Linear and Non-Linear Model for Statistical Localization of Landmarks

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Abstract

This paper presents and compares 3 methods for the statistical localization of partially occulted landmarks. In many real applications, some information is visible in images and some parts are missing or occulted. These parts are estimated by 3 statistical approaches : a rigid registration, a linear method derived from PCA, which represents spatial relationships, and a non linear model based upon Kernel PCA. Applied to the cephalometric problem, the best method exhibits a mean error of 3.3 mm, which is about 3 times the intra-expert variability.

1. Introduction

The goal of orthodontic and orthognatic therapy is to improve the interrelationships among craniofacial tissues. A cephalogram is a two-dimensional X-Ray image of the sagital skull projection [1][2]. It is used to evaluate these relationships. Cephalometric landmarks are bony landmarks and are first located on the radiograph. Distances and angles among these landmarks are compared with normative values to diagnose a patient's deviation from ideal form and to evaluate craniofacial growth characteristics, skeletal and dental disharmonies. It is also used to evaluate results and stability of various treatment approaches. This task is challenging and has been the subject of previous research [3][11]. Our goal is the realization of a computer vision system to obtain an objective and reproducible cephalometric analysis. Indeed, large inter-expert and intra-expert variability has been noticed [2]. The main source of errors is the precise identification of landmarks. The two main causes are the subjectivity in the interpretation of the landmark definitions and the positional repeatability of human experts. Landmarks are difficult to distinguish on images and interpretation needs a long training time. In a computerized method, the formal descriptions of landmarks used by clinicians are not directly transposable: we then use a statistical approach to provide an initial estimation of landmark positions, using statistical models and training sets.

During the past decade, there has been a lot of work in shape based approaches, for segmentation, registration or identification tasks. In fact, any application where the geometric comparison of objects is required needs a shape analysis. The pioneers of the subject of shape analysis are Kendall [9] and Bookstein [8]. In these works, shape is defined as the remaining information after alignment (rotation, translation, scale) between two objects. In image analysis, Pentland [13] has defined modal analysis and a similar idea has been used by Cootes [10] in the Active Shape Model (ASM) and Active Appearance Model (AAM). They both involve a Principal Component Analysis (PCA) to build a statistical shape model. In this model, the mean object and the variation around this mean position are both represented. AAM was used for cephalometric purpose by Hutton [11] without sufficient accuracy.

Other methods related to this problem use elastic registration to align an image with a model. The model can be an image [6], an atlas [5] or a set of landmarks [4]. Elastic registration is a powerful tool based upon physical models such as solid or fluid deformations and includes complex and non linear model. Yet, the variability of the shape is not represented.

Some works on Kernel PCA [12] are very close to our method. Briefly, Kernel PCA maps the input data in a Feature Space (F-Space) using a non linear mapping. PCA is performed in the F-Space. The mean shape is given by the eigenvectors corresponding to the largest eigenvalues. In a classification problem, classification is done in the F-Space. In a localization problem, mean shape in the F-space must be back-projected in the input space. The choice of the mapping and the back projection are difficult problems and are still open issues.

In this paper, we present a comparison of 3 methods to localize landmarks and their application to the cephalometric problem. The first method is a simple affine registration, the second method is based upon a linear PCA and the third one is a non linear method close to PCA.

2. Methods

In the cephalometric problem, orthodontists have annotated cephalograms with 14 landmarks (cephalometric points) on a training set of radiographs. We also use an a-priori knowledge, which is a common

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knowledge in all cephalometric analysis : there is an unknown spatial relation between the cranial contour and the cephalometric points. The main problem in the cephalometric analysis is to discover this relation. Fortunately, the cranial contour can be automatically detected and extracted from the image [14], and then sampled (16 points). Our training set of points is composed of the 14 cephalometric points and the sampled version of the cranial contour. From this data-set, a mean shape model is computed. To retrieve landmarks on a new image, the cranial contour is detected and sampled, cephalograms are registered and the mean shape model is used to estimate the position.

2.1. Linear affine model

The problem is to compute a mean shape from a training set of points. First, all the sets of the training base have to be aligned. Procustes Analysis is a common tool to register two sets. It is a one to one mapping. To avoid this mapping, we have approximated the cranial contour by an ellipse, with the following parameters :

x_g, y_g : center of ellipse,

θ : angle between first principal axis and Ox,

a,b : length of the principal axis.

The coordinates of cephalometric points are expressed in the coordinate space defined by the center x_g, y_g and the vectors a and b along the principal axis of the ellipse.

Let $X_i=(x_i,y_i)$ $i \in 1..n$, be the points of the cranial contour. We can write :

$$X_g = \begin{pmatrix} x_g \\ y_g \end{pmatrix} = \frac{1}{n}\sum_{i=1}^{n} X_i \ ,$$

$$\mu_{pq} = \frac{1}{n}\sum_{i=1}^{n}(x_i - x_g)^p (y_i - y_g)^q \ ,$$

$$\tan(2*\theta) = \frac{2\,\mu_{11}}{\mu_{02} - \mu_{20}} \ .$$

Intersections between the cranial contour and the principal axis defined by the angles θ and θ+2π define the the unit vectors a and b.

Let $C_i^j = (cx_i^j, cy_i^j)$, $i \in 1..m$, $j \in 1..P$, be the i^{th} cephalometric point of j^{th} image. The mean shape C_i of the i^{th} cephalometric points is then defined by :

$$\hat{C}_i = \frac{1}{P}\sum_{j=1}^{P}\left(\begin{pmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{pmatrix} * \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}\right)^{-1} * \begin{pmatrix} x_i^j - x_G \\ y_i^j - y_G \end{pmatrix} .$$

On an unseen image, the cranial contour is detected and is fitted with an ellipse and the 5 parameters x_g, y_g, θ , a and b are computed. The estimated landmarks are then :

$$\begin{pmatrix} x_i \\ y_i \end{pmatrix} = \left(\begin{pmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{pmatrix} * \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} * (\hat{C}_i)\right) + \begin{pmatrix} x_G \\ y_G \end{pmatrix} .$$

2.2. Linear PCA model

In the previous method, spatial relations between cephalometric points are not examined although they seem to be quite important for the expert. The linear PCA method defined here is an elegant way to take into account spatial relations between landmarks and between landmarks and contour, and can also estimate the unknown part (cephalometric points) of the partially visible or occulted model (cranial contour).

Let $X_i = (x_{1i}, y_{1i}, x_{2i}, y_{2i}, .. x_{ni}, y_{ni}) \in R^{2n}$ be the locations of the n cephalometric points on the i^{th} cephalogram, C_i be the locations of the m points of the sampled cranial contour on the i^{th} cephalogram, and $T_i=(X_i,C_i)$ the concatenation of X_i and C_i.

To compute a model with this training set, the first step is to align all these samples. This is realized with an iterative version of the Procuste analysis.

Using PCA, we can write $T_i \approx \overline{T} + \phi\, b$ where :

$\overline{T} = \frac{1}{p}\sum_{i=1}^{p} T_i$ is the mean shape of the pattern,

$\phi = (\phi_1 \mid \phi_2 \mid \phi_3 \mid \ldots \mid \phi_t)$ is a (n+m)*t matrix composed with the eigenvectors of the (n+m)*(n+m) covariance matrix S of the centered data: $S = \frac{1}{p-1}\sum_{i=1}^{p}(T_i - \overline{T})(T_i - \overline{T})^T$,

b is a vector of dimension t : $b = \phi^T (T_i - \overline{T})$.

The dimension t of the vector b is the number of eigenvectors with the largest eigenvalues. In classical use of PCA, t is chosen by $\sum_{i=1}^{t}\lambda_i \geq 0.95\sum_{i=1}^{n+m}\lambda_i$, i.e. only eigenvectors that explain sufficiently the standard deviation are kept. The vector b of dimension t is a good approximation for the original data set and any set of n+m points can be represented or retrieved with the t (t<n+m) values of the vector b. Then, PCA can be seen as a denoising method.

Now, on a new image, the cranial contour is extracted. To compare this contour with the model, we need to align it with the model. The transformation to align the n points (x_i, y_i) with the model (x'_i,y'_i) is defined by a least square process :

$$T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \alpha & -\beta \\ \beta & \alpha \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \end{pmatrix},$$

$$\alpha = \frac{\frac{1}{n}\sum x_i x_i' + \frac{1}{n}\sum y_i y_i'}{\frac{1}{n}\sum x_i^2 + \frac{1}{n}\sum y_i^2} = \frac{x.x'}{|x|^2},$$

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$$\beta = \frac{\frac{1}{n}\sum x_i y_i' - \frac{1}{n}\sum y_i x_i'}{\frac{1}{n}\sum x_i^2 + \frac{1}{n}\sum y_i^2} = \frac{\frac{1}{n}\sum x_i y_i' - \frac{1}{n}\sum y_i x_i'}{|x|^2}.$$

Once the registration of a new form onto the model is done, the original idea of the paper is the following one : if some information is present, if some points (saying t=n points) are known, the remaining unknown m points can be determined using PCA, under the hypothesis that the first n eigenvalues of the covariance matrix explain the training dataset.

Let C=(x,y) be the points of the cranial contour and X be the cephalometric points. Without any approximations, we can write :

$$\begin{pmatrix} C_1 \\ C_2 \\ \vdots \\ C_n \\ X_1 \\ X_m \end{pmatrix} = \begin{pmatrix} \overline{C}_1 \\ \overline{C}_2 \\ \\ \overline{C}_n \\ \overline{X}_1 \\ \overline{X}_m \end{pmatrix} + \begin{pmatrix} \phi_{1,1} & \phi_{1,2} & \cdot\cdot & \phi_{1,n+m} \\ \phi_{2,1} & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ \phi_{n+m,1} & \phi_{n+m,2} & \cdot\cdot & \phi_{n+m,n+m} \end{pmatrix} \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \\ b_{n+1} \\ b_m \end{pmatrix}$$

This is a linear system with n+m equations and n+2m unknown (X_1,X_m,b_1,..,b_{m+n}) that can not be resolved. Since PCA represents the dataset with less values, we can write, using t=n :

$$\begin{pmatrix} C_1 \\ C_2 \\ \vdots \\ C_n \\ X_1 \\ X_m \end{pmatrix} = \begin{pmatrix} \overline{C}_1 \\ \overline{C}_2 \\ \vdots \\ \overline{C}_n \\ \overline{X}_1 \\ \overline{X}_m \end{pmatrix} + \begin{pmatrix} \phi_{1,1} & \phi_{1,2} & \cdot\cdot & \phi_{1,n} \\ \phi_{2,1} & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ \phi_{n+m,1} & \phi_{n+m,2} & \cdot\cdot & \phi_{n+m,n} \end{pmatrix} \begin{pmatrix} b_1 \\ b_2 \\ \dots \\ b_n \end{pmatrix}.$$

Now the unknown vector (b_1,..,b_n) is given by the n first equations of the system. Notice that if we choose t<n, the system is now overdetermined (t<n), a least square method is used to resolve the system. The m last equations are a linear system with m equations and m unknowns X_1..X_m :

$$\begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix} = \begin{pmatrix} \phi_{1,1} & \phi_{1,2} & \cdots & \phi_{1,n} \\ \phi_{2,1} & \phi_{2,2} & \cdots & \phi_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ \phi_{n,1} & \phi_{n,2} & \cdots & \phi_{n,n} \end{pmatrix}^{-1} \left(\begin{pmatrix} C_1 \\ C_2 \\ \vdots \\ C_n \end{pmatrix} - \begin{pmatrix} \overline{C}_1 \\ \overline{C}_2 \\ \vdots \\ \overline{C}_n \end{pmatrix} \right),$$

and $$\begin{pmatrix} X_1 \\ \vdots \\ X_m \end{pmatrix} = \begin{pmatrix} \phi_{1+n,n} & \phi_{1+n,2} & \cdots & \phi_{1+n,n} \\ \vdots & \vdots & \ddots & \vdots \\ \phi_{n+m,1} & \phi_{n+m,2} & \cdots & \phi_{n+m,n} \end{pmatrix} \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix}.$$

In this framework, a linear approximation of spatial relations between cranial contour and cephalometric points are explicitly determined from the eigenvectors of the covariance matrix. In the same way, relations between landmarks are approximated.

2.3. Non Linear model

In this model, we project the location of the initial data in a new space, following the idea of Kernel methods .

Let P_i be the points of the cranial contour. The origin of the sampling is given by the point P_0 with the higher curvature, i.e. a point near the intersection of the curve and of the nasal bone. Let the set of vectors R be defined by each pair of points P_i and P_j of the sample cranial contour.

$$R = \left\{ (O_i, \vec{v}_i) \middle| \forall (j,k) \in \{1...n\}, j < k, \exists\ i / O_i = P_j \ and \vec{v}_i = \overrightarrow{P_i P_k} \right\}$$

Let M $(M_x, M_y, 1)$ be a point. The coordinates α_ι of M in R are defined by the scalar product (or projection) of M and each vectors of R, i.e. $\alpha_i = \left\langle \overrightarrow{O_i M} \middle| \vec{v}_i \right\rangle$.

Then, we can write :

$$\alpha = (\alpha_i) = \left(v_{ix} \quad v_{iy} \quad -(O_{ix}v_{ix} + O_{iy}v_{iy}) \right) \begin{pmatrix} M_x \\ M_y \\ 1 \end{pmatrix} = \left(\mathrm{A} \right) \begin{pmatrix} M_x \\ M_y \\ 1 \end{pmatrix}.$$

The coordinates α_i are the projection (mapping) of the point M on each vector built using the cranial contour. Our model represents the position and the variability of these new values. The mean E(α) and the standard deviation σ(α) of the location of each landmark are computed and stored. As the number of coefficients α_i is large n(n+1)/2, and because all the coefficients are not relevant for each landmark in the image, only the coefficients which are really useful (i.e. with small standard deviation) are stored.

To determine the unknown position of the landmark X in a new cephalogram, we can write:

$$E[\alpha] = A \cdot X$$

where A is computed by detecting the cranial contour and the vector α on the new cephalogram. X is the unknown cephalometric point and is computed with a weight least square inversion matrix, using the constraint X_z=1.

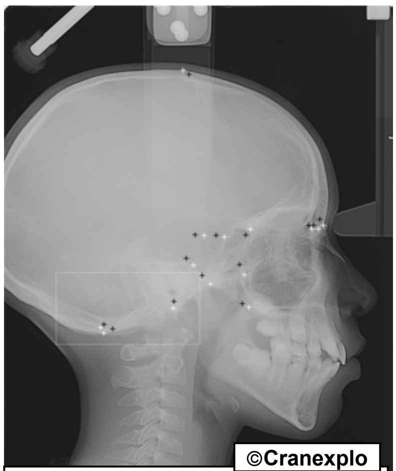

Figure 1 : Annoted Points (black) and estimation (white)

3. Results

We have now 227 cephalograms annotated by an orthodontist. All the cephalograms have been processed by the 3 methods. The methods have failed on one cephalogram because the cranial contour was not detected properly. To test and compare the accuracy of these methods, this training set is divided in a first set of 60 cephalograms, randomly chosen, used to compute the model and a second set used to test the methods. We have measured the mean error between the real position and the estimated position of landmarks, on the x-axis and on the y-axis. Results are summarized in the table 1.

Table 1 : Mean Error

Error (mm)	x	y
Method 1	3.4	3.5
Method 2	3.1	2.6
Method 3	2.7	2.0

For the cephalometric use, these values must be compared with intra and inter experts variability. For practical reasons, only the intra-expert variability have been estimated to 1.1mm on a small set (15) of cephalograms. In literature, inter-expert is often larger (5 times). Then, results of the statistical process are quite accurate, but not enough for clinical usage.

4. Conclusion

In this paper, we have addressed the problem of locating occulted parts of objects described by some landmarks. When only some of the landmarks are visible or extracted from an image, the remaining unknown landmarks must be estimated. A statistical method solves easily this problem. 3 methods have been described and compared.

Rigid registration or Procustes Analysis aligns (translation, rotation, scale) two or more sets of points, which allows to compute a mean model. This mean model is simply registered on a new image using the extracted (visible) landmarks.

Classical shape analysis uses PCA to determine a mean value and a set of allowed deformation modes and parameters. These modes are the eigenvectors of the covariance matrix, which described the spatial relationships between landmarks. The deformation parameters are computed from the extracted landmarks, and unknown landmarks are determined from these values. Spatial relationships and the shape of the actual image are then taken into account by this method.

At last, a non linear method maps the initial data in a large feature space using a projection. For each landmark, only the few useful axes of the space are used to estimate the unknown landmarks by a least square inversion.

Applied to the cephalometric problem, the set of landmarks is composed of cephalometric points (unknown) and of the cranial contour, which is automatically extracted. Results are quite satisfactory, but accuracy is not sufficient for clinical use.

Improvement of this accuracy is possible, and we are working on a non linear-method which is a mixture of the second and the third method : it will introduce spatial relationships in non-linear model and optimize the non linear mapping for the cephalometric problem.

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6. References

[1] J.A.MacNamara : A method of Cephalometric Evaluation. American Journal of Orthodontics, 86, pp. 449-469, 1984.
[2] M.Cretot : L'image téléradiographique en céphalometrie, Edition CPD, 1989.
[3] D.N.Davis, C.J.Taylor : A Blackboard Architecture for Automatic Cephalogram Analysis, Medical Informatics, Vol 16, pp. 137-149, 1991.
[4] F. L. Bookstein: Principal Warps : Thin-Plate Splines and the Decomposition of Deformations, IEEE PAMI, 11(6), pp. 567-585, 1989.
[5] G. Subsol, J.Ph. Thirion, N. Ayache. « A general scheme for automatically building 3D morphometric anatomical atlases: application to a skull atlas ». Medical Image Analysis, 2(1), pp. 37-60, 1998.
[6] R. Woods et al. "Automated Image Registration: I. General Methods and Intrasubject, Intramodality Validation". Journal of Computer Assisted Tomography, 22(1), pp. 139-152, 1998.
[7] G.E. Christensen, R.D. Rabbitt, M.I. Miller, Deformable Templates Using Large Deformation Kinematics, Image Processing (5), pp. 1435-1447, 1996.
[8] F.L. Bookstein. On the cephalometrics of skeletal change. Am. J. Orthod 83, pp. 177-182, 1987.
[9] D.G. Kendall. Shape manifolds, procrustean metrics and complex projectives spaces, Bull. Lond. Math. Soc., 16, pp. 81-121, 1984.
[10] T.F.Cootes, G.J. Edwards, C.J.Taylor. Active Appearance Models, IEEE PAMI, 23(6), pp. 681-685, 2001.
[11] T.J. Hutton, S. Cunningham, P. Hammond. An Evaluation of Active Shape Models for the Automatic Identification of Cephalometric Landmarks, Eur J Orthod, London, 22(5), pp. 499, 2000.
[12] S. Mika, B. Scholkopf, A. Smola, K. Muller, M. Scholz, G. Ratsch. Kernel PCA and De-Noising in Feature Spaces. Advances in Neural Information Processing Systems, 11, pp. 536-542, 1999.
[13] S. Sclaroff, A.P. Pentland. Modal Matching for Correspondence and Recognition. IEEE PAMI, 17(6), pp. 545-561,1995.
[14] M. Desvignes, B. Romaniuk, J. Robiaille, M. Revenu, M.J. Deshayes, Détection et modélisation du contour crânien sur des radiographies, 18ième GRETSI, Toulouse, pp. 121-123, 2001.