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Efficient Time Domain Threshold for Sparse Channel Estimation in OFDM System

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Abstract

A novel efficient time domain threshold based sparse channel estimation technique is proposed for orthogonal frequency division multiplexing (OFDM) systems. The proposed method aims to realize effective channel estimation without prior knowledge of channel statistics and noise standard deviation within a comparatively wide range of sparsity. Firstly, classical least squares (LS) method is used to get an initial channel impulse response (CIR) estimate. Then, an effective threshold, estimated from the noise coefficients of the initial estimated CIR, is proposed. Finally, the obtained threshold is used to select the most significant taps. Theoretical analysis and simulation results show that the proposed method achieves better performance in both BER (bit error rate) and NMSE (normalized mean square error) than the compared methods, has good spectral efficiency and moderate computational complexity.

Keywords: Sparse channel estimation; least squares; OFDM; time domain threshold.

1. Introduction

Orthogonal frequency division multiplexing (OFDM) technique is widely used in wireless communication system thanks to its advantages of high data
transmission rate over multipath fading channel. In OFDM systems, accurate channel state information (CSI) is necessary for coherent detection at the receiver. Therefore, how to optimally use the precious limited communication system resources to obtain effective CSI is the main challenge in channel estimation field.

Channel estimation can be carried out in either frequency or time domain. In frequency domain, least squares (LS) and minimum mean square error (MMSE) are the most common channel estimation methods [1]. Generally, MMSE method can achieve better channel estimation performance than LS method, but MMSE method requires prior knowledge of channel statistics and noise variance. Moreover, it has an increased computational complexity [1, 2]. Comparatively, the complexity of LS is low and it is popular to combine LS method with different interpolation algorithms to realize effective channel estimation [2, 3, 4]. Among them, linear interpolation with lowest computational cost is the most classical one [2, 3, 4, 5, 6]. However, its performance relies on comparatively high percentage of pilots and it is vulnerable to noise, especially when the channel is sparse. For sparse channel, the channel impulse response (CIR) is characterized by a few dominant channel taps and majority of zero or nearly zero channel taps. In time domain, it is possible to estimate the sparse channel with limited number of pilots by fully considering the maximum channel length and the characteristics of channel sparsity. The most significant taps (MST) method, firstly proposed in [7], is popular in estimating sparse channel. Following that, different thresholds [8, 9] have been proposed for MST based methods. However, all these methods have drawbacks. For example, a statistical threshold is derived for pilot aided sparse channel estimation in [8], which needs prior knowledge of channel statistics and has reduced spectral efficiency. [9] uses twice of the noise power as threshold, which can hardly be optimal [6]. The noise standard deviation based threshold [10] is also not optimal [11]. In order to overcome the drawbacks of the above sparse channel estimation methods, recently, the theoretical optimal threshold is derived for sparse Rayleigh channel estimation in OFDM system [12], which relies on the accurate knowledge of the channel tap power delay profile and the received pilot energy to noise ratio. In order to make the threshold more robust to communication environment, a sub-optimal threshold is proposed by assuming a uniform power delay profile [12], which is proven to be robust with the true power delay profile and depends only on the received signal to noise ratio (SNR).
This paper proposes a novel sparse channel estimation method with effective time-domain threshold. The proposed method depends only on the estimated noise standard deviation and needs no prior knowledge of channel statistics and noise standard deviation, it can realize effective channel estimation within a wide range of sparsity rate (the ratio of non-zero taps to the considered channel length) [13].

The rest of this paper is organized as follows. In Section 2, the system model for sparse channel estimation in OFDM system is introduced and the noise statistics are analyzed. Based on the characteristics of the sparse channel, an effective time domain threshold is proposed for the improvement of channel estimation performance in Section 3. Some simulation results comparing the performance of the proposed method and other existing methods and the computational complexity analysis are provided in Section 4. Finally, conclusions are drawn in Section 5.

2. System Model

Consider a \( N \) subcarriers OFDM system. \( M \) pilots with index \( k_0, k_1, \ldots, k_{M-1} \) are employed to estimate a sparse channel. The received pilot vector can be expressed by [10]:

\[
y_p = X_p F_{M \times L_{cp}} h + v_p
\]  

where \( X_p = diag[x_{k_0}, x_{k_1}, \ldots, x_{k_{M-1}}] \) is the diagonal matrix of transmitted pilots with each element having same power and being normalized; \( y_p = [y_{k_0}, y_{k_1}, \ldots, y_{k_{M-1}}]^T \) is the received pilot vector; \( h = [h_0, h_1, \ldots, h_{L_{cp}-1}]^T \) (\( L_{cp} \) is the length of Cyclic Prefix and if a CIR with length of \( L \) is considered, then \( L \leq L_{cp} \)) is the sparse channel vector; \( v_p = [v_{k_0}, v_{k_1}, \ldots, v_{k_{M-1}}]^T \) is the complex additive white Gaussian noise (AWGN) vector with zero mean and covariance matrix \( \sigma_n^2 I_M, \sigma_t^2 = N \sigma_n^2 \) [7], where \( \sigma_t^2 \) is the variance of the AWGN in time domain. The partial Fourier matrix \( F_{M \times L_{cp}} \) is obtained by selecting the rows of Fourier matrix with index \( k_0, k_1, \ldots, k_{M-1} \) and the first \( L_{cp} \) columns as follows:

\[
F_{M \times L_{cp}} = \begin{bmatrix}
W_N^{k_00} & W_N^{k_01} & \cdots & W_N^{k_0(L_{cp}-1)} \\
W_N^{k_10} & W_N^{k_11} & \cdots & W_N^{k_1(L_{cp}-1)} \\
\vdots & \vdots & \ddots & \vdots \\
W_N^{k_{M-1}0} & W_N^{k_{M-1}1} & \cdots & W_N^{k_{M-1}(L_{cp}-1)}
\end{bmatrix}
\]
where \( W_N^{kpq} = e^{-j2\pi kpq/N}, 0 \leq p \leq M - 1, 0 \leq q \leq L_{cp} - 1 \). Let \( A = X_p F_{M \times L_{cp}} \), (1) becomes
\[
y_p = Ah + v_p
\]
where \( A \) is generally known as measurement matrix.

The main task is to reconstruct the \( K (K \ll L_{cp}) \) sparse channel taps from a few number of pilots with almost no perceptual loss in performance.

An initial estimate of the channel vector \( h \) is firstly obtained by LS:
\[
\hat{h}_{LS} = (A^H A)^{-1} A^H y_p
\]
Combining (3) and (4), the following formula is obtained:
\[
\hat{h}_{LS} = (A^H A)^{-1} A^H Ah + (A^H A)^{-1} A^H v_p
\]

In this paper, we consider the case where \( L_{cp} \leq M < N \) (\( N \) is an integer multiple of \( M \)) and the pilots are uniformly distributed, in this case \( A^H A = MI_{L_{cp}} \), (5) can be rewritten as [7]:
\[
\hat{h}_{LS} = h + n
\]
where \( n = \frac{1}{M} A^H v_p \), it is still an AWGN vector, with covariance matrix \( C \) expressed as:
\[
C = E(nn^H) = \frac{1}{M} \sigma_n^2 I_{L_{cp}} = \frac{N}{M} \sigma_n^2 I_{L_{cp}}
\]
From (6) and (7), the estimated CIR by LS is highly affected by the noise when channel is sparse. Therefore, it is important to denoise the estimated CIR by an appropriate threshold. To do this, the universal threshold, firstly proposed in [11], and then widely used in compressed channel sensing [10, 14, 15] can be employed:
\[
\lambda = \sqrt{2 \ln L_{cp}} \sigma_n
\]
An accurate standard deviation \( \sigma_n \) of each element in the noise vector \( n \), is necessary in practical communication. However, it is difficult to obtain an effective estimate of standard deviation of each element in \( n \) when the noise vector \( n \) and the channel vector \( h \) are present together. In the following, we propose an efficient threshold based on the estimated CIR and the characteristics of the sparse channel.

The amplitude of complex white Gaussian noise \( n_i \) in \( i^{th} \) element of vector \( n \) follows Rayleigh distribution with standard deviation \( \sigma_n = \sqrt{2}\sigma \),
where $\sigma$ is the standard deviation of either the real or imaginary parts of $n_i$. The cumulative distribution function of Rayleigh distribution is expressed as:

$$F(x) = 1 - e^{-x^2/2\sigma^2}$$

(9)

When $F(x) = 0.5$, the corresponding value of $x$ is the median of $|n_i|$, therefore the relation between the standard deviation of $|n_i|$ and its median value can be written as [16]:

$$\sigma_n = \sqrt{2\sigma} = \sqrt{2^\frac{\text{median}(|n_i|)}{\ln 4}}$$

(10)

For sparse channel, the majority of coefficients in CIR are noise, therefore it is possible to obtain an approximated estimation of noise standard deviation $\hat{\sigma}_n'$:

$$\hat{\sigma}_n' = \sqrt{2\hat{\sigma}'} = \sqrt{2^\frac{\text{median}(|\hat{h}_{LS}|)}{\ln 4}}$$

(11)

However, the presence of channel taps results in bias on the estimated noise standard deviation, especially when the SNR is high and the channel is not sufficiently sparse. In general, we have $\hat{\sigma}_n' > \sigma_n$. If the majority number of channel taps in $\hat{h}_{LS}$ are removed, the remaining coefficients can be regarded approximately as noise coefficients, which can be used to get a better estimate of the noise standard deviation $\hat{\sigma}_n''$. With this estimated noise standard deviation $\hat{\sigma}_n''$, an effective threshold can be obtained.

3. Proposed Threshold for Sparse Channel Estimation

The main framework of the proposed threshold for sparse channel estimation is shown in Fig 1. The different steps are described as follows.

Step.1. LS is used to get an initial CIR estimate with length $L_{cp}$ (4).

Step.2. To get a good estimate of the noise standard deviation, a threshold is needed to eliminate the majority of channel taps in the initial estimated CIR. An initial rough estimate of noise standard deviation can be obtained from the coefficients of sparse channel vector $\hat{h}_{LS}$ by (11). Then, $T = \sqrt{2\ln L_{cp}}\hat{\sigma}_n'$ is used as a threshold to eliminate the majority of channel taps present in the estimated CIR. By comparing with $T$, the vector of noise coefficients (the coefficients $h_{LS}[j]$ with amplitude equal or smaller than $T$) denoted by $c$ ($c = [c[0], c[1], \ldots, c[L_{cp'} - 1]], L_{cp'} < L_{cp}$) is extracted.
Step 3. With the vector of noise coefficients $c$ (with no or much fewer channel taps than in $\hat{h}_{LS}$), $\hat{\sigma}_n''$ is estimated by $\hat{\sigma}_n'' = \sqrt{\frac{2}{\ln 4}} \sqrt{\text{median}(|c|)}$. Then, an effective threshold is obtained by $\eta = \sqrt{\frac{2 \ln L_{cp}}{L_{cp}}} \hat{\sigma}_n''$. The final estimated CIR is given by:

$$\hat{h}[n] = \begin{cases} 
\hat{h}_{LS}[n], & |\hat{h}_{LS}[n]| > \eta \\
0, & |\hat{h}_{LS}[n]| \leq \eta
\end{cases} \quad 0 \leq n \leq L_{cp} - 1$$

(12)

4. Simulations

In this section, simulations are carried out to evaluate the estimation performance of the proposed method and compare it with that of other existing methods. We consider two QPSK modulated OFDM systems for two different channel models:

1) An OFDM system with 1024 subcarriers, of which 64 subcarriers are pilots. The length of cyclic prefix is $L_{cp} = 64$.

2) An OFDM system with 1024 subcarriers, of which 256 subcarriers are pilots. The length of cyclic prefix is $L_{cp} = 256$.

For the first OFDM system, the channel model is a simplified version of DVB-T channel model whose channel impulse response is given in Table 1 [7].

For the second OFDM system, we use the ATTC (Advanced Television Technology Center) and the Grand Alliance DTV laboratory’s ensemble E model whose CIR is given by [7]:

$$h[n] = \delta[n] + 0.3162\delta[n-2] + 0.1995\delta[n-17] + 0.1296\delta[n-36] + 0.1\delta[n-75] + 0.1\delta[n-137].$$

(13)
Table 1: CIR for the first OFDM system

<table>
<thead>
<tr>
<th>Delay (OFDM samples)</th>
<th>Gain</th>
<th>Phase(rad)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.2478</td>
<td>-2.5694</td>
</tr>
<tr>
<td>1</td>
<td>0.1287</td>
<td>-2.1208</td>
</tr>
<tr>
<td>3</td>
<td>0.3088</td>
<td>0.3548</td>
</tr>
<tr>
<td>4</td>
<td>0.4252</td>
<td>0.4187</td>
</tr>
<tr>
<td>5</td>
<td>0.4900</td>
<td>2.7201</td>
</tr>
<tr>
<td>7</td>
<td>0.0365</td>
<td>-1.4375</td>
</tr>
<tr>
<td>8</td>
<td>0.1197</td>
<td>1.1302</td>
</tr>
<tr>
<td>12</td>
<td>0.1948</td>
<td>-0.8092</td>
</tr>
<tr>
<td>17</td>
<td>0.4187</td>
<td>-0.1545</td>
</tr>
<tr>
<td>24</td>
<td>0.3170</td>
<td>-2.2159</td>
</tr>
<tr>
<td>29</td>
<td>0.2055</td>
<td>2.8372</td>
</tr>
<tr>
<td>49</td>
<td>0.1846</td>
<td>2.8641</td>
</tr>
</tbody>
</table>

The coefficients in (13) and the gains in Table 1 represent the standard deviation of the corresponding zero mean complex Gaussian random variable. In the simulations, one OFDM sample period is assumed to be the same as the unit delay of the channel, the CIR is static for each OFDM symbol duration and each OFDM symbol has a newly generated Rayleigh channel. Additionally, the channel tap gains are obtained by multiplying the CIR coefficients with zero mean unit variance complex Gaussian random variables. Moreover, 10 OFDM symbols are considered for each iteration; and there are totally 800 iterations. Therefore, 8000 independent channel realizations have been considered in each simulation.

The simulations focus on the performance of bit error rate (BER) and normalized mean square error (NMSE) comparison between the proposed method, the classical method (frequency domain LS method with linear interpolation) [2, 3, 4, 5, 6], LS method with ideal channel knowledge (exact number of channel taps and their position), LS method with MST proposed by Minn et al [7] (MST method uses a fixed number of MST, which is double of the designed number of channel taps as recommended in [7]) and LS method with the threshold proposed by Kang et al [9] (for convenient comparisons, the exact noise standard deviation is used for the threshold proposed by Kang et al).

Fig 2 illustrates the BER performance of different algorithms of the first OFDM system. With only 6.25% of pilots, the proposed method outper-
Figure 2: BER performance comparison of the first OFDM system

forms the classical frequency domain method with 25.3% of pilots in the overall considered $E_b/N_0$. Meanwhile, with the same percentage (6.25%) of pilots, compared with LS method with the threshold proposed by Kang et al and LS method with MST proposed by Minn et al, the proposed method achieves the same BER performance with at least 1dB gain in high $E_b/N_0$ (13dB-30dB). Moreover, the proposed method has almost the same performance as LS method with ideal channel knowledge in the majority of considered $E_b/N_0$ (8dB-30dB). Additionally, LS method without threshold has the poorest performance, there is at least 2dB gap in $E_b/N_0$ between LS method without threshold and LS method with ideal channel knowledge for a same BER. Comparatively, the known CSI (instantaneous channel frequency response is known) has the best BER performance, however, for the majority of considered $E_b/N_0$ (8dB-30dB), the performance gap between the proposed method and known CSI is less than 1dB in $E_b/N_0$.

In the following, we consider the NMSE of the channel frequency response defined by [17]:

$$NMSE = \frac{E[\|g - \hat{g}\|_2^2]}{E[\|g\|_2^2]}$$  \hspace{1cm} (14)$$

where $g = [g_0, g_1, ..., g_{N-1}]^T$, $g_k = \sum_{l=0}^{L_{cp}-1} h_l e^{-j2\pi lk/N}$, $0 \leq k \leq N - 1$ and $\hat{g}$ is
an estimate of $g$.

Fig 3 shows that the NMSE performance of the first OFDM system has similar trends as that of the BER performance except that the differences between algorithms are much more obvious. For example, when the NMSE reaches $10^{-3}$, there is an approximate 4dB gains in $E_b/N_0$ for the proposed method compared with LS method with threshold proposed by Kang et al.

In Fig 4, the NMSE performance of different algorithms of the second OFDM system is compared. The proposed method maintains at least 4dB performance advantage in the all considered $E_b/N_0$ for a same NMSE compared with LS method with MST proposed by Minn et al and LS method with the threshold proposed by Kang et al, which is bigger than in the first OFDM system. Furthermore, in the second OFDM system, the NMSE performance gap between the proposed method and LS method with ideal channel knowledge is slightly smaller than in the first OFDM system due to more accurate estimation on noise standard deviation and changes on $\sigma_n$ (See (7), the $\sigma_n$ will be reduced with the increase of the number of pilots).

The noise standard deviation estimation plays a central role in the proposed method. In order to show the performance of the proposed noise standard deviation estimation method, we consider the absolute relative error on the estimated standard deviation $\varepsilon = \frac{|\hat{\sigma}_n - \sigma_n|}{|\sigma_n|}$ ($\sigma_n$ is the true noise
standard deviation for each element of vector $n$ and $\hat{\sigma}_n$ is the estimated one) of different estimation methods for both DVB and DTV channels with 8000 independent channel realizations. As can be seen from Table 2, for both channel models, when $E_b/N_0$ increases, there is slight changes on the relative estimation error of standard deviation estimation for both the median based estimation method $\hat{\sigma}'_n = \sqrt{2 \over \text{median}(|\hat{h}_{LS}|)}$ in formula (11) (used in the first threshold to eliminating the majority of channel taps) and the proposed method (the improved standard deviation estimation after eliminating the majority of channel taps). We can also see that the relative estimation error of the proposed method is smaller than the median based estimation method. Furthermore, the classical standard deviation estimation method $\hat{\sigma}''_n = \sqrt{1 \over L_{cp}} \sum_{i=0}^{L_{cp}-1} |\hat{h}_{LS}[i] - \text{mean}(\hat{h}_{LS})|^2$ and Bayesian estimation method [18] provide biased estimation, especially for high $E_b/N_0$, due to the presence of non-zero channel taps.

From the above analysis of simulation results, we observe that even though the sparsity rate [13] $K \over T_{cp}$ has changed significantly (2.34% for the second channel and 18.75% for the first channel), the proposed method still maintains good performance of both the noise standard deviation estimation and channel estimation. Therefore, we can draw the conclusion that even without prior knowledge of channel statistics and noise standard deviation,
Table 2: Absolute relative error of different noise standard deviation estimation methods

<table>
<thead>
<tr>
<th>Channel Model</th>
<th>DVB Model</th>
<th>DTV Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>Proposed method</td>
<td>0.0860</td>
<td>0.0808</td>
</tr>
<tr>
<td>Median method</td>
<td>0.1650</td>
<td>0.1785</td>
</tr>
<tr>
<td>Classical method</td>
<td>3.3607</td>
<td>12.4768</td>
</tr>
<tr>
<td>Bayesian method</td>
<td>3.5083</td>
<td>12.9357</td>
</tr>
</tbody>
</table>

The proposed method can still work within a wide range of sparsity rate.

Table 3: (Complex) Computational complexity comparison

<table>
<thead>
<tr>
<th>Alg</th>
<th>Proposed Method</th>
<th>Classical Method</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LS+Thr</td>
<td>FFT</td>
</tr>
<tr>
<td>Comp</td>
<td>$O(\frac{M}{2} \log_2 M + M)$</td>
<td>$O(\frac{N}{2} \log_2 N)$</td>
</tr>
</tbody>
</table>

In Table 3, the computational complexity (for simplicity only complex multiplications are considered) comparison for the proposed method and the classical frequency domain method is presented. The proposed method is composed of three algorithms, which are LS algorithm (time domain), threshold estimation algorithm and FFT algorithm. The LS and threshold estimation have a total complexity of $O(\frac{M}{2} \log_2 M + M)$ (in the case of this paper, LS algorithm can mainly be realized by a $M$ size IFFT computation), while the FFT method has complexity of $O(\frac{N}{2} \log_2 N)$. Therefore, the total computational complexity of the proposed method is around $O(N \log_2 N)$. Obviously, the proposed method has higher computational complexity than that of the classical method, which is the combination of LS (frequency domain) and linear interpolation algorithms and has computational complexity of $O(N)$. However, for the same performance of BER and NMSE the proposed method allows to use much fewer pilots than that of the classical frequency domain method, thus it has better spectral efficiency.

5. Conclusion

Channel estimation methods with good estimation performance without requiring prior knowledge of channel statistics and noise standard deviation will significantly benefit the practical wireless communications. In this paper, an effective sparse channel estimation method based on LS is proposed in OFDM system. In this method, a novel effective time domain threshold
depending only on the effective noise standard deviation estimated from the noise coefficients obtained by eliminating the channel coefficients with an initial estimated threshold is proposed. Both theoretical analysis and simulation results show that the proposed method can achieve better performance in both BER and NMSE than the compared methods within a wide range of sparsity rate, has good spectral efficiency and moderate computational complexity. The proposed two-step threshold estimation technique is general, other threshold, like the suboptimal threshold proposed in [12] can be used in the same way as the universal threshold used in this paper.

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References


