-evry.fr

F. Pommereau
LACL, Univ. Paris Est, 61 av. du général de Gaulle, 94010 Créteil, France, E-mail: pommereau@univ-paris12.fr

are constructed from building blocks and a semantics should support the modular construction of systems. Modularity has been a somewhat weak point of Petri net theory: a Petri net is defined as a whole, and not in the first instance obtained by composing subnets. This is totally different in process algebras where systems are described by process terms, which are by definition built from subterms. The semantics of a process term is obtained from the semantics of its subterms and is compositional by nature. However, the standard process algebras reduce concurrency to interleaving arguing that interleaving is simpler than true concurrency and just as expressive. On the other hand, interleaving based models are less suitable for a top-down design, since they describe systems using actions that are assumed to be instantaneous and indivisible.

Both Petri nets and process algebras approaches have increasingly influenced each other, and considerable effort has been made to combine their respective merits. The *Petri Box Calculus* (*PBC*) [4,5], which evolved later to the *Petri Net Algebra* (*PNA*) [6], offers a very general solution to combine process algebras and Petri nets. However, in practical situations, the standard low-level Petri net model on which PBC is based leads to huge nets which are well defined mathematically but difficult to grasp intuitively. As a consequence, the appreciated feature of Petri nets of representing graphically system properties such as concurrency, sequentiality, conflicts, etc., vanishes because of the size of Petri nets necessary to describe the considered system. To address this problem, high-level Petri net models have been proposed, such as *predicate/transition nets* [42], *coloured Petri nets* [47], or *algebraic nets* [76].

In particular, the *Calculus of Modular Multilabelled Nets* (*M-nets*) [8,51] was introduced in order to combine the compositionality of process algebras and the explicit representation of concurrency of Petri nets in a common high-level framework. Indeed, M-nets are considered as the coloured counterpart of the PBC. Actually, M-nets and PBC are related through an operation of *unfolding* which takes an M-net N and yields an equivalent low-level net N_ℓ. N can be seen as an abbreviation of N_ℓ, and N_ℓ as the semantics of N. M-nets support various composition operations (parallel, sequence, choice, iteration, synchronisation, restriction, etc.), which are essentially the same as in PBC. Indeed, it was one of the main aims in the design of the M-net model to ensure that the unfolding of a composed net coincides with the composition of the unfoldings of its parts. The PBC and M-nets are implemented in the PEP tool [41] which allows to edit, simulate and verify systems using model-checking. Moreover, the SNAKES toolkit [73], a tool specifically dedicated to work with variants of M-nets and PBC, allows to quickly implement new operations for those models, thus providing a framework to prototype and experiment with new variants.

This paper surveys the research related to M-nets since they were introduced. The next section defines the Petri net aspects of the model, including the refinement (meta-)operation which allows to substitute M-net transitions by arbitrary M-nets. The low-level net model is also introduced in order to state the consistency between high and low levels. Section 3 presents several extensions of the core model: the parameterised refinement allowing to

exchange information between different abstraction levels of a system; recursion which is like a repetitive refinement; and buffered communication providing a simple scheme to share data between different parts of an M-net. Section 4 shows how an algebra of M-nets is built using their annotations (for synchronous and asynchronous communication) and the refinement (for the control flow operations). The introduced operations are consistent with those existing in PBC. Section 5 reviews several applications or further extensions of the M-nets algebra. This includes a definition of an M-net semantics of the parallel specification language B(PN)^2 and several other extensions like the introduction of object oriented paradigms, the modelling of mobility, preemption (suspend/resume and abort of subsystems) or timing constraints. Finally, section 6 presents a case study using M-nets to model and verify a timed railroad crossing system.

2 The model of M-nets

2.1 Basic definitions and notations

We start with the definition of multisets which are widely used in the following. Let E be a set. A *multiset* over E is a function $\mu : E \to \mathbb{N}$ which associates to each element of E its number of occurrences in μ. A multiset μ is finite if so is the set $\{x \in E \mid \mu(x) \neq 0\}$. Sometimes, we will use for multisets an extended set notation; for instance, $\{x, y, y\}$ will denote the multiset μ such that $\mu(x) = 1$, $\mu(y) = 2$ and $\mu(z) = 0$ for all $z \in E \setminus \{x, y\}$. We will denote by $\emptyset$ the empty multiset. We also introduce the following notations for μ, μ_1 and μ_2 multisets over E and $n \in \mathbb{N}$:

- the symbols $+$, $-$, and $*$ denote, respectively, the sum, the difference of multisets and the multiplication of a multiset by a natural number; formally, for x in E: $(\mu_1 + \mu_2)(x) \stackrel{\text{df}}{=} \mu_1(x) + \mu_2(x)$, $(\mu_1 - \mu_2)(x) \stackrel{\text{df}}{=} \max(0, \mu_1(x) - \mu_2(x))$ and $(n * \mu)(x) \stackrel{\text{df}}{=} n * \mu(x)$;
- we write $x \in \mu$ if $\mu(x) > 0$, and $\mu_1 \subseteq \mu_2$ if $\forall x \in E : \mu_1(x) \leq \mu_2(x)$;
- we denote by $\mathsf{mult}(E)$ the set of all the multisets over E, and by $\mathsf{mult}_f(E)$ the set of the finite multisets over E.

A (*low-level*) *Petri net* is a directed bipartite graph whose nodes are *places* or *transitions*. The places may be *marked*, *i.e.*, may carry *tokens*. The *input arcs* of a transition t come from the set of *pre-places* of t and the *output arcs* of t go to its *post-places*. A transition is *activated* if its pre-places are marked by sufficiently many tokens. It may then be *fired* in which case some tokens are removed from each pre-place and some other produced in each post-place. The arcs are *weighted*, *i.e.*, annotated by a natural number (where 0 corresponds to the absence of the arc) indicating the number of tokens which are transported through the arc during the firing of a transition. Places and transitions may also carry labels. Formally:

Definition 1 A *(low-level) labelled Petri net* N is a quadruple (S, T, W, λ) where:

- S is a set of places and T is a set of transitions, with $S \cap T = \emptyset$;

- $W : (S \times T) \cup (T \times S) \to \mathbb{N}$ is the weight function on arcs;
- λ is the labelling function on $S \cup T$.

A marking of N is a function $M : S \to \mathbb{N}$ which associates to each place the number of tokens it carries.

For a place or a transition $x \in S \cup T$, we define ${}^\bullet x \stackrel{\text{df}}{=} \{y \in S \cup T \mid W(y,x) > 0\}$ and $x^\bullet \stackrel{\text{df}}{=} \{y \in S \cup T \mid W(x,y) > 0\}$. The marking of a place defines a local state, in such a way that the global state of the net is represented by the set of all such local states. The dynamic behaviour of such a net is given by the transition rule:

Definition 2 Let $N = (S, T, W, \lambda)$ be a labelled Petri net and M its marking. A transition $t \in T$ is *activated* at M iff $\forall s \in S : M(s) \geq W(s,t)$. The *firing* of t produces the visible action $\lambda(t)$ and gives rise to the new marking M' defined by $\forall s \in S : M'(s) \stackrel{\text{df}}{=} M(s) - W(s,t) + W(t,s)$.

The transition rule illustrates the property of locality of nets: only the part ${}^\bullet t \cup t^\bullet$ of the global state is involved in the firing of the transition t. In concurrent systems, the actions (represented by the occurrences of transitions) may appear concurrently, *i.e.*, independently of each other. If these occurrences are described by an arbitrary interleaving of actions, then each sequence of independent actions is a sequence of occurrences of the system and the corresponding semantics is called a *sequential* or an *interleaving* semantics. If the occurrence of a finite multiset of actions is allowed, then the corresponding semantics is a *step* or a *concurrent* semantics [3, 9, 43, 44]. If the occurrences of actions are partially ordered, then the corresponding semantics is a *partial order* or *true concurrency* semantics [66, 79, 81].

The model based on low-level nets is particularly interesting because it is supported by various implemented tools (see [69, 80] for a presentation of many tools) and may be analyzed using methods devoted to such nets or using efficient algorithms of *model checking*. However, these nets are not often used directly because the specifications of real size systems are in general too large to be understandable. The designers often prefer to use high-level versions of Petri nets which provide a better abstraction and which may be automatically *unfolded* to low-level nets before being analysed.

Figure 1 represents two equivalent nets (in the sense that they describe the same behaviours), the net on the left is high-level and the net on the right is its *unfolding* (so, it is a low-level net).

The annotations used for the high-level nets have the following meanings:

- the places are *typed*, *i.e.*, have associated sets of values (s has the type $\{\bullet\}$, s' has the type $\{1,2,3\}$ and s'' has the type $\{2,3\}$);
- the tokens are values respecting the types of the places (s' carries the token 2 and s'' carries the token 3);
- the arcs are annotated with *values* or *variables*; the scope for the variables is bound to a transition and its adjacent arcs (so, the variables around a transition have only a local meaning and may be consistently renamed);
- the transitions may carry Boolean expressions called *guards* playing the role of firing conditions: a transition can fire only with tokens which make

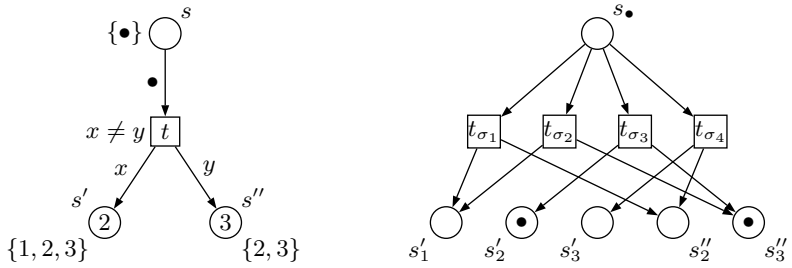


Fig. 1 A high-level net (on the left) and its unfolding (on the right) whose labelling has been omitted. The σ_i's are the modes of t in the high-level net, the indexes of the s'_i's and the s''_j's correspond to the values in the types of s' and s''.

the guard true (this is the case for the transition t taking, for instance, $x = 2$ and $y = 3$, but not for $x = y = 2$). The guard is often used in order to *compute* values during a firing. Here, since x and y are free variables (not bound on an input arc), the transition actually computes non-deterministically a number in $\{1, 2, 3\}$ for x and another one in $\{2, 3\}$ for y, choosing them distinct (because of the guard).

The transition rule in a high-level net is more complicated than that used at the low-level: in order to fire a transition t it is necessary to take tokens in the pre-places of t, to associate them to the variables around t and to ensure that the guard evaluates to true. Moreover, only values allowed by the types of places may be produced. The mappings, called *bindings*, associating a value to each variable exist independently of the marking of the net and of the guard of t. (However, we shall only consider the variables involved in t when showing a binding intended to be used for t.) For t in figure 1, the bindings are of the form $(x \mapsto i, y \mapsto j)$ where i and j are arbitrary values. A binding is called a *mode* if it allows to evaluate the guard to true and if the value corresponding to each variable appearing in an annotation of an arc belongs to the type of the adjacent place. So, the modes of t in figure 1 are $\sigma_1 \stackrel{\text{df}}{=} (x \mapsto 1, y \mapsto 2)$, $\sigma_2 \stackrel{\text{df}}{=} (x \mapsto 1, y \mapsto 3)$, $\sigma_3 \stackrel{\text{df}}{=} (x \mapsto 2, y \mapsto 3)$ and $\sigma_4 \stackrel{\text{df}}{=} (x \mapsto 3, y \mapsto 2)$, but not $\sigma'_1 \stackrel{\text{df}}{=} (x \mapsto 1, y \mapsto 1)$ nor $\sigma'_2 \stackrel{\text{df}}{=} (x \mapsto 2, y \mapsto 2)$ because they do not respect the guard of t, and neither $\sigma'_3 \stackrel{\text{df}}{=} (x \mapsto 0, y \mapsto 2)$ because it does not respect the type of s'.

These high-level aspects are expressible in the low-level nets through the operation of *unfolding* defined as follows:

- each high-level place is unfolded to as many low-level places as there are values in its type (so, s' gives rise to the low-level places s'_1, s'_2 and s'_3);
- the marking is obtained in such a way that, for instance, the token 2 in s' leads to a (black) token in the place s'_2;
- each high-level transition is unfolded to as many low-level transitions as there are modes associated to it (so, t generates the low-level transitions t_{σ_1} to t_{σ_4});
- the arcs are obtained consistently with the modes. So, according to the mode $\sigma_1 = (x \mapsto 1, y \mapsto 2)$, t_{σ_1} is connected to s'_1, with the weight 1

corresponding to the multiplicity of x in the annotation of the high-level arc, and to s''_2, with the weight 1 corresponding to the multiplicity of y in the annotation of the high-level arc.

The unfolding allows to express systems in the high-level domain with the guarantee that they have a representation in the low-level one, which is needed for their automated analysis. (Notice, however, that allowing infinite types for the high-level places gives rise to infinite low-level nets.)

2.2 Static and dynamic aspects of M-nets

We consider the following pairwise disjoint sets:

- *Val* is the set of *values* (in particular, *Val* contains the "black token" $\bullet$, natural numbers, Boolean values, etc.);
- *Var* is the set of *variables* (*Var* is assumed large enough to allow renaming each time it is necessary in order to avoid name clashes);
- $\mathbb{A}$ is the set of high-level *action symbols*, provided with a bijection $\widehat{\ }$, called *conjugation*, such that for all $act \in \mathbb{A} : \widehat{act} \neq act$ and $\widehat{\widehat{act}} = act$. Each symbol $act \in \mathbb{A}$ has an *arity* $\mathsf{ar}(act)$ and we have $\mathsf{ar}(\widehat{act}) = \mathsf{ar}(act)$. The terms $act(x_1, \ldots, x_{\mathsf{ar}(act)})$ and $\widehat{act}(x_1, \ldots, x_{\mathsf{ar}(act)})$ (where $x_i \in Val \cup Var$ for $1 \leq i \leq \mathsf{ar}(act)$) are (high-level) *actions* and they are said to be *elementary* if all their arguments (the x_i's) are values;
- $\mathbb{A}_\ell$ corresponds to $\mathbb{A}$ in the low-level domain; it is the set of all elementary actions constructed from $\mathbb{A}$ and *Val*. (Notice that $\mathbb{A}_\ell$ is closed under $\widehat{\ }$.)
- $\mathbb{X}$ contains the *hierarchical symbols*, which will be used to denote abstract actions used to label transitions to be refined;
- the symbols e, i and x denote the *status* of places used to label places in order to guide net compositions; for a net N, we denote by N^{e}, N^{i} and N^{x}, respectively, the set of its *entry* places (labelled by e), *internal* places (labelled by i) and *exit* places (labelled by x).

The *boxes* are labelled low-level nets with some structural constraints:[1] the entry places have no input arcs (symmetrically, the exit places have no output arcs), there always exists entry and exit places and each transition has at least one pre-place and one post-place.

Definition 3 A *box* $N = (S, T, W, \lambda)$ is a low-level labelled Petri net such that:

- for each place $s \in S$ we have $\lambda(s) \in \{\mathsf{e}, \mathsf{i}, \mathsf{x}\}$;
- for each transition $t \in T$, we have $\lambda(t) \in \mathsf{mult}_f(\mathbb{A}_\ell)$ (if t is a *communication* transition) or $\lambda(t) \in \mathbb{X}$ (if t is a *hierarchical* transition);
- N is ex-restricted: $N^{\mathsf{e}} \neq \emptyset \neq N^{\mathsf{x}}$;
- N is ex-oriented: $\forall t \in T, \forall s \in N^{\mathsf{e}}, \forall s' \in N^{\mathsf{x}} : W(t,s) = 0 \ \wedge \ W(s',t) = 0$;
- N is T-restricted: $\forall t \in T, \exists s, s' \in S : W(s,t) > 0 \ \wedge \ W(t,s') > 0$.

[1] These conditions were required in PBC [4] but relaxed in PNA [6]. We will use the original definition which leads to a simpler, more intuitive, model.

With this definition, in particular, we are sure to have non-empty entry and exit interfaces (N^{e} and N^{x}) in every box N. The communication and the hierarchical interfaces, which are composed of transitions, may be empty. These interfaces will be crucial for defining composition operations on boxes, see section 4. The transition rule for boxes is that of low-level labelled Petri nets.

M-nets are high-level boxes. In order to introduce them, we need essentially to enrich the annotations of low-level nets.

Definition 4 An *M-net* is a triple $N = (S, T, \iota)$, where S is the set of places, T is the set of transitions (with $S \cap T = \emptyset$) and ι is the annotation function on $S \cup T \cup (S \times T) \cup (T \times S)$, such that:

- for each place $s \in S$, $\iota(s)$ is a pair $\lambda(s).\alpha(s)$ where $\lambda(s) \in \{\mathsf{e}, \mathsf{i}, \mathsf{x}\}$ gives the status of s and $\alpha(s) \subseteq \mathit{Val}$, with $\alpha(s) \neq \emptyset$, gives its type;
- for each transition $t \in T$, $\iota(t)$ is a pair $\lambda(t).\gamma(t)$, where:
  - either $\lambda(t)$ is a finite multiset of high-level actions, or $\lambda(t) \in \mathbb{X}$;
  - $\gamma(t)$ is the guard of t, which is a Boolean expressions on Var and Val. We denote by $\mathsf{var}(t)$ (a subset of Var) the set of variables appearing in the annotations of t and its arcs;
- for each arc (s, t), $\iota(s, t)$ is a multiset of structured annotations on $\mathit{Var} \cup \mathit{Val}$, representing the values consumed during a firing of t; similarly, the values produced during a firing of t are represented by the annotation $\iota(t, s)$;
- N is ex-restricted, ex-oriented and T-restricted (like for boxes, using $\iota(x, y) = \emptyset$ or $\iota(x, y) \neq \emptyset$ instead of $W(x, y) = 0$ or $W(x, y) > 0$ respectively).

The structured annotations are formalised in [37] and illustrated in the following, see for instance figures 5 and 6. They include, depending on the context, constants and variables, but also more complex terms (introduced later on, possibly including distinguished symbols ζ and φ) that encode, for each mode, different sets of values.

In the figures, the hierarchical transitions are represented using double lines (the hierarchical symbols being the capital letters X, Y, etc.). Entry places are depicted with an incoming double arrow, exit places with an outgoing double arrow and internal places with no double arrow. Also, the notations are often simplified: an empty communication label or an empty (true) guard are generally omitted, arcs with empty annotations are never represented, the singleton multisets are replaced by their unique element, the places are not always named, etc. The purpose of these simplifications is to alleviate the presentation in order to focus on the aspects serving directly the understanding.

We will formalise now different notions allowing us to define the unfolding and the transition rule of M-nets. This will allow us to state the property of consistency of the behaviour of an M-net with respect to the behaviour of its unfolding.

A *binding* of a transition t is a substitution $\sigma : \mathsf{var}(t) \to \mathit{Val}$. If x is an entity (expression, action, etc.) which depends on the variables in $\mathsf{var}(t)$, we denote by $\sigma(x)$ the evaluation of x under σ. A transition t with a guard

$\gamma(t)$ may be fired with a binding σ only if, for all $g \in \gamma(t)$, $\sigma(g)$ is *true*. Moreover, if $\forall s \in S : \sigma(\iota(s,t)) \in \mathsf{mult}(\alpha(s)) \ \wedge \ \sigma(\iota(t,s)) \in \mathsf{mult}(\alpha(s))$, *i.e.*, if the annotations on the arcs evaluated under σ respect the types of the places, we say that σ *activates* t, *i.e.*, that σ is a *mode* for t.

A *marking* of an M-net (S,T,ι) is a function $M : S \to \mathsf{mult}(\mathit{Val})$ which associates to each place $s \in S$ a multiset of values from $\alpha(s)$. We distinguish the *entry* marking for which $M(s) = \alpha(s)$ if $\lambda(s) = \mathsf{e}$ and $M(s) = \emptyset$ otherwise, and the *exit* marking for which $M(s) = \alpha(s)$ if $\lambda(s) = \mathsf{x}$ and $M(s) = \emptyset$ otherwise. The typical expected behaviour of an M-net is to start its execution from its entry marking, finishing, if ever, with its exit marking.

The *unfolding* operation, unf, associates a low-level labelled net $\mathsf{unf}(N)$ to any M-net N and also the marking $\mathsf{unf}(M)$ of $\mathsf{unf}(N)$ to any marking M of N. Examples are given in the figures 1 and 2.

Definition 5 Let $N \stackrel{\text{df}}{=} (S,T,\iota)$ be an M-net. The *unfolding* of N is the low-level labelled net $\mathsf{unf}(N) \stackrel{\text{df}}{=} (\mathsf{unf}(S), \mathsf{unf}(T), W, \lambda)$ such that:

- $\mathsf{unf}(S) \stackrel{\text{df}}{=} \{s_v | s \in S, v \in \alpha(s)\}$, and for all $s_v \in \mathsf{unf}(S)$ we have $\lambda(s_v) = \lambda(s)$;
- $\mathsf{unf}(T) \stackrel{\text{df}}{=} \{t_\sigma | t \in T, \sigma \text{ is a mode for } t\}$, and for all $t_\sigma \in \mathsf{unf}(T)$ we have $\lambda(t_\sigma) = \sigma(\lambda(t))$;
- $W(s_v, t_\sigma) \stackrel{\text{df}}{=} \sum_{x \in \iota(s,t) \wedge v = \sigma(x)} \iota(s,t)(x)$ and analogously for $W(t_\sigma, s_v)$.

Let M be a marking of N. The marking $\mathsf{unf}(M)$ of $\mathsf{unf}(N)$ is defined as follows: for all places $s_v \in \mathsf{unf}(S)$, $\mathsf{unf}(M)(s_v) \stackrel{\text{df}}{=} M(s)(v)$. So, the number of (black) tokens in s_v is the number of occurrences of v in s.

Such a definition of the unfolding corresponds to the intuition presented in figures 1 and 2: the places and the transitions are unfolded separately, the arcs are derived from the corresponding bindings and the marking of the low-level net reflects directly that of the M-net. We have the following static property:

Proposition 1 *The unfolding of an M-net N is a box.*

The dynamic aspects of the net are captured though to the following definition of the transition rule.

Definition 6 A transition t may fire at a marking M and for a mode σ iff for all places $s \in S$, we have $\sigma(\iota(s,t)) \leq M(s)$. The firing of t produces the action $\sigma(\lambda(t))$ and leads to a new marking M' defined by $\forall s \in S : M'(s) \stackrel{\text{df}}{=} M(s) - \sigma(\iota(s,t)) + \sigma(\iota(t,s))$.

Like for low-level nets, it is possible to consider various kinds of semantics (interleaving, step or truly concurrent). It is also possible to prove the consistency property relating the behaviour of a high-level net with that of its unfolding [8].

Proposition 2 *A transition t may be fired in the M-net N at a marking M for the mode σ, leading to the marking M' iff t_σ may be fired in $\mathsf{unf}(N)$ at the marking $\mathsf{unf}(M)$, leading to the marking $\mathsf{unf}(M')$. The elementary actions corresponding to the executed transition in both nets are the same: $\sigma(\lambda(t)) = \lambda(t_\sigma)$.*

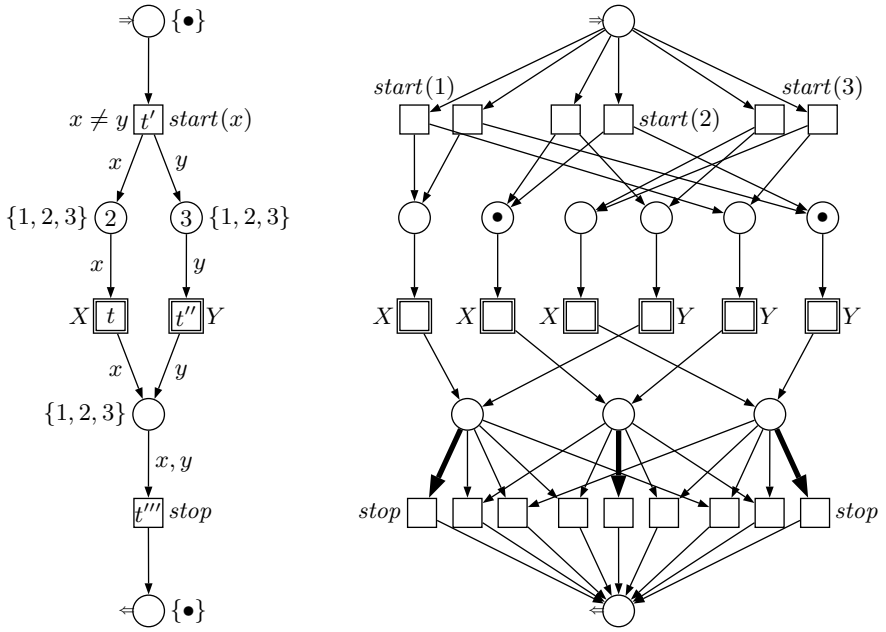


Fig. 2 An M-net with hierarchical and communication transitions (on the left) and its unfolding (on the right). In the latter, thick arcs have the weight 2 (all the other arcs having the weight 1); this corresponds to the modes of t''' that bind x and y to the same value. The bottom-most transitions are all labelled by *stop* and the top-most transitions are grouped according to their labels: $start(1)$ for the two left-most ones, $start(2)$ for the two in the middle and $start(3)$ for the two right-most ones.

2.3 M-net refinement

One of the essential features of a modular Petri net model is the ability of expressing various levels of abstraction, *i.e.*, the possibility to specify systems in an incremental way using successive refinements. It is also important to be able to define the operations on the model in a simple and homogeneous way. These features can be provided thanks to a *transition refinement (substitution)*, allowing one to replace a transition (describing an abstract, simple, behaviour) by a net (describing a concrete, detailed, behaviour). Approaches like, for instance, [5,6], propose very general solutions for low-level (box-like) nets. However, for a long time, a similar flexibility had not been obtained for high-level nets. Indeed, the approaches introducing the concept of hierarchy like for instance [46] or [12] in coloured nets, [25] in algebraic nets, or [45] in M-nets, work only in very restricted cases. These results were generalised in [37] which removed most of the previous restrictions.

The *substitution* of a hierarchical transition t (labelled by X) in an M-net N by an M-net N' means that t is replaced in N by a copy of N'; this is denoted by $N[X \leftarrow N']$. In order to do this, the transition t is removed and its pre-places are combined with the entry places of N' while its post-places are combined with the exit places of N'. In other words, ${}^\bullet t$ is identified with

N'^{e} while $t^\bullet$ is identified with N'^{x}. This is performed in such a way that any execution of t, for any mode, in N is replaced in $N[X \leftarrow N']$ by an execution of N' from its entry marking to its exit marking. We wish also that all the transitions labelled X can be replaced *simultaneously*, which allows the net N' to be executed (possibly concurrently) for each replaced transition.

The basic mechanism used here (see also [36,37,78]) in order to guarantee the consistency with respect to the unfolding of the M-net refinement and that of boxes, is that of *labelled trees*. It is a generalisation of the Cartesian product used by other authors in order to define combinations of places.

In the case of boxes, the labelled trees are used in order to generate the places combining the pre-places and the post-places of refined transitions with the entry and exit places of the refining nets. The main difference in the case of M-nets is that only one place is generated during each combination, but its type is a set of labelled trees. These have at most two levels, one level corresponds to what comes from the refined net N and the second level corresponds to what comes from the refining net N'. Labelled trees like that shown in figure 3 are used in order to generate new structured values belonging to the types of interface places in the refined net (*i.e.*, the places which are connected to one or more hierarchical transitions).

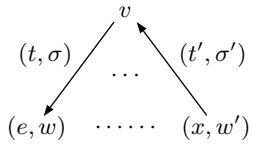


Fig. 3 Illustration of the scheme of the value trees of the interface places.

This M-net refinement allows to overcome most of the restrictions present in previous approaches. In particular, arbitrary types are allowed for the places around hierarchical transitions and for the entry/exit places of refining nets. Moreover, arbitrary arc annotations and side-loops on hierarchical transitions are also admitted.

Let $N \stackrel{\text{df}}{=} (S, T, \iota)$ be an M-net, $X \in \mathbb{X}$ be a hierarchical symbol, $(N_i)_{i\in I}$ be a family of M-nets and $(X_i)_{i\in I}$ be a family of hierarchical symbols. We use the following notations:

- $\mathbb{X}_N \stackrel{\text{df}}{=} \{X_i \mid i \in I\}$ is the set of hierarchical symbols of N;
- $T^X \stackrel{\text{df}}{=} \{t \in T \mid \lambda(t) = X\}$ is the set of hierarchical transitions labelled by X;
- $T^{\mathbb{X}_N} \stackrel{\text{df}}{=} \bigcup_{i\in I} T^{X_i}$ is the set of hierarchical transitions of N;
- for each set $R \subseteq S \cup T$, we consider the sets $R^\bullet \stackrel{\text{df}}{=} \cup_{x\in R} x^\bullet$, ${}^\bullet R \stackrel{\text{df}}{=} \cup_{x\in R} {}^\bullet x$ of successors and predecessors of the nodes in R;
- $\widetilde{N} \stackrel{\text{df}}{=} N[X_i \leftarrow N_i \mid i \in I] \stackrel{\text{df}}{=} (\widetilde{S}, \widetilde{T}, \widetilde{\iota})$ is the net N in which each transition labelled by X_i is replaced by a copy of N_i, for all $i \in I$. All these substitutions are simultaneous. We call N the *refined* net, the N_i's the *refining* nets and $\widetilde{N}$ the *resulting* net.

The places of the resulting net $\widetilde{N}$ belong to one of two possible categories:

- either they are the places of the refined net N (some of them are the *interface* places if they are connected to one or more hierarchical transitions);
- or they are copies of internal places of the refining nets N_i.

This way, each place s of N is also a place of the refined net, with the same status. The only difference is in its type which is then the set of labelled trees constructed from the values in the type of s and from the values coming from the types of entry/exit places of the refining nets N_i. The new type of s, $\tilde{\alpha}(s)$, is the set of all labelled trees (up to isomorphism) of the form represented in figure 3 where:

- the root is labelled by a value $v \in \alpha(s)$;
- the edges are labelled by pairs (t, σ) where t is a hierarchical transition of N, adjacent to s and σ is a mode for t;
- the direction of the edges corresponds to the direction of the arc between t and s in N (down if there is an arc (s, t), up otherwise);
- the leaves are labelled by pairs (s_i, w_j) where s_i is an entry or exit place of the refining net N_i and w_j is a value in the type of s_i.

For example, the new values of an interface place connected as shown in figure 4 in the refined net, are of the form depicted on the right. We assume that the transition σ_1 is a mode for t_1 that is refined by an M-net N_1 having a unique entry place e with the value $\bullet$ in the type of e. Respectively, σ_2 is a mode for t_2 that is refined by an M-net N_2 having a unique exit place x with the value $\bullet$ in the type of x. The root of the value is labelled by a value from the original type of s and the leaves by the values of the places e and x.

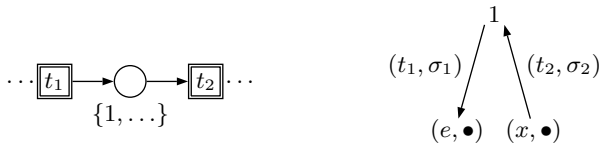


Fig. 4 An interface place in a refined net and an example of a value tree in the type of this place in the refined net.

More precisely, as illustrated in figure 5, for all $i \in I$, for all $t \in s^\bullet$ labelled by X_i and for all modes σ of t such that v belongs to $\sigma(\iota(s, t))$, there is in the tree an edge labelled (t, σ) going down to a leaf labelled by (e, w) where $e \in {N_i}^{\mathsf{e}}$ and $w \in \alpha(e)$. Symmetrically, for all $i \in I$, for all $t' \in {}^\bullet s$ labelled by X_i and for all modes σ' of t' such that v belongs to $\sigma'(\iota(s, t'))$, there is an edge labelled (t', σ') which goes up from a leaf labelled by (x, w') where $x \in {N_i}^{\mathsf{x}}$ and $w' \in \alpha(x)$. Notice also, that for the places s in N which are not connected to any hierarchical transition to be refined (*i.e.*, if $T^{\mathbb{X}_N} \cap ({}^\bullet s \cup s^\bullet) = \emptyset$) the trees are reduced to their roots and we have $\tilde{\alpha}(s) \stackrel{\text{df}}{=} \alpha(s)$.

The internal places of the refining nets N_i lead to the set S^i of internal places of the resulting net, with $S^i \stackrel{\text{df}}{=} \{t.s_i \mid t \in T^{X_i} \wedge s_i \in {N_i}^{\mathsf{i}}\}$. The status of these places is still internal and their types are composed of values of the

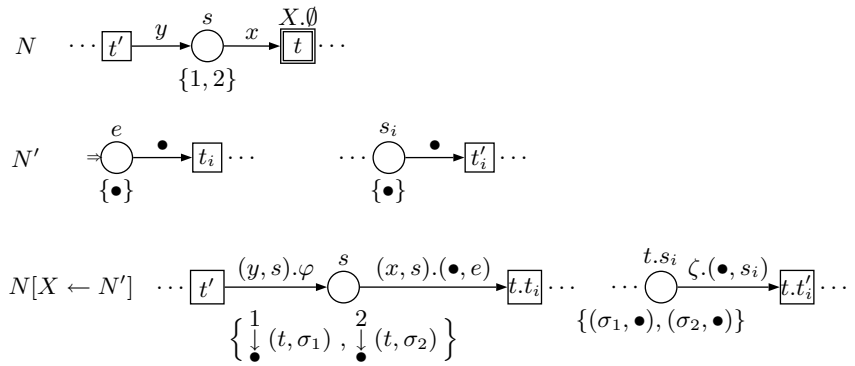


Fig. 5 Place types and arc inscriptions, assuming that σ_1 and σ_2 are the only modes for t in N.

form $\sigma.v$ (trees reduced to their roots) where σ is a mode for t and v is a value from $\alpha(s_i)$.

The transitions of the resulting net $\widetilde{N}$ may be of two kinds:

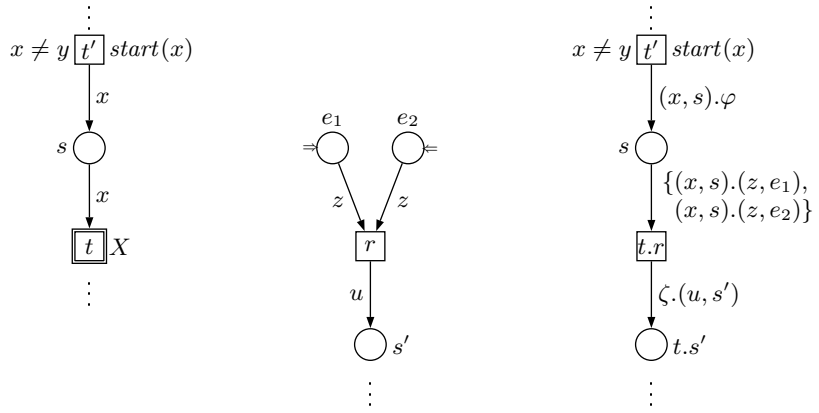
- the transitions $t \in T \setminus T^{\mathbb{X}_N}$ from N which are not substituted and whose inscriptions are unchanged, *i.e.*, $\tilde{\iota}(t) \stackrel{\text{df}}{=} \iota(t)$;
- the transitions copied from each N_i, belonging to $T^i \stackrel{\text{df}}{=} \{t.t_i \mid t \in T^{X_i} \wedge t_i \in T_i\}$.

The arcs may be of three different kinds:

- those connecting places coming from N to transitions which are not substituted in N;
- those connecting places coming from N with transitions coming from N_i;
- and finally, those connecting internal places and transitions both coming from N_i.

These three kinds of arcs are illustrated on the right hand side in figure 6:

- the transition t' was untouched by the refinement, but the place s has now a type composed of labelled trees. The firing of t' has to produce in s one instance of each tree belonging to the type of s. Each root corresponds to one value produced in s by the firing of t' in N. The notation $(x, s).\varphi$ corresponds to this case and the distinguished symbol φ means that the matching leaves of the tree may be arbitrarily labelled;
- the transition $t.r$ has to consume from s one instance of each tree whose root is labelled by the value corresponding to the binding of x during the firing of t in N. Each tree has a leaf labelled by the value consumed (in e_1 or e_2) via z during the firing of r in N'. The notations $(x, s).(z, e_1)$ and $(x, s).(z, e_2)$ correspond to this case;
- finally, the transition $t.r$ has to produce in $t.s'$ one instance of each value $\sigma.v$ where v is a value produced via u during the firing of r in N' and σ is a mode for t in N. The notation $\zeta.(u, s')$ corresponds to this case and the



a fragment of N a fragment of N' a fragment of $N[X \leftarrow N']$

Fig. 6 Illustration of different kinds of structured annotations.

distinguished symbol ζ denotes that the first part of the token matches any value allowed by the place types.

The formal definition of the refinement, of the evaluation of arcs and also of the transition rule of a refined net follows exactly this intuition. It is presented in the next section in a more general context. It guarantees a number of properties (*cf.* [37]), for instance:

Theorem 1 *Let N, N_i, N'_j and N''_k (for $i \in I$, $j \in J$, and $k \in K$) be M-nets. The following properties hold.*

- Domain preservation: *$N[X_i \leftarrow N_i \mid i \in I]$ is an M-net.*
- Consistency *with the low-level model via the unfolding: up to isomorphism,*

$$\mathsf{unf}(N[X_i \leftarrow N_i \mid i \in I]) = \mathsf{unf}(N)[X_i \leftarrow \mathsf{unf}(N_i) \mid i \in I].$$

- Expansion law: *if $J \subseteq I$, $I \cap K = \emptyset$ and $\{Y_k \mid k \in K\} \cap \{X_i \mid i \in I\} = \emptyset$, then, up to isomorphism:*

$$\begin{aligned} &N[X_i \leftarrow N_i \mid i \in I][X_j \leftarrow N'_j, Y_k \leftarrow N''_k \mid j \in J, k \in K] \\ &\quad = N[X_i \leftarrow N_i[X_j \leftarrow N'_j, Y_h \leftarrow N''_h \mid j \in J, h \in K], \\ &\qquad\qquad Y_k \leftarrow N''_k \mid i \in I, k \in K]. \end{aligned}$$

The expansion law expresses the commutativity of refinements and is a generalisation of the following property for M-nets N, N', N'' and N''':

$$\begin{aligned} &N[X \leftarrow N'][X \leftarrow N'', Y \leftarrow N'''] \\ &\quad = N[X \leftarrow N'[X \leftarrow N'', Y \leftarrow N'''], Y \leftarrow N''']. \end{aligned}$$

The refinement is very useful in the process of modelling a system in a structured way. It is also essential for a uniform synthesis of the control-flow operators in the algebra of M-nets as we will see in section 4.

3 Extensions of M-nets

3.1 Parameterised substitution

In [39,53,64] the syntax of the parallel specification language B(PN)^2 [10] having a Petri net semantics has been extended by procedures. The mechanism to manage the different instances of a procedure used in [39,53,64] requires the usage of "net parameters" in the refinement in order to be able to distinguish between different active instances. Indeed, if in a sequential environment the instances of a procedure are always totally ordered and can be managed by a stack, it is not the case in a parallel environment, where several active instances of a procedure may be generated by concurrent calls and may evolve concurrently. These instances are distinguished by different modes of the same hierarchical transition, as for instance, the modes $\sigma_1 = (x \mapsto 1)$ and $\sigma_2 = (x \mapsto 2)$ of the transition t in figure 7. The procedure body is modelled by the parameterised M-net $N'(\mathsf{id})$, where the parameter id can take values in $\{1, 2\}$ and is used to identify the instances. The result of the refinement is presented on the right of figure 7.

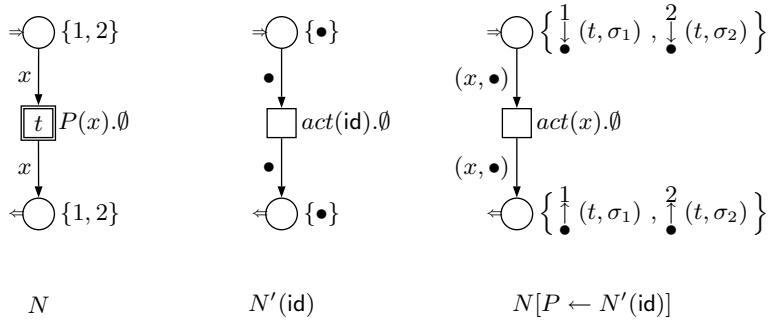


Fig. 7 Parameterised refinement: management of the instances of a procedure P whose body is given by N'.

The parameterised refinement [37] introduces a mechanism that allows to identify the variables from the environment of a hierarchical transition (for instance, x in the net N in figure 7) with the corresponding parameters (such as id in $N'(\mathsf{id})$). Moreover, the solution proposed in [37] is not limited to this particular case but introduces a general scheme which works in all the circumstances.

To start with, we introduce *parameterised M-nets* [37] in which we will use the following extensions:

- Par denotes the set of net parameters, disjoint from Val and Var. Each parameter $\psi \in Par$ is assumed to have a non-empty type denoted $set(\psi)$;
- Ψ is a *list of parameters* of the form $\psi_1, \ldots, \psi_n$, where $\forall i \in \{1, \ldots, n\}$: $\psi_i \in Par$ (with $i \neq j \Rightarrow \psi_i \neq \psi_j$). The set of parameters $\{\psi_1, \ldots, \psi_n\}$ associated to Ψ is denoted by $Par(\Psi)$;
- a *substitution* $\kappa : Par \to Var \cup Par \cup Val$ is denoted by $(\psi_1 \mapsto \nu_1, \ldots, \psi_n \mapsto \nu_n)$; it is elementary if $\nu_i \in set(\psi_i)$, for all $i \in \{1, \ldots, n\}$;

- each action $act(\nu_1, \ldots, \nu_{\mathsf{ar}(act)})$ has arguments which are values, variables or parameters, *i.e.*, for all $j \in \{1, \ldots, \mathsf{ar}(act)\}$, $\nu_j \in Val \cup Var \cup Par(\Psi)$;
- each hierarchical symbol $X \in \mathbb{X}$ has an arity $\mathsf{ar}(X)$ representing the number of its arguments. The term $X(\nu_1, \ldots, \nu_{\mathsf{ar}(X)})$ represents a *hierarchical parameterised action* where all the $\nu_j \in Val \cup Var \cup Par(\Psi)$, for $j \in \{1, \ldots, \mathsf{ar}(X)\}$, are the arguments of X.

Definition 7 A parametrised M-net N is a quadruple (S, T, ι, Ψ) (often denoted by $N(\Psi)$), where S, T and ι are as in definition 5 and satisfy the following conditions:

- the net has an *arity* which is the size of the list $Par(\Psi)$;
- for each transition $t \in T$, $\lambda(t)$ is either a finite multiset of parameterised actions or a hierarchical parameterised action. The parameters which appear in $\lambda(t)$ belong to $Par(\Psi)$;
- the parameters never appear in guards nor in structured annotations of arcs.[2]

For a given substitution, the dynamic behaviour of such a net is similar to that without the parameters. Also, the semantics of a parameterised M-net [37] does not depend on the names of the parameters nor on the variables in the environment of each transition; they may be consistently renamed at will.

The unfolding of a parameterised M-net is a family of low-level nets (rather than one low-level net as in the case without the parameters) $\mathsf{unf}(N)$: $\kappa \rightarrow \mathsf{unf}(N, \kappa)$ where κ is an elementary substitution such that each choice of values of parameters specified by κ leads to a (generally different) low-level net. As for M-nets without parameters, we have the consistency of the behaviour of a parameterised M-net with that of each low-level net corresponding to it [37].

Proposition 3 *A transition t can fire in a parameterised M-net N at a marking M with the binding σ and leads to a marking M' iff t_σ can fire in all $\mathsf{unf}(N, \kappa)$ at $\mathsf{unf}(M)$ and leads to the marking $\mathsf{unf}(M')$. The parameterised or hierarchical actions produced during this firing are the same in the high-level net N and in each low-level one $\mathsf{unf}(N, \kappa)$.*

Using the concepts and the notations introduced in the previous section (S^i, T^i, $\widetilde{\alpha}(s)$, ζ, φ, etc.), the parameterised refinement is defined as follows:

Definition 8 Let $N = (S, T, \iota, \Psi)$ and $N_i = (S_i, T_i, \iota_i, \Psi_i)$, for $i \in I$, be M-nets. The refinement $N[X_i \leftarrow N_i(\psi_{i,1}, \ldots, \psi_{i,ar(X_i)}) \mid i \in I]$ (with $i \neq j \Rightarrow X_i \neq X_j$) is defined as the M-net $\widetilde{N} \stackrel{\text{df}}{=} (\widetilde{S}, \widetilde{T}, \widetilde{\iota}, \Psi)$, with the same[3] set of parameters as N, such that:

- $\widetilde{S} \stackrel{\text{df}}{=} \big(\bigcup_{i \in I} S^i\big) \cup S$,

[2] This last restriction were removed in [27].

[3] The N_i's do not introduce new parameters in the refined net, as their parameters are all instantiated by variables, values or parameters of N.

- $\widetilde{T} \stackrel{\text{df}}{=} \big(\bigcup_{i\in I} T^i\big) \cup (T \setminus T^{X_I})$,
- $\tilde{\iota}(\tilde{s}) \stackrel{\text{df}}{=} \begin{cases} \lambda(\tilde{s}).\tilde{\alpha}(\tilde{s}) \text{ if } \tilde{s} \in S, \\ \mathsf{i}.\tilde{\alpha}(\tilde{s}) \quad \text{if } \tilde{s} = t.s_i \in S^i, \end{cases}$
- $\tilde{\iota}(\tilde{t}) \stackrel{\text{df}}{=} \begin{cases} \iota(t) \quad \text{if } \tilde{t} = t \in (T \setminus T^{\mathbb{X}_N}) \\ \varrho_i(\lambda_i(t_i)).\gamma(t) \cup \gamma_i(t_i) \\ \qquad \text{if } \tilde{t} = t.t_i \in T^i, \lambda(t) = X_i(\nu_{i,1}, \ldots, \nu_{i,ar(X_i)}) \text{ and if } \varrho_i \\ \qquad \text{is a substitution } (\psi_{i,1} \mapsto \nu_{i,1}, \ldots, \psi_{i,ar(X_i)} \mapsto \nu_{i,ar(X_i)}), \end{cases}$
- $\tilde{\iota}(\tilde{s},\tilde{t}) \stackrel{\text{df}}{=} \begin{cases} \sum\limits_{a\in\iota(s,t)} \iota(s,t)(a) * \{(a,s).\varphi\} \\ \qquad \text{if } \tilde{s} = s \in S \text{ and } \tilde{t} = t \in (T \setminus T^{\mathbb{X}_N}) \\ \sum\limits_{a\in\iota(s,t)} \sum\limits_{e_i\in N_i{}^{\mathrm{e}}} \sum\limits_{b\in\iota_i(e_i,t_i)} \iota(s,t)(a) * \iota_i(e_i,t_i)(b) * \{(a,s).(b,e_i)\} \\ \qquad \text{if } \tilde{s} = s \in S \text{ and } \tilde{t} = t.t_i \in T^i \text{ for an } i \in I \\ \sum\limits_{b\in\iota_i(s_i,t_i)} \iota_i(s_i,t_i)(b) * \{\zeta.(b,s_i)\} \\ \qquad \text{if } \tilde{s} = t.s_i \in S^i \text{ and } \tilde{t} = t.t_i \in T^i \text{ for an } i \in I \\ \emptyset \text{ otherwise,} \end{cases}$

The inscriptions $\tilde{\iota}(\tilde{t},\tilde{s})$ of the arcs in $\widetilde{T} \times \widetilde{S}$ are defined analogously.

The main difference between the parameterised refinement and the non parameterised one concerns the label of a transition in the refined net. It contains an application of a substitution ϱ_i which defines the correspondence between the arguments $(\nu_{i,1}, \ldots, \nu_{i,ar(X_i)})$ of the hierarchical transition of N and the parameters $(\psi_{i,1}, \ldots, \psi_{i,ar(X_i)})$ of N_i. The rest of the definition is exactly the same as explained for the non parametrised case and illustrated in figure 5: in particular, the values of the types of the places (labelled trees) and the arc annotations are constructed exactly in the same way.

Theorem 2 *The properties stated for the non parameterised refinement hold in the parameterised context, in particular:*

- *domain preservation;*
- *consistency with the low-level domain via the unfolding;*[4]
- *expansion law.*

3.2 Recursion

The recursion $\mu\{X_i.N_i \mid i \in I\}N$, means "replace in N all the X_i's by N_i's, and restart on the result *ad infinitum*". This operation has been defined first for low-level nets [4,5], where the recursion $\mu\{X_i.N_i \mid i \in I\}N$ was interpreted as a kind of "limit" of the refinements $N[X_i \leftarrow N_i][X_i \leftarrow N_i]\cdots$ or $N[X_i \leftarrow N_i[X_i \leftarrow N_i[\cdots]]]$. This concept is formalised in [37] in the case of M-nets, including the possibility of parametrised recursion. The semantics of the recursion is given by expressing explicitly the form of the result instead of as a limit like in [4,5].

[4] Remember that the unfolding of a parametrised M-net gives a family of low-level nets.

There are several similarities between the refinement and the recursion, but the latter is more complex. The transitions are here finite sequences $t.t_1 \dots t_j$ of arbitrary length (but satisfying some constraints), instead of sequences of length 1 or 2 like t or $t.t_i$ as used in definition 8. Moreover, the types of the places contain values which are labelled trees of arbitrary height, possibly infinite, instead of the trees of height 1 or 2 as in definition 8.

Intuitively, a sequence of transitions $t.t_1 \dots t_{j-1}.t_j$ expresses a sequence of refinements which begins by the hierarchical transition t in N, continues with the hierarchical transitions t_1, ..., t_{j-1} in M-nets N_{i_1}, ..., $N_{i_{j-1}}$ and finishes with a transition t_j (hierarchical with a label $X_j \notin \mathbb{X}_N$ or non-hierarchical) in N_{i_j}. Each sequence of that form leads to a transition in the recursive M-net $\widetilde{N}$.

As for the refinement, the places $\tilde{s}$ may be of the form $\pi.s$ and we distinguish two cases:

- the places coming from N, for which π is empty;
- internal places copied from the nets N_i for which $\pi \stackrel{\text{df}}{=} t_1.t_2 \cdots t_{n-1}$ is not empty.

The new values of these places have the general shape shown in figure 8. The root is labelled by $\sigma_1.\sigma_2 \dots \sigma_{n-1}.u$ where u is a value from $\alpha(s)$ if π is empty or from $\alpha_i(s)$ if $\pi = t_1.t_2 \cdots t_{n-1}$ and t_{n-1} is labelled by X_i. Each σ_i is a mode for the hierarchical transition t_i corresponding to a given depth of the recursion. The other labels are analogous to that of refinement, their exact description is given in [37].

As for the refinement, the definition of the recursion is complex, but it has the advantage to work in any circumstance. In particular, it allows hierarchical transitions to be connected to entry or exit places of the nets N_i (or both), which was not the case with previous attempts.

We have the following properties [37]:

Theorem 3

1. Domain preservation: *if N and all the N_i's, for $i \in I$, are M-nets, then $\mu\{X_i.N_i \mid i \in I\}N$ is also an M-net.*
2. Consistency *of the recursion with the low-level nets via the unfolding: for M-nets N and N_i, for $i \in I$, and for all choices κ of values for the*

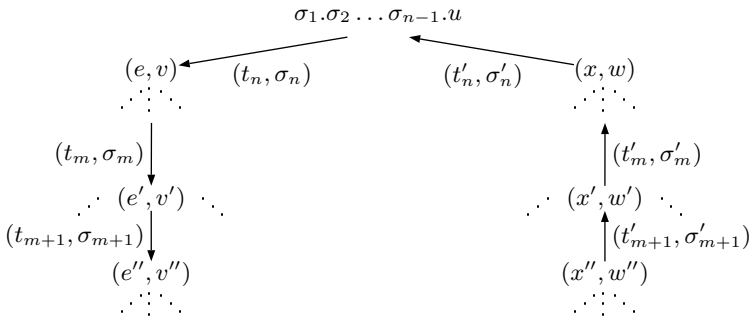


Fig. 8 The general shape of value trees in a recursive M-net.

parameters, we have, up to isomorphism:

$$\mathsf{unf}(\mu\{X_i.N_i \,|\, i{\in}I\}N, \kappa) = \\ \mu\{X_i(\kappa_i).\mathsf{unf}(N_i, \kappa_i) \,|\, i{\in}I, \kappa_i \textit{ is elementary }\}\mathsf{unf}(N, \kappa).$$

3. Substitution property: *for M-nets N and N_i ($i \in I$), we have, up to isomorphism:*

$$\mu\{X_i.N_i \mid i \in I\}N = N[X_i \leftarrow \mu\{X_j.N_j \mid j \in I\}N_i \mid i \in I].$$

In particular, the third property generalises the standard fix point equation $\mu\{X.N\}N = N[X \leftarrow \mu\{X.N\}N]$.

This approach of the recursion produces potentially infinite nets, a finite representation of which was investigated in [26].

3.3 Buffered communication

An extension introduced in [54,71] allows M-nets to support buffered communication. The key of this extension is a new kind of places, called *buffer places*, which are automatically merged when several M-nets are combined, *i.e.*, during the refinement.[5] Consider for instance two M-nets N_1 and N_2 being refined in a net N; if a transition t_1 from N_1 produces a token in a buffer place and if a transition t_2 from N_2 consumes a token from its own buffer place, after the refinement, the place merging ensures that these transitions use the same buffer place, making possible the communication between t_1 and t_2, even if they are not originated from the same M-net.

More precisely, we consider a set $\mathbb{B}$ of *buffer symbols* such that each $b \in \mathbb{B}$ is associated with a type $\mathsf{type}(b) \subseteq \mathit{Val}$. Each buffer place s is labelled by a $b \in \mathbb{B}$ and we have $\iota(s) \stackrel{\text{df}}{=} b.\mathsf{type}(b)$. Then, the refinement and the recursion (parameterised or not) are redefined as follows: (1) we use the original definition considering the buffer places as internal ones; (2) for all $b \in \mathbb{B}$, we merge the buffer places sharing the same label $b \in \mathbb{B}$, their markings being added.

The extension with buffered communication turns out to be very useful and generally allows to express complex systems using smaller and simpler parts. Many applications of M-nets rely on the buffered communication extension. In particular, we will see later on how the semantics of a parallel specification language can be simplified, thanks to buffer places, by using just combinations of basic nets having only one transition.

4 M-net algebra

The model by which M-nets are inspired, the Petri Box Calculus (PBC) [4,5] and its more recent versions [6,34], is a process algebra provided with a low-level Petri net semantics. It has a syntactical domain of *box expressions* and

[5] As shown later on, various M-nets composition operations can be defined. Since they are based on the refinement, they do not require any change to support buffered communication.

a semantical domain of *boxes*. Each syntactical operator has a corresponding operation (with the same name) in the net domain. The semantics can be obtained by associating a box to a box expression and then through the standard definition of concurrent evolutions (processes) [3,44] in order to obtain, for instance, a partial order semantics. The semantics can also be obtained directly from a box expression following the rules of the structured operational semantics following the approach from [6].

4.1 M-expressions

Even though the model of M-nets has been developed first in the semantical domain, it also has a syntactical domain, called the algebra of *M-expressions* [58].

$$E ::= E_N \mid E \fatsemi E \mid E \,\Box\, E \mid E \parallel E \mid [E * E * E] \mid E[X \leftarrow E] \mid E[f]$$
$$\mid E \,\mathsf{tie}\, b \mid E \,\mathsf{sy}\, a \mid E \,\mathsf{rs}\, a \mid E \,\mathsf{sc}\, a$$

The syntax of the algebra of M-expressions E is presented above, where $a \in \mathbb{A}$, $b \in \mathbb{B}$, and f is a renaming function on actions, hierarchical and buffer symbols. The operators comprise: *sequence* $E_1 \fatsemi E_2$ (the execution of E_1 is followed by that of E_2); *choice* $E_1 \,\Box\, E_2$ (either E_1 or E_2 can be executed); *parallel composition* $E_1 \| E_2$ (E_1 and E_2 can be executed concurrently); *iteration* $[E_1 * E_2 * E_3]$ (E_1 is executed once (initialisation), E_2 can be executed an arbitrary number of times (loop), and then is followed by E_3 (termination)); *refinement* $E_1[X \leftarrow E_2]$ (events labelled by hierarchical symbol X in E_1 are replaced by E_2); *renaming* $E_1[f]$ (elements of E_1 are consistently renamed by f); *buffer restriction* $E_1 \,\mathsf{tie}\, b$ (the buffer place b and the related communications become private to E_1); *synchronisation* $E_1 \,\mathsf{sy}\, a$ (all multi-way synchronisations involving the actions a or $\widehat{a}$ are made possible); *restriction* $E_1 \,\mathsf{rs}\, a$ (events involving a or $\widehat{a}$ may no longer be executed) and *scoping* $E_1 \,\mathsf{sc}\, a$ (the synchonisation followed by the restriction w.r.t. a).

We assume that any M-net N has an associated constant expression E_N in the M-expression domain. Often, this N is a very simple M-net composed of only one transition, one entry and one exit place, and possibly connected buffer places, as for instance in [18,53].

The correspondence between the syntactical domain of M-expressions and that of M-nets is given compositionally through the semantical function mnet. For the base case we define $\mathsf{mnet}(E_N) \stackrel{\text{df}}{=} N$, and the semantics of the operators is as follows, where bin stands for any binary operator in $\{\fatsemi, \|, \Box\}$ and una stands for any unary operator in $\{\mathsf{sy}\, a, \mathsf{rs}\, a, \mathsf{sc}\, a, \mathsf{tie}\, b, [f]\}$, the net operations appearing on the right being defined in the next sections:

$$\begin{aligned}
\mathsf{mnet}(E_1 \,\mathsf{bin}\, E_2) &\stackrel{\text{df}}{=} \mathsf{mnet}(E_1) \,\mathsf{bin}\, \mathsf{mnet}(E_2) \\
\mathsf{mnet}(E_1[X \leftarrow E_2]) &\stackrel{\text{df}}{=} \mathsf{mnet}(E_1)[X \leftarrow \mathsf{mnet}(E_2)] \\
\mathsf{mnet}(E_1 \,\mathsf{una}) &\stackrel{\text{df}}{=} \mathsf{mnet}(E_1) \,\mathsf{una} \\
\mathsf{mnet}([E_1 * E_2 * E_3]) &\stackrel{\text{df}}{=} [\mathsf{mnet}(E_1) * \mathsf{mnet}(E_2) * \mathsf{mnet}(E_3)]
\end{aligned}$$

A fragment of the algebra of M-expressions has also been provided with a structured operational semantics [18,58]. It was shown that, for a restricted class of M-expressions (used for the semantics of the parallel specification language B(PN)^2), the semantics obtained using the operational rules from the initial state of an M-expression was equivalent to the step semantics of the corresponding M-net.

4.2 Control flow operations

Originally, at the level of boxes as well as at that of M-nets, the control flow operations, such as sequence, parallel composition, choice or iteration, were defined through auxiliary operations of net composition, see for instance [4, 8]. The refinement allows to define these operations in a uniform way. One uses for that *operator nets* like those shown in figure 9.

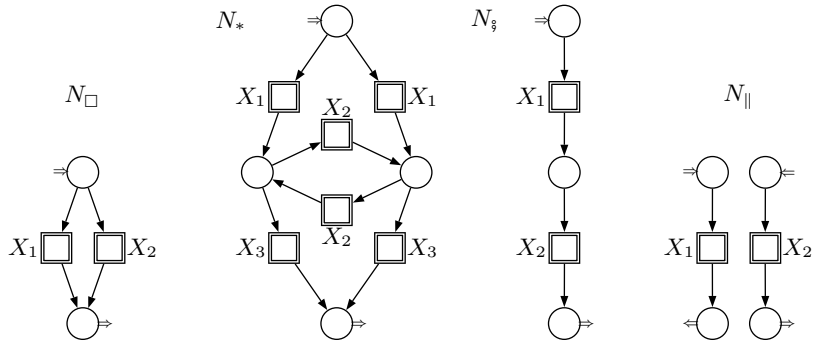


Fig. 9 The operator nets for the definition of the M-net control flow operations.

Definition 9 For arbitrary M-nets N_1, N_2 and N_3, we define:

$N_1 \Box N_2$	$\stackrel{\text{df}}{=} N_\Box[X_1 \leftarrow N_1, X_2 \leftarrow N_2]$	choice
$[N_1 * N_2 * N_3]$	$\stackrel{\text{df}}{=} N_*[X_1 \leftarrow N_1, X_2 \leftarrow N_2, X_3 \leftarrow N_3]$	iteration
$N_1 \fatsemi N_2$	$\stackrel{\text{df}}{=} N_\fatsemi[X_1 \leftarrow N_1, X_2 \leftarrow N_2]$	sequence
$N_1 \parallel N_2$	$\stackrel{\text{df}}{=} N_\parallel[X_1 \leftarrow N_1, X_2 \leftarrow N_2]$	parallel

4.3 Synchronous communications

The formal definitions of the operations presented in this section may be found in [8]; we give here an informal presentation.

The synchronisation $N\,\mathsf{sy}\,a$ adds new transitions to the net N. It may be seen as a CCS-like synchronisation extended to multi-sets of actions with arbitrary arity. Intuitively, one can consider the operation of synchronisation on an M-net as a result of an application, possibly repeatedly, of a certain number of partial synchronisations between pairs of transitions having in

their labels conjugate actions a and $\widehat{a}$. The repetition of such binary synchronisations with respect to a, for all pairs of transitions which contain conjugate actions, and until no new transition can be added, results in the synchronisation of N with respect to a, $N \,\mathsf{sy}\, a$. For instance, the transitions t_1 and t_2 in the fragment of the M-net N in figure 10 may synchronise together w.r.t. *start*. The corresponding basic synchronisation generates a new transition t via a renaming of the variables in the surrounding of t_1 and t_2 and a unification of the arguments of the actions $start(x)$ and $\widehat{start}(z)$, *e.g.*, with $\{x \mapsto z\}$. The guard of this new transition is the disjunction of the guards of t_1 and t_2, and the multiset of synchronous actions is the sum of those of t_1 and t_2 minus $start(x)$ and $\widehat{start}(z)$, both substituted by the unification discovered for the conjugated actions. The result of this binary synchronisation is shown on the right in figure 10.

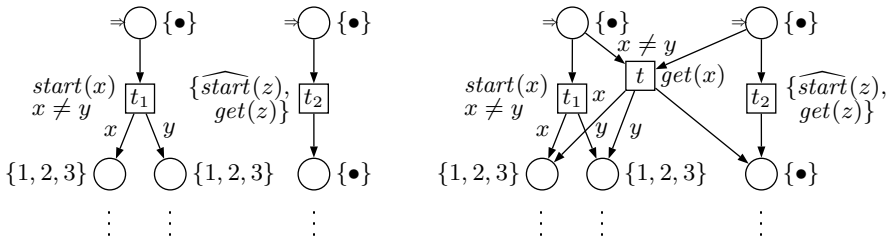


Fig. 10 A fragment of an M-net N and its synchronisation w.r.t. *start* (where z has been substituted with x).

The restriction of an M-net with respect to a removes from the net all the transitions which contain in their labels actions involving a or $\widehat{a}$. For instance, the result of $(N \,\mathsf{sy}\, start) \,\mathsf{rs}\, start$ removes from the net represented on the right in figure 10 the transitions t_1 and t_2 (but preserves t); it corresponds also to the scoping $N \,\mathsf{sc}\, start$.

The algebraic properties of the restriction are stated by the theorem 4 (equality of nets being considered up to isomorphism). The same properties are obtained in M-nets for the synchronisation (equality of nets being considered up to isomorphism and up to the equivalence which identifies duplicated transitions [8,35]). This theorem is a corollary of a similar one for a version of M-nets using algebraic data types [51].

Theorem 4 *For an M-net N and $a_1, a_2 \in \mathbb{A}$, we have:*

- *commutativity of* sy*:* $(N \,\mathsf{sy}\, a_1) \,\mathsf{sy}\, a_2 = (N \,\mathsf{sy}\, a_2) \,\mathsf{sy}\, a_1$
- *idempotence of* sy*:* $(N \,\mathsf{sy}\, a_1) \,\mathsf{sy}\, a_1 = N \,\mathsf{sy}\, a_1$
- *commutativity of* rs*:* $(N \,\mathsf{rs}\, a_1) \,\mathsf{rs}\, a_2 = (N \,\mathsf{rs}\, a_2) \,\mathsf{rs}\, a_1$
- *idempotence of* rs*:* $(N \,\mathsf{rs}\, a_1) \,\mathsf{rs}\, a_1 = N \,\mathsf{rs}\, a_1$

Thanks to the commutativity of sy and rs, we shall use the extended notations $N \,\mathsf{sy}\, \mathcal{A}$, $N \,\mathsf{rs}\, \mathcal{A}$ and $N \,\mathsf{sc}\, \mathcal{A}$, for $\mathcal{A} \subseteq \mathbb{A}$.

4.4 Buffered communication

The buffer restriction of an M-net N w.r.t. $b \in \mathbb{B}$ is the M-net $N \,\mathsf{tie}\, b$ which is obtained from N by replacing b by i in the place s_b having this status. This is to say that the buffer place b is made internal. This is illustrated in figure 11. If N has no such place s_b, applying $\mathsf{tie}\, b$ leaves the net untouched. We then obtain the following results.

Theorem 5 *Let N be an M-net, $a \in \mathbb{A}$ and $b_1, b_2 \in \mathbb{B}$. Then:*

- *commutativity of* tie*:* $(N \,\mathsf{tie}\, b_1) \,\mathsf{tie}\, b_2 = (N \,\mathsf{tie}\, b_2) \,\mathsf{tie}\, b_1$
- *idempotence of* tie*:* $(N \,\mathsf{tie}\, b_1) \,\mathsf{tie}\, b_1 = N \,\mathsf{tie}\, b_1$
- *commutativity of* tie *and* sy*:* $(N \,\mathsf{tie}\, b_1) \,\mathsf{sy}\, a = (N \,\mathsf{sy}\, a) \,\mathsf{tie}\, b_1$

The last result, which holds up to isomorphism, shows the orthogonality between buffered and synchronous communication schemes. Moreover, thanks to the commutativity of tie, we shall use the extended notation $N \,\mathsf{tie}\, \mathcal{B}$, where $\mathcal{B} \subseteq \mathbb{B}$.

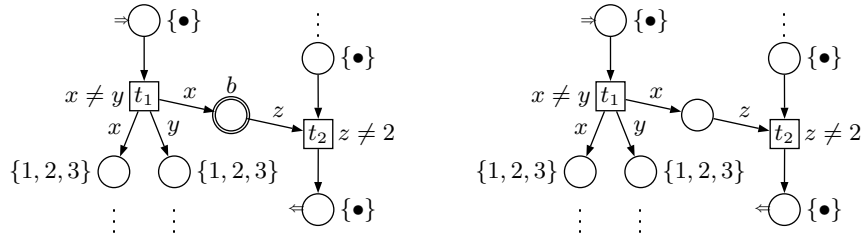


Fig. 11 A fragment of an M-net N with a buffer place b (depicted with a double line) and its version with b removed by the buffer restriction $N \,\mathsf{tie}\, b$.

4.5 Relabelling

The relabelling $N[f]$ of an M-net N is like N where the action symbols, the hierarchical symbols and the buffer names are consistently renamed using the function f which must be a bijection on $\mathbb{A} \cup \mathbb{X} \cup \mathbb{B}$ such that $f(\mathbb{A}) = \mathbb{A}$, $f(\mathbb{X}) = \mathbb{X}$ and $f(\mathbb{B}) = \mathbb{B}$.

4.6 Consistency of the operations

The consistency of the operations of the M-net algebra with their low-level counterparts is crucial. It means that the unfolding of an M-net N obtained from a composition of M-nets N_i, $i \in I$, using operations of the M-net algebra, is equivalent to the low-level net obtained by composing the unfoldings of the N_i's using the corresponding operations in the low-level domain. It allows in particular to model a concurrent system in the high-level framework (where it is easier) and to obtain automatically (thanks to the unfolding) its low-level equivalent where properties may be efficiently verified.

Theorem 6 *The operations of the M-net algebra are consistent with the low-level ones through the unfolding.*

For the control-flow operations, this is a corollary of the consistency of the refinement. For the synchronisation and the restriction, the consistency with the low-level model is non-trivial to show, see [8]. It was obtained up to an equivalence which identifies duplicated transitions (*i.e.*, having the same label, the same connectivity and same annotations, up to consistent renaming of variables). Consistency of the buffer restriction was shown in [54, 71].

The consistency of the M-nets operations was also studied in a more general context allowing the presence of net parameters not only as arguments of synchronous or hierarchical actions but also in the guards and in the arc inscriptions [27].

5 Applications of M-nets

5.1 Semantics of parallel specification languages

The definition of M-nets was accompanied in [7,8] by their application to a formal semantics of the parallel specification language B(PN)^2: Basic Petri Net Programming Notation [10]. B(PN)^2 comprises in a simple syntax most traditional concepts of parallel programming including nested parallel composition, iteration, guarded commands, and communication via both handshake and buffered communication channels, as well as shared variables. Originally, B(PN)^2 incorporated no procedures, but this important feature was added at the M-net level first in [39,64] and then in [53]. The main difficulty when dealing with procedures consisted in the treatment of their parameters, which may be passed by value or by reference. The approach from [53] was based on M-net refinement and synchronisation operations inheriting in this way all properties of the M-net model, in particular the consistency with the low-level. Another semantics based on parameterised M-expressions was proposed in [52, 70] exploiting the introduction of the buffered communication and the properties of the M-expression algebra. With the experience acquired with the modelling of B(PN)^2, M-nets could also be used in [40] for a semantics of SDL: Specification and Description Language [21].

5.2 Semantics of object orientation

The works in [61–63] provided first attempts in expressing object oriented concepts using M-nets. They were improved in [16,17,20] by allowing the definitions of classes with their own fields (attributes and methods), single class inheritance, polymorphism and dynamic binding. This led to the definition of an extension of B(PN)^2, called BOON (Basic Object Oriented Notation), having a syntax inspired from Java and C++, and a fully compositional semantics in terms of M-nets. It may be seen as an alternative to other Petri net based formalisms capable to express object oriented concepts, which generally use more complex net classes. This is the case, for

instance, for *Object Petri Nets* (OPN) [59], which uses net tokens, or for CO-OPN [13] and CLOWN [22], which use algebraic Petri nets (nets extended with algebraic data types).

5.3 Semantics of mobility

In order to be able to express mobility, dedicated process algebras have been designed, among which one of the most popular is certainly the π-calculus [67]. The basic device for expressing mobility in this framework is to pass a *reference* (or a *channel*) to a process through a communication, allowing the recipient to use then the new channel for further interactions with other processes.

On the other hand, Petri nets are generally not considered as naturally suitable for expressing processes with changing structure, such as communicating agents which can be dynamically linked to other agents, possibly depending on previous communications. In this respect, the approaches from [28,29,31,32] show that M-nets enriched with read arcs can be successfully applied to the modelling of systems with such a mobility feature and thus to emulate the π-calculus. The basic idea is to use buffer places to store the current value of a channel and to update this value during the evolution. The originality of this approach is that it allows to build a Petri net semantics of a mobile system in a compositional way, using some simple composition rules and only a few basic nets.

These ideas were applied in [14,15] to give a high-level net semantics to the security protocol language (SPL) [23] inspired from π-calculus-like process algebras. Subsequently, Needham-Schröder protocol was considered to illustrate how this semantics could be used in order to establish the violation of the authentication property

M-nets were also used in [30,33] for a semantics of various versions of Klaim language [24] implementing the mobility using the features of multiple data tuple spaces distributed over network nodes. This work has been applied in a case study about the modelling and verification of multi-agents systems [2].

5.4 Semantics of preemption

An extension of M-nets was proposed in [55,57] in order to provide them with preemption capabilities. The preemption of a process is an interruption of its execution. It is called a *suspension* when it is temporary and followed by a *resuming*, while it is called an *abortion* when the process is killed. This extension was applied in [56] to give a semantics of exceptions in $\mathrm{B(PN)}^2$. Two unary operators were introduced: the first one, π_s, allows an M-net to be suspended and the other, π_a, to be aborted. It turned out that defining these operators in the algebraic framework of M-nets required to introduce priorities between the transitions. But it was showed that they were used carefully enough to avoid obtaining the expressive power of Turing machines; instead they allowed a logarithmic simplification of the expression of the

preemption operators. A similar compression was observed in *Place Charts Nets* introducing abortion in Petri nets [48].

The M-nets extended with priorities and preemption operators are called *preemptible M-nets* (*PM-nets*). A PM-net is a pair (N, ρ) where N is an M-net and ρ is a binary relation over the transitions of N which is called the *priority relation*, following the scheme from [11]. Having $(t_1, t_2) \in \rho$ is denoted $t_1 \prec t_2$ and means that the enabling of t_2 disables t_1, *i.e.*, that t_2 has the priority over t_1. The refinement is then adapted in order to allow a priority over a hierarchical transition t_X, *e.g.*, $t_X \prec t$, to be propagated to all the transitions of an M-net N' refining t_X, which ensures that $t_X.t' \prec t$ holds for all the transitions t' of N'. This allows to define the suspension operator in a simple way as $\pi_s(N) \stackrel{\text{df}}{=} N_s[X \leftarrow N]\,\mathsf{sc}\,sleep$ where N_s is the net shown on the left of figure 12. Thanks to the priority $t_X \prec t_2$, all the transitions in the net refining t_X are suspended as long as t_2 remains enabled. The abortion operator is defined in a similar way using the net N_a given on the right of figure 12. The main difference is that the looping transition t_2 is exploited to remove the tokens in each place of the PM-net on which π_a is applied. This is made possible by adding to these places emptying transitions (holding the action *empty*) which are synchronised with t_2. Finally, the abortion of N is obtained thanks to the priority $t_3 \prec t_2$ in N_a, which ensures that all the tokens are removed before the net $\pi_a(N)$ can produce its final marking.

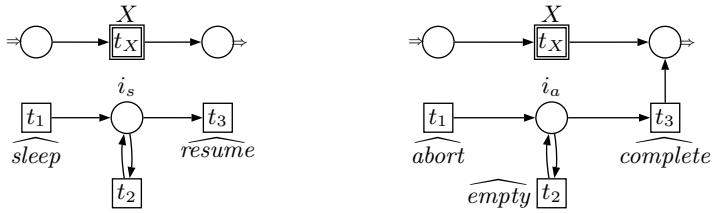


Fig. 12 On the left, the operator net N_s used to define the suspension operator. On the right, the operator net N_a for the abortion operator.

Another approach to preemption has been proposed in [74], which consists in a framework where a syntactic level (PBC-like) allows to restrict the concurrency to communicating sequential threads of executions. Preemption is here limited to an exception mechanism, but a thread cannot be suspended or aborted from another thread. The simplification of the concurrency scheme and the restricted preemption allows for a more simpler implementation without priorities.

5.5 Semantics of time constrained specifications

The *causal time* approach is a way to introduce timing features in an otherwise untimed model [38]. In particular, we shall consider coloured Petri nets as in [77], or M-nets as in [19,54,71]. The idea behind causal time is to *use the expressive power of the model*, this amounts to give an *explicit representation of clocks* in the modelled systems. In the case of Petri nets, it is

possible to introduce *counters* and a distinguished *tick transition* whose role is to simultaneously increment the counters. Thus, they become the timing reference and can be used as clock-watches by the processes. This is quite similar to what exists, *e.g.*, in timed automata [1] except that, in this later case, clocks are real values which are continuously increased by a process (the passing of time) external to the system. Using the features of M-nets allows for a definition of a causal clock which acts like a server for the rest of the system: each tick counter is associated with an identifier which can be allocated or freed and may be used to set or check the current value of the counter from any part of the system. Transmitting the counter identifiers is made easy by using the buffer places.

Using the causal time approach has been further simplified in [72, 74] by using a PBC-like syntax allowing to specify time constrained system without the need to manage the identifiers of counters. In particular, [74] allows to define multiple independent clock ticks, each being associated to various *watch* or *timeout* counters allowing respectively to measure or to constrain the passing of time. This work also avoids to statically define boundaries to some counters, which was used to model timeouts in the previous approaches.

It was shown in [19] that the causal time approach is highly relevant since the model checking can be more efficient using a general purpose model-checker for high-level Petri nets (MARIA [65]) than using well known tools for timed automata (Kronos [82] and UPPAAL [60]). This case study was performed on a railroad crossing problem which is presented in the next section. A method to use the concurrent semantics of Petri nets with a notion of time region was proposed in [72], allowing for more efficient model-checking of causally timed systems. Another approach has been proposed in [75] to handle counters of ticks using Petri nets equipped with integer variables. A compact state space construction was provided in order to give a representation of the reachable states. This construction aggregates in one unique symbolic state possibly infinitely many concrete states that differ only by the values of the integer variables.

6 Example

This section presents an example of a specification using M-nets and causal time in the version proposed in [19, 71]. The system of interest is a railroad crossing composed of n_t track sections (each being occupied by only one train) and of a pair of gates which can prevent cars from crossing the tracks when a train is present.

The trains are independent and at the beginning none is present in the crossing. Each of them starts far from the railroad crossing; it triggers a signal *app* when it approaches close enough to the gates. From this point, it reaches the gates after at least a_m and at most a_M time units. Then, it passes inside the gates during at least e_m and at most e_M time units and finally leaves the gates triggering a signal *exit*.

The gates are initially open. They close in at least g_m and at most g_M time units after receiving a signal *down*. We assume that they require the same delay for opening after receiving a signal *up*. It may happen that the

gates receive the signal *down* when they are already going up; in this case also, the time needed in order to close is within the same bounds.

A controller receives the signals from the trains and reacts by sending signals to the gates in at least c_m and at most c_M time units. It must ensure the *safety property* which states that if a train is present at the crossing, then the gates must be closed. (In this example, we do not address the *availability property*, *i.e.*, the gates are open as much as possible.)

These various parts of the system are modelled using the three M-nets depicted at the top of figure 13. The M-net given at the bottom of this figure is the causal clock N_k. The number $n_c \stackrel{\text{df}}{=} 2n_t + 1$ of tick counters in clock N_k depends on the number n_t of tracks in the system because we use two counters for each train, with the following static allocation (which improves the efficiency of the verification).

The counter 0 is reserved to the controller, and its maximal value is $max_0 \stackrel{\text{df}}{=} c_M$. This counter is reset when a train is approaching (see the transition t_{13}) and is used in order to ensure that the signal *down* is sent to the gates after at least c_m ticks (transition t_{14}). The maximum number of ticks allowed here, c_M, is enforced in the guard of the transition t_{20} in the clock. The same counter is also used when the last train leaves the crossing (transitions t_{15} and t_{17}). Notice that if we have had different constraints in these two cases, we should have used two different counters. (This is not an intrinsic limitation of causal time but rather a limitation of the simple clock we use.)

The counter 1 is reserved to the gates and its maximal value is $max_1 \stackrel{\text{df}}{=} g_M$. It is reset when the gates receive the signal to go down (transition t_5) and it ensures that the gates are down after at least g_m and at most g_M ticks (see the transition t_7 and the guard of t_{20}). The same counter is used in order to ensure the opening of the gates under the same timing constraints.

For each track i, for $i \in \{1, \ldots, n_t\}$, we use two distinct counters: $2i$ and $2i+1$, with $max_{2i} \stackrel{\text{df}}{=} a_M$ and $max_{2i+1} \stackrel{\text{df}}{=} e_M$. When a train approaches, at least a_m and at most a_M ticks can occur between the sending of the signal *app* and the arriving of the train between the gates. This constraint is ensured by the counter $2i$ (transitions t_1 and t_2). The counter $2i+1$ ensures that there must be at least e_m and at most e_M ticks between the crossing of the road by a train and its leaving sending the signal *exit* (transitions t_2 and t_3). In particular, t_2 fires when the train enters the crossing; the counter $2i$ must then indicate a value greater than a_m (thanks to $t \geq a_m$ in the guard t_2) and the counter $2i+1$ is set to zero.

The complete system is then specified as the parallel composition of its constituting nets followed by a scoping w.r.t. all the visible actions. In particular, the actions i_t, i_c and i_g are used to allow a simultaneous initialisation of all the components. So we have:

$$N_{rc} \stackrel{\text{df}}{=} (N_t \| N_g \| N_c \| N_k)\ \mathsf{sc}\ \{i_t, i_g, i_c, app, exit, down, up, clock\}\ .$$

This system can be modelled using the PEP toolkit [41] which allows for editing and composing M-nets. For the model checking of M-nets, PEP relies on the unfolding, which is unfortunately intractable in this case because of the infinite type of the place Time in N_k. Bounding the type, is not enough since

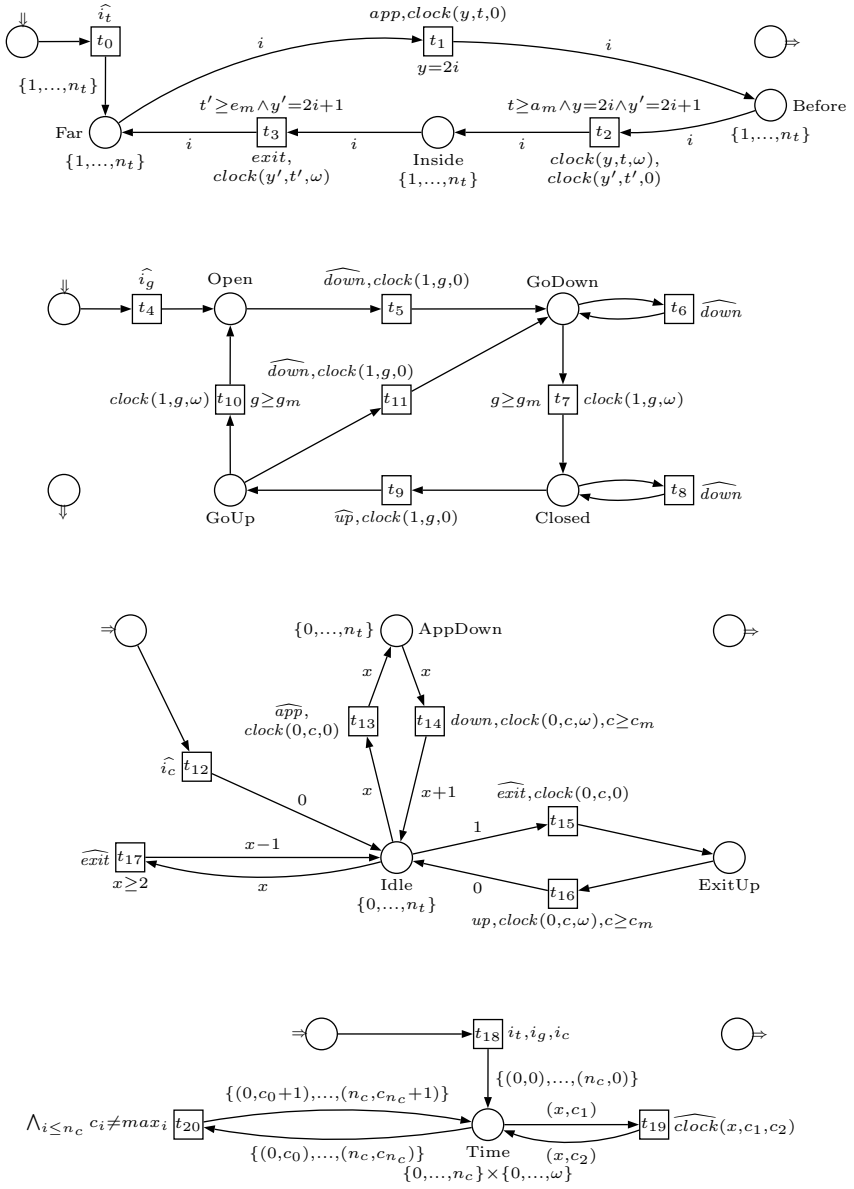


Fig. 13 The nets N_t, N_g, N_c and N_k (from top to bottom, if taken separately) or their parallel composition (if taken as a single net).

it still has many values which do not correspond to actual markings but lead to a huge unfolding, intractable as well. This explains why MARIA [65] was used in [19] instead of PEP. After the publication of [19], the tool PUNF [49, 50] became available and integrated in PEP, allowing to overcome this limitation.

7 Conclusion

We presented the model of M-nets, based on labelled high-level Petri nets and provided with various composition operations. We shown that these features make it suitable for bottom-up design by using its process algebra like compositions, as well as for top-down design by using refinement allowing to replace transitions by arbitrary M-nets. Thanks to the operation of unfolding, M-nets are consistent with the Petri Box Calculus (PBC). This characteristics offers a strong formal basis and allows for efficient model checking. Indeed, PBC is based one safe (1-bounded) Petri nets for which optimised techniques exist.

The main extensions of M-nets allow to express parameterised refinement, recursion and buffered communication. The main applications are the definitions of semantics for parallel specification languages, the modelling of object oriented, mobile, preemptive or timed systems. A more detailed example is developed for illustrating this last application. It concerns a railroad crossing system and allows to show how M-net modularity may facilitate the modelling of such time constrained critical systems.

Future works about M-nets will aim at improving the process algebraic aspects, trying to introduce most of the high-level features of M-nets in PBC-like algebras, exploiting, in particular, the possibilities of abstraction offered by buffered communication (see, *e.g.*, [72, 74] as first steps). Moreover, work is in progress concerning an efficient verification method for M-nets. The main idea consists in avoiding going through the unfolding way; instead, staying at the high-level allows to envisage approaches based on abstraction (like in [75]) and exploiting the modular aspect of the verified systems.

Acknowledgements

The authors would like to warmly thank the anonymous referees who provided extremely detailed reports that where really useful in improving the paper.

References

1. R. Alur and D. Dill. *A theory of timed automata.* TCS 126(2). Elsevier, 1994.
2. N. Benaissa, B. Djafri, G. Hutzler and H. Klaudel. *Towards modelling and verification of mobile agent systems.* INADIS/IBERAMIA-SBIA-SBRN'2006. Springer, 2006.

3. E. Best and R. Devillers. *Sequential and concurrent behaviour in Petri net theory.* TCS 55. Elsevier, 1987.
4. E. Best, R. Devillers and J.G. Hall. *The box calculus: a new causal algebra with multi-label communication* APN'92, LNCS 609. Springer, 1992.
5. E. Best, R. Devillers and J. Esparza. *General refinement and recursion for the box calculus.* STACS'93, LNCS 665. Springer, 1993.
6. E. Best, R. Devillers and M. Koutny. *Petri net algebra.* EATCS Monographs on TCS. Springer, 2001.
7. E. Best, H. Fleischhack, W. Frączak, R.P. Hopkins, H. Klaudel and E. Pelz. *An M-net semantics of B(PN)^2.* SCT'95. Springer, 1995.
8. E. Best, W. Frączak, R.P. Hopkins, H. Klaudel and E. Pelz. *M-nets: an algebra of high level Petri nets, with an application to the semantics of concurrent programming languages.* Acta Informatica 35. Springer, 1998.
9. E. Best and C. Fernández. *Nonsequential processes.* EATCS Monographs on TCS 13. Springer, 1988.
10. E. Best and R.P. Hopkins. *B(PN)^2 – a basic Petri net programming notation.* PARLE '93, LNCS 694. Springer, 1993.
11. E. Best and M. Koutny. *Petri net semantics of priority systems.* TCS 96(1). Elsevier, 1992.
12. E. Best and T. Thielke. *Refinement of coloured Petri nets.* FCT'97, LNCS 1279. Springer, 1997.
13. O. Biberstein, D. Buchs and N. Guelfi. *Object-oriented Nets with algebraic specifications: The CO-OPN/2 formalism.* APN on Object-Orientation, LNCS 2001. Springer, 2000.
14. R. Bouroulet, H. Klaudel and E. Pelz. *A semantics of security protocol language (SPL) using a class of composable high-level Petri nets.* ACSD'04. IEEE, 2004.
15. R. Bouroulet, H. Klaudel and E. Pelz. *Modelling and verification of authentication using enhanced net semantics of SPL (security protocol language).* ACSD'06. IEEE, 2006.
16. C. Bui Thanh and H. Klaudel. *Encapsulation in an object oriented notation based on modular Petri nets.* Workshop on Simulation with Petri nets (satellite of ESMc'2003). Eurosis, 2003.
17. C. Bui Thanh and H. Klaudel. *Object oriented modelling with high-level modular Petri nets.* IFM'04, LNCS 2999. Springer, 2004.
18. C. Bui Thanh, H. Klaudel and F. Pommereau. *Box calculus with coloured buffers.* DASD/ASTC'04. SCS, 2004.
19. C. Bui Thanh, H. Klaudel and F. Pommereau. *Petri nets with causal time for system verification.* MTCS'02, ENTCS 68.5. Elsevier, 2002.
20. C. Bui Thanh. *Modèles Orientés-Objet pour la Vérification de Systèmes Concurrents.* Ph.D. Thesis, Univ. Paris 12. Créteil, 2004.
21. CCITT. *Specification and description language.* CCITT Z.100. ICCTT, 1992.
22. A. Chizzoni. *CLOWN: class orientation with nets.* Master thesis, Univ. of Milan, 1996.
23. F. Crazzolara and G. Winskel. *Events in security protocols.* ACM Conf. on Computer and Communications Security. ACM Press, 2001.
24. R. De Nicola, G. Ferrari and R. Pugliese. *Klaim: a kernel language for agents interaction and mobility.* IEEE Transactions on Software Engineering 24(5). IEEE, 1998.
25. R. Devillers and H. Klaudel. *Refinement and recursion in a high level Petri box calculus.* STRICT'95, WiC. Springer, 1995.
26. R. Devillers and H. Klaudel. *Solving Petri net recursions through finite representation.* ACST'2004. ACTA Press, 2004.
27. R. Devillers and H. Klaudel. *Synchronous and asynchronous communications in composable parameterized high-level Petri nets.* Fundamenta Informaticae 3(66). IOS Press, 2005.
28. R. Devillers, H. Klaudel and M. Koutny. *Context-based process algebras for mobility.* ACSD'04. IEEE, 2004.

29. R. Devillers, H. Klaudel and M. Koutny. *Petri net semantics of the finite π-calculus.* FORTE'2004, LNCS 3235. Springer, 2004.
30. R. Devillers, H. Klaudel and M. Koutny. *A Petri net semantics of a simple process algebra for mobility.* EXPRESS/CONCUR'05, ENTCS 154(3). Elsevier, 2006.
31. R. Devillers, H. Klaudel and M. Koutny. *Petri net semantics of the finite π-calculus terms.* Fundamenta Informaticae 70(2006)3. IOS Press, 2006.
32. R. Devillers, H. Klaudel and M. Koutny. *A Petri net translation of π-calculus terms.* ICTAC'06, LNCS 4281. Springer, 2006.
33. R. Devillers, H. Klaudel and M. Koutny. *Modelling mobility in high-level Petri nets.* ACSD'07. IEEE, 2007
34. R. Devillers, H. Klaudel, M. Koutny and F. Pommereau. *Asynchronous box algebra.* Fundamenta Informaticae 54. IOS Press, 2003.
35. R. Devillers, H. Klaudel and E. Pelz. *An algebraic box calculus.* JALC 5(2). Univ. of Magdeburg, 2000.
36. R. Devillers, H. Klaudel and R.-C. Riemann. *General refinement for high level Petri nets.* FST-TCS'97, LNCS 1346. Springer, 1997.
37. R. Devillers, H. Klaudel and R.-C. Riemann. *General parameterised refinement and recursion for the M-net calculus.* TCS 300. Elsevier, 2003.
38. R. Durchholz. *Causality, time and deadlines.* Data & Knowledge Engineering, 6. North-Holland, 1991.
39. H. Fleischhack and B. Grahlmann. *A Petri net semantics for B(PN)^2 with procedures.* PDSE'97. IEEE, 1997.
40. H. Fleischhack and B. Grahlmann. *A compositional Petri net semantics for SDL.* ATPN'98, LNCS 1420. Springer, 1998.
41. B. Grahlmann and E. Best. *PEP – More than a Petri net tool.* TACAS'96, LNCS 1055. Springer, 1996.
42. H. Genrich and H.J. Lautenbach. *System modelling with high-level Petri nets.* TCS 13. Elsevier, 1981.
43. H.J. Genrich, K. Lautenbach and P.S. Thiagarajan. *Elements of general net theory.* ACGNTPS'80, LNCS 84. Springer, 1980.
44. U. Goltz and W. Reisig. *The non-sequential behaviour of Petri nets.* Information and Control 57/2-3. 1983.
45. B. Grahlmann. *Parallel programs as Petri nets..* Ph.D. Thesis, Univ. Hildesheim. Germany, 1999.
46. P. Huber, K. Jensen and R.M. Shapiro. *Hierarchies in coloured Petri nets.* APN'90, LNCS 483. Springer, 1990.
47. K. Jensen. *Coloured Petri nets. Basic concepts, analysis methods and practical use.* EATCS Monographs on TCS 1. Springer, 1992.
48. M. Kishinevsky, J. Cortadella, A. Kondratyev, L. Lavagno, A. Taubin and A. Yakovlev. *Coupling asynchrony and interrupts: place chart nets and their synthesis.* ICATPN'97, LNCS 1248. Springer, 1997.
49. V. Khomenko. *Punf documentation and user guide.* 2002. `http://homepages.cs.ncl.ac.uk/victor.khomenko/home.formal/tools/punf`
50. V. Khomenko and M. Koutny. *Branching processes of high-level Petri nets.* TACAS'2003, LNCS 2619. Springer, 2003.
51. H. Klaudel. *Modèles algébriques, basés sur les réseaux de Petri, pour la sémantique des langages de programation concurrents.* Ph.D. Thesis, Univ. Paris 11. Orsay, 1995.
52. H. Klaudel. *Parameterized M-expression semantics of parallel procedures.* DAPSYS'00. Kluwer, 2000.
53. H. Klaudel. *Compositional high-level Petri net semantics of a parallel programming language with procedures.* SCP 41(3). Elsevier, 2001.
54. H. Klaudel and F. Pommereau. *Asynchronous links in the PBC and M-nets.* ASIAN'99, LNCS 1742. Springer, 1999.
55. H. Klaudel and F. Pommereau. *A concurrent and compositional Petri net semantics of preemption.* IFM'00, LNCS 1945. Springer, 2000
56. H. Klaudel and F. Pommereau. *A concurrent semantics of static exceptions in a parallel programming language.* ICATPN'01, LNCS 2075. Springer, 2001

57. H. Klaudel and F. Pommereau. *A class of composable and preemptible high-level Petri nets with an application to multi-tasking systems.* Fundamenta Informaticae, 50(1). IOS Press, 2002.
58. H. Klaudel and R.-C. Riemann. *High level expressions and their SOS semantics.* CONCUR'97, LNCS 1243. Springer, 1997.
59. C. Lakos. *Object oriented modelling with object Petri nets.* APN'97, LNCS 2001. Springer, 1997.
60. K.G. Larsen, P. Pettersson and W. Yi. *UPPAAL in a nutshell.* International Journal on Software Tools and Technology Transfer, 1(1-2). Springer, 1997.
61. J. Lilius. *OOB(PN)^2: an object-oriented Petri net programming notation.* 2^{nd} Workshop on Object-Oriented Programming and Models of Concurrency, 1996.
62. J. Lilius. *OB(PN)^2: An object based Petri net programming notation.* Euro-Par'96, LNCS 1123. Springer, 1996.
63. J. Lilius. *OB(PN)^2: an object based Petri net programming notation.* APN'01, LNCS 2001. Springer, 1997.
64. J. Lilius and E. Pelz. *An M-net semantics for B(PN)^2 with procedures.* IS-CIS'96, 1. Middle East Technical Univ., 1996.
65. M. Mäkelä. *MARIA: modular reachability analyser for algebraic system nets.* Online manual, `http://www.tcs.hut.fi/maria`, 1999.
66. A. Mazurkiewicz. *Trace theory.* APN'87, Part II, LNCS 255. Springer, 1987.
67. R. Milner, J. Parrow and D. Walker. *A calculus of mobile processes. Part I and II.* Information and Computation 100. Elsevier, 1992.
68. C.A. Petri. *Kommunikation mit Automaten.* Schriften des Instituts für instrumentelle Mathematik. Universität Bonn, 1962.
69. The CPN group, University of Aarhus, Denmark. *The Petri net world, Petri nets tool database.* `http://www.daimi.au.dk/PetriNets/tools/db.html`
70. F. Pommereau. *FIFO buffers in tie sauce.* DAPSYS'00. Kluwer, 2000.
71. F. Pommereau. *Modèles composables et concurrents pour le temps-réel.* Ph.D. Thesis, Univ. Paris 12. Créteil, 2002.
72. F. Pommereau. *Causal time calculus.* FORMATS'03. LNCS 2791. Springer, 2004.
73. F. Pommereau. *SNAKES is the net algebra kit for editors and simulators.* `http://www.univ-paris12.fr/lacl/pommereau/soft/snakes`
74. F. Pommereau. *Versatile boxes: a multi-purpose algebra of high-level Petri nets.* DADS/SCSC'07, SCS/ACM, 2007.
75. F. Pommereau, R. Devillers and H. Klaudel. *Efficient reachability graph representation of Petri nets with unbounded counters.* Infinity'07. ENTCS. Elsevier, to appear.
76. W. Reisig. *Petri nets and algebraic specifications.* TCS 80. Elsevier, 1991.
77. G. Richter. *Counting interfaces for discrete time modeling.* Tech. report 26, GMD. September 1998.
78. R-C. Riemann. *Modelling of concurrent systems: structural and semantical methods in the high-level Petri net calculus.* Ph.D. Thesis, Univ. Paris 11. Orsay, 1999.
79. P. H. Starke. *Processes in Petri nets.* EIK 17/8-9. 1981.
80. H. Störrle. *An evaluation of high-end tools for Petri-nets.* Tech. Report Bericht 9802. Ludwig Maximilians Univerität München, 1998.
81. W. Vogler. *Modular construction and partial order semantics of Petri nets.* LNCS 625. Springer, 1992.
82. S. Yovine. *Kronos: A verification tool for real-time systems.* International Journal of Software Tools for Technology Transfer 1(1/2). Springer, 1997.