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J.A. Otero[a*], R. Rodríguez-Ramos[b], G. Monsivais[c], C. Stern[d] and F. Lebon[e]

[a]Instituto de Cibernética, Matemática y Física (ICIMAF), 15 No. 551, Vedado, C.P. 10-400, La Habana, Cuba; [b]Facultad de Matemática y Computación, Universidad de La Habana, San Lazaro y L, Vedado, C.P. 10-400, La Habana, Cuba; [c]Instituto de Física, Universidad Nacional Autónoma de México, A.P. 20-364, 01000 México, D.F, México; [d]Facultad de Ciencias, Universidad Nacional Autónoma de México, D.F., 04510, México; [e]Laboratoire de Mécanique et d'Acoustique, Université Aix-Marseille 1, 31 Chemin Joseph-Aiguier, 13402 Marseille Cedex 20, France

We study the propagation of shear horizontal waves between the interface of piezoelectric and piezomagnetic half-spaces with a magneto-electro-mechanical imperfect contact. Mechanical, electrical and magnetical imperfections are modelled by means of a spring, a capacitor and a inductor, respectively. A general expression for the dispersion relation not reported previously in literature is given in an explicit form, with the diverse limit cases analysed in detail. In some of these limit cases, new expressions are also obtained which predict the existence of interfacial waves. In the other cases, when already reported results exist, a comparison with them is done. Some physical interpretations are derived from the limit cases. The influence of mechanical, electrical and magnetical imperfect contacts are shown in some numerical examples.

Keywords: interfacial wave; dispersion relation; imperfect bonding; piezoelectric; piezomagnetic

Recently, Yang Z. and Yang J. [1] and Nan et al. [2] studied the shear horizontal piezoelectric waves propagating along the interface between two semi-infinite half-spaces. The propagation of an interfacial shear wave in two bonded semi-infinite piezoelectric and piezomagnetic materials with a perfect interface has been studied by Soh and Liu [3]. The existence of certain waves propagating near an imperfectly bonded interface between two half-spaces of different piezoelectric ceramics has been studied by Fan, Yang and Xu [4]. In Ref. [4], imperfection is considered by means of a discontinuity of the displacement (spring model). The influence of elastic imperfect bonding on interface waves guided by piezoelectric/piezomagnetic composites has been presented by Melkumyan and Mai [5] and Huang et al. [6]. Besides, some other works related to interface waves in composite materials with elastic imperfect contact at the interface have been published by Darinskii and Weihnacht [7], Bai et al. [8]. The imperfect bonding sometimes exists in devices, but little is known about its influence on the wave propagations in it. Otero et al. [9] studied dispersion relations of surface SH-waves in heterostructures with elastic imperfect bonding at the

*Corresponding author. Email: jaotero@icimaf.cu

interface. The mechanical imperfection of the interface should also induce electric and/or magnetic defects in the interface piezoelectric/piezomagnetic due to their magneto-electro-mechanical coupling. More recently, the propagation of shear horizontal waves between the interface of two piezoelectric material with an electro-mechanical imperfect contact has been studied by Otero et al. [10]. Here, the mechanical and electrical imperfections have been modelled by means of a spring and capacitor ([11]), respectively, where the imperfect interface affects the interfacial wave.

In the present work, we have considered the existence of magnetic, electric and mechanic imperfections at the interface between piezoelectric and piezomagnetic half-spaces. The presence of magnetical [12], electrical [11] and mechanical imperfections are modelled by means of an inductor, a capacitor and a spring, respectively. The capacitance (inductance) of the capacitor (inductor) is a measure of the electrical (magnetical) imperfection.

Let us consider piezoelectric and piezomagnetic half-spaces with an interface. The xz plane is the interface at $y = 0$. The space $y > 0$ is occupied by piezoelectric and the space $y < 0$ by piezomagnetic. Both materials have 6mm hexagonal symmetry and polarization in the z-axis direction. For the case of shear horizontal (SH) waves, the displacement vector $\mathbf{u} = (u_x, u_y, u_z)$, the electric potential φ and the magnetic potential ψ are given by $u_x = u_y = 0$, $u_z = u(x, y, t)$, $\varphi = \varphi(x, y, t)$ and $\psi = \psi(x, y, t)$. Auxiliary functions $\bar{\varphi}$ and $\bar{\psi}$ can be introduced by $\varphi = \bar{\varphi} + eu/\varepsilon$ and $\psi = \bar{\psi} + fu/\mu$, where $e = e_{15}$, $f = f_{15}$, $\varepsilon = \varepsilon_{11}$ and $\mu = \mu_{11}$ are the piezoelectric, piezomagnetic, dielectric permittivity, magnetic permeability, respectively. The governing equations for SH waves in the half-spaces are [4–6]

$$\begin{aligned} \bar{c}_A \nabla^2 u_A = \rho_A \ddot{u}_A, \quad \nabla^2 \bar{\varphi}_A = 0, \quad \nabla^2 \psi_A = 0, \qquad y > 0, \\ \bar{c}_B \nabla^2 u_B = \rho_B \ddot{u}_B, \quad \nabla^2 \varphi_B = 0, \quad \nabla^2 \bar{\psi}_B = 0, \qquad y < 0, \end{aligned} \tag{1}$$

where $\bar{c}_A = c_A + e_A^2/\varepsilon_A$, $\bar{c}_B = c_B + f_B^2/\mu_B$ and $c = c_{44}$ is the elastic constant. $\nabla^2 \equiv \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the two-dimensional Laplacian. The subscript A (B) indicate quantities in piezoelectric (piezomagnetic) half-space. For interfacial wave, it is required that u, ϕ and ψ vanish at $y = \pm\infty$; which in turn implies that there are solutions of Equation (1) of the form

$$\begin{aligned} u_A &= U_A \exp(-\eta_A y) \cos(\xi x - \omega t), \qquad y > 0, \\ \bar{\varphi}_A &= \Phi_A \exp(-\xi y) \cos(\xi x - \omega t), \qquad y > 0, \\ \psi_A &= \Psi_A \exp(-\xi y) \cos(\xi x - \omega t), \qquad y > 0, \end{aligned} \tag{2}$$

$$\begin{aligned} u_B &= U_B \exp(\eta_B y) \cos(\xi x - \omega t), \qquad y < 0, \\ \phi_B &= \Phi_B \exp(\xi y) \cos(\xi x - \omega t), \qquad y < 0, \\ \bar{\psi}_B &= \Psi_B \exp(\xi y) \cos(\xi x - \omega t), \qquad y < 0, \end{aligned} \tag{3}$$

where U_A, Φ_A, Ψ_A, U_B, Φ_B and Ψ_B are undetermined constants, ω and ξ are the frequency and wave number, respectively, and

$$\eta_A = \xi \sqrt{1 - \frac{v^2}{v_A^2}} > 0, \qquad \eta_B = \xi \sqrt{1 - \frac{v^2}{v_B^2}} > 0, \tag{4}$$

where $v_A = \sqrt{\bar{c}_A/\rho_A}$ $\left(v_B = \sqrt{\bar{c}_B/\rho_B}\right)$ is the bulk shear wave speed in piezoelectric (piezomagnetic) half-space and $v = \omega/\xi$ is the phase velocity. The stress component $T \equiv T_{zy}(x, y, t)$, electric displacement $D \equiv D_y(x, y, t)$ and magnetic induction $B \equiv B_y(x, y, t)$ in piezoelectric (piezomagnetic) half-space are denoted by T_A (T_B), D_A (D_B) and B_A (B_B), respectively; and can be expressed by

$$
\begin{aligned}
T_A = \bar{c}_A \frac{\partial u_A}{\partial y} + e_A \frac{\partial \bar{\varphi}}{\partial y} &= -\left(\bar{c}_A \eta_A U_A \exp(-\eta_A y) + e_A \xi \Phi_A \exp(-\xi y)\right) \cos(\xi x - \omega t), \\
D_A = -\varepsilon_A \frac{\partial \bar{\varphi}_A}{\partial y} &= \varepsilon_A \xi \Phi_A \exp(-\xi y) \cos(\xi x - \omega t), \\
B_A = -\mu_A \frac{\partial \psi_A}{\partial y} &= \mu_A \xi \Psi_A \exp(-\xi y) \cos(\xi x - \omega t),
\end{aligned}
\tag{5}
$$

$$
\begin{aligned}
T_B = \bar{c}_B \frac{\partial u_B}{\partial y} + f_B \frac{\partial \bar{\psi}}{\partial y} &= \left(\bar{c}_B \eta_B U_B \exp(\eta_B y) + f_B \xi \Psi_B \exp(\xi y)\right) \cos(\xi x - \omega t), \\
D_B = -\varepsilon_B \frac{\partial \varphi_B}{\partial y} &= -\varepsilon_B \xi \Phi_B \exp(\xi y) \cos(\xi x - \omega t), \\
B_B = -\mu_B \frac{\partial \bar{\psi}_B}{\partial y} &= -\mu_B \xi \Psi_B \exp(\xi y) \cos(\xi x - \omega t).
\end{aligned}
\tag{6}
$$

As mentioned, we consider that the condition for the magneto-electro-mechanical imperfect contact at $y = 0$ is given by [4,11,12]

$$
\begin{aligned}
T_A = T_B &= K_u (u_A - u_B), \\
D_A = D_B &= K_\varphi \left(\varphi_B - \bar{\varphi}_A - \tfrac{e_A}{\varepsilon_A} u_A \right), \\
B_A = B_B &= K_\psi \left(\bar{\psi}_B + \tfrac{f_B}{\mu_B} u_B - \varphi_A \right).
\end{aligned}
\tag{7}
$$

The first conditions in (7) describe an elastic interface with spring imperfect parameter K_u, in other words imperfection takes place only in the form of displacement discontinuity. Perfect contact is revealed when the spring imperfect parameter approaches to infinity, while the debonding contact takes place as this imperfect parameter approaches to zero. It may look natural that the mechanical weakening of the interface should also induce the decrease of the electric and/or magnetic contacts due to their magneto-electro-mechanical coupling. The second condition in (7) describes an electric interface with electric capacitor parameter K_φ, i.e. electric potential at the interface is discontinuous. The capacitor represents an interface electric barrier. The capacity of the capacitors determines the degree of the electrical imperfection. When the capacity is equal to zero, the interface is a complete electric barrier. As the capacity increases the degree of electrical imperfection decreases, and perfect electrical interface prevails as the capacity approaches infinity. The third condition in (7) describes a magnetic interface with magnetic inductor parameter K_ψ, i.e. magnetic potential at the interface is discontinuous. The inductance of the inductors determines the degree of the magnetical imperfection. When the inductance is equal to zero, the interface is a complete magnetic barrier. As the inductance increases the degree of magnetical imperfection decreases, and perfect magnetical interface prevails as the inductance approaches infinity.

A substitution of (2), (3), (5) and (6) into (7) yields to a homogeneous system of linear algebraic equations for U_A, U_B, Φ_A, Φ_B, Ψ_A and Ψ_B. For non-trivial solutions, the determinant of the principal matrix must be equal to zero, which leads to the following dispersion relation

$$
P_A\, P_B + K_u\ (P_A + P_B) = 0,
\tag{8}
$$

where

$$P_A = \bar{c}_A \left(\eta_A - \xi \Upsilon_\varphi \frac{\bar{K}_A^2}{\varepsilon_A} \right), \quad \bar{K}_A^2 = \frac{e_A^2}{\bar{c}_A \varepsilon_A},$$
$$P_B = \bar{c}_B \left(\eta_B - \xi \Upsilon_\psi \frac{\bar{K}_B^2}{\mu_B} \right), \quad \bar{K}_B^2 = \frac{f_B^2}{\bar{c}_B \mu_B}, \tag{9}$$
$$\Upsilon_\varphi = \frac{K_\varphi}{K_\varphi \left(\frac{1}{\varepsilon_A} + \frac{1}{\varepsilon_B} \right) + \xi}, \quad \Upsilon_\psi = \frac{K_\psi}{K_\psi \left(\frac{1}{\mu_A} + \frac{1}{\mu_B} \right) + \xi}.$$

Expression (8) determines the phase velocity v and since it changes with frequency, the waves are dispersive. Note that $K_u/\bar{c}$, K_φ/ε and K_ψ/μ have dimensions of wave number (1/m).

Let us consider the following limit cases:

(i) $0 < K_u < \infty$, $K_\varphi \to \infty$ and $K_\psi \to \infty$. The interface has perfect electrical and magnetical interactions and partial mechanic interaction. Equation (8) reduces to

$$\xi \left(\lambda_A - \frac{\varepsilon_B}{\varepsilon_A + \varepsilon_B} \bar{K}_A^2 \right) \left(\lambda_B - \frac{\mu_A}{\mu_A + \mu_B} \bar{K}_B^2 \right) \bar{C}_A \bar{C}_B$$
$$+ K_u \left[\left(\lambda_A - \frac{\varepsilon_B}{\varepsilon_A + \varepsilon_B} \bar{K}_A^2 \right) \bar{C}_A + \left(\lambda_B - \frac{\mu_A}{\mu_A + \mu_B} \bar{K}_B^2 \right) \bar{C}_B \right] = 0, \tag{10}$$

where $\lambda_A = \sqrt{1 - v^2/v_A^2}$ and $\lambda_B = \sqrt{1 - v^2/v_B^2}$. This result appears in [5].

(ii) $K_u \to 0$ and $0 < K_\varphi, K_\psi < \infty$. The interface has partial electric and magnetic interactions but has no mechanical interaction. The two roots of (8) are

$$v = v_A \sqrt{1 - \left(\frac{\bar{K}_A^2 \Upsilon_\varphi}{\varepsilon_A} \right)^2}, \quad v = v_B \sqrt{1 - \left(\frac{\bar{K}_B^2 \Upsilon_\psi}{\mu_B} \right)^2}. \tag{11}$$

As far as the authors know this is a new result.

(iii) $K_u \to 0$ and $K_\varphi \to \infty$. The interface has perfect electric interaction but has no mechanical interaction (mechanical debonding). Equation (11.1) reduces to

$$v \equiv v_{AL} = v_A \sqrt{1 - \frac{\bar{K}_A^4 \varepsilon_B^2}{(\varepsilon_A + \varepsilon_B)^2}}. \tag{12}$$

This result appears in [13].

(iv) $K_u \to 0$ and $K_\varphi \to 0$. In this case, the interface has no mechanical and electrical interaction (electro-mechanical debonding). Equation (11.1) reduces to $v^2 = v_A^2$, which do not correspond to an interfacial wave because it is not an evanescent wave towards $\pm\infty$.

(v) $K_u \to 0$ and $K_\psi \to \infty$. The interface has perfect magnetic interaction but has no mechanical interaction. Equation (11.2) reduces to

$$v \equiv v_{BL} = v_B \sqrt{1 - \frac{\bar{K}_B^4 \mu_A^2}{(\mu_A + \mu_B)^2}}. \tag{13}$$

This result appears in [13].

(vi) $K_u \to 0$ and $K_\psi = 0$. In this case, the interface has no mechanical and magnetical interaction (magneto-mechanical debonding). Equation (11.2) reduces to $v^2 = v_B^2$, which do not correspond to an interfacial wave.

(vii) $K_u \to \infty$ and $0 < K_\varphi, K_\psi < \infty$. The interface has perfect mechanical interaction and partial electric and magnetic interactions. Expression (8) reduces to

$$\bar{C}_A\sqrt{1-\frac{v^2}{v_A^2}} + \bar{C}_B\sqrt{1-\frac{v^2}{v_B^2}} = \frac{e_A^2}{\varepsilon_A^2}\Upsilon_\varphi + \frac{f_B^2}{\mu_B^2}\Upsilon_\psi. \tag{14}$$

We can see that the left-hand side of (14) is monotonically decreasing with respect to v. Therefore, this term reaches its maximum $\bar{C}_A + \bar{C}_B$ at $v = 0$ and its minimum $\bar{C}_B\sqrt{1-(v_A/v_B)^2}$ at $v = v_A$ when $v_A < v_B$. In case that $v_B < v_A$, we obtain the minimum $\bar{C}_A\sqrt{1-(v_B/v_A)^2}$ at $v = v_B$. Consequently, for the existence of the interfacial wave the following conditions must be satisfied:

$$\bar{C}_B\sqrt{1-\frac{v_A^2}{v_B^2}} < \frac{e_A^2}{\varepsilon_A^2}\Upsilon_\varphi + \frac{f_B^2}{\mu_B^2}\Upsilon_\psi < \bar{C}_A + \bar{C}_B, \quad v_A < v_B, \tag{15}$$

$$\bar{C}_A\sqrt{1-\frac{v_B^2}{v_A^2}} < \frac{e_A^2}{\varepsilon_A^2}\Upsilon_\varphi + \frac{f_B^2}{\mu_B^2}\Upsilon_\psi < \bar{C}_A + \bar{C}_B, \quad v_B < v_A. \tag{16}$$

(viii) $K_u \to \infty$, $K_\varphi \to \infty$ and $K_\psi \to \infty$. The interface has perfect mechanical, electrical and magnetical interaction (perfect bonding). Expression (8) reduces to

$$\bar{C}_A\sqrt{1-\frac{v^2}{v_A^2}} + \bar{C}_B\sqrt{1-\frac{v^2}{v_B^2}} = \frac{e_A^2}{\varepsilon_A}\frac{\varepsilon_B}{\varepsilon_A+\varepsilon_B} + \frac{f_B^2}{\mu_B}\frac{\mu_A}{\mu_A+\mu_B}. \tag{17}$$

This expression appears in [3,13]. The conditions for the existence of the interfacial wave are

$$\bar{C}_B\sqrt{1-\frac{v_A^2}{v_B^2}} < \frac{e_A^2}{\varepsilon_A}\frac{\varepsilon_B}{\varepsilon_A+\varepsilon_B} + \frac{f_B^2}{\mu_B}\frac{\mu_A}{\mu_A+\mu_B} < \bar{C}_A + \bar{C}_B, \quad v_A < v_B, \tag{18}$$

$$\bar{C}_A\sqrt{1-\frac{v_B^2}{v_A^2}} < \frac{e_A^2}{\varepsilon_A}\frac{\varepsilon_B}{\varepsilon_A+\varepsilon_B} + \frac{f_B^2}{\mu_B}\frac{\mu_A}{\mu_A+\mu_B} < \bar{C}_A + \bar{C}_B, \quad v_B < v_A. \tag{19}$$

(ix) $K_u \to \infty$, $K_\varphi \to 0$ and $K_\psi \to 0$. The interface has perfect mechanical interaction but has no electrical and magnetical interaction (magnetical and electrical debonding). In this case, $v^2 = v_A^2$ and $v^2 = v_B^2$.

(x) $K_u \to \infty$, $K_\varphi \to \infty$ and $K_\psi \to 0$. The interface has perfect mechanical and electrical interaction but has no magnetical interaction. Expression (8) reduces to

$$\bar{C}_A\sqrt{1-\frac{v^2}{v_A^2}} + \bar{C}_B\sqrt{1-\frac{v^2}{v_B^2}} = \frac{e_A^2}{\varepsilon_A}\frac{\varepsilon_B}{\varepsilon_A+\varepsilon_B}. \tag{20}$$

As far as the authors know this is a new result. The conditions for the existence of the interfacial wave are

$$\bar{C}_B\sqrt{1-\frac{v_A^2}{v_B^2}} < \frac{e_A^2}{\varepsilon_A}\frac{\varepsilon_B}{\varepsilon_A+\varepsilon_B} < \bar{C}_A+\bar{C}_B, \quad v_A < v_B, \tag{21}$$

$$\bar{C}_A\sqrt{1-\frac{v_B^2}{v_A^2}} < \frac{e_A^2}{\varepsilon_A}\frac{\varepsilon_B}{\varepsilon_A+\varepsilon_B} < \bar{C}_A+\bar{C}_B, \quad v_B < v_A. \tag{22}$$

(xi) $K_u \to \infty$, $K_\varphi \to 0$ and $K_\psi \to \infty$. The interface has perfect mechanical and magnetical interaction but has no electrical interaction. Expression (8) reduces to

$$\bar{C}_A\sqrt{1-\frac{v^2}{v_A^2}}+\bar{C}_B\sqrt{1-\frac{v^2}{v_B^2}} = \frac{f_B^2}{\mu_B}\frac{\mu_A}{\mu_A+\mu_B}. \tag{23}$$

As far as the authors know this is a new result. The conditions for the existence of the interfacial wave are

$$\bar{C}_B\sqrt{1-\frac{v_A^2}{v_B^2}} < \frac{f_B^2}{\mu_B}\frac{\mu_A}{\mu_A+\mu_B} < \bar{C}_A+\bar{C}_B, \quad v_A < v_B, \tag{24}$$

$$\bar{C}_A\sqrt{1-\frac{v_B^2}{v_A^2}} < \frac{f_B^2}{\mu_B}\frac{\mu_A}{\mu_A+\mu_B} < \bar{C}_A+\bar{C}_B, \quad v_B < v_A. \tag{25}$$

In order to illustrate the effect of the mechanical, electrical and magnetical imperfections in the propagation of the wave, we study the interfacial shear horizontal waves between the interface of two half-spaces.

In Figure 1, numerical results of Equations (11.1) and (11.2) are presented to illustrate dispersion curves $v = v(\xi)$. Under the above considerations, the medium A is $BaTiO_3$ with material parameters $c_A = 4.39 \times 10^{10}$ N/m^2, $e_A = 11.4$ C/m^2, $\varepsilon_A = 9.82 \times 10^{-11}$ F/m, $\mu_A = 5 \times 10^{-6}$ Ns^2/C^2, $\rho_A = 5.7 \times 10^3$ kg/m^3 and $v_A = 3165.998$ m/s. The medium B is $CoFe_2O_4$ and the values of the material constants are $c_B = 45.3 \times 10^9$ N/Am, $f_B = 550$ N/m^2, $\varepsilon_B = 0.08 \times 10^{-9}$ C/m^2, $\mu_B = 1.57 \times 10^{-4}$ Ns^2/C^2, $\rho_B = 5.3 \times 10^3$ kg/m^3 and $v_B = 2985.081$ m/s. The phase velocity given by Equation (11.1) depends on the electric imperfect parameter (K_φ). Figure 1a shows the phase velocity for some values of K_φ/ε_A (1/m) $= 0.2, 0.5, 1, 5, 10, 20, 50, 100$. The dispersion curves related to these cases are caught between the phase velocities $v = v_A$ for K_φ/ε_A (1/m) $= 0$ (Case iv) and $v = v_{AL} = 3165.992977$ m/s for $K_\varphi/\varepsilon_A \to \infty$ (Case iii). The dispersion curves start from the same initial point because the phase velocity given by the Equation (11.1) for $\xi = 0$ is independent of the electric imperfect parameter when it is different from zero. All the curves of Figure 1a are above of v_B then the solution of Equation (11.1) does not describe interfacial waves. On the other hand, the phase velocity given by Equation (11.2) depends on the magnetic imperfect parameter (K_ψ). We have taken the following values K_ψ/μ_A (1/m) $= 0.2, 0.5, 1, 5, 10, 20, 50, 100$ in Figure 1b for the illustration. Notice that the curves are restricted between the values $v = v_B$ for K_ψ/μ_A (1/m) $= 0$ (Case vi) and $v = v_{BL} = 2985.078511$ m/s for $K_\psi/\mu_A \to \infty$ (Case v). Therefore, the solution given by Equation (11.2) represents interfacial waves.

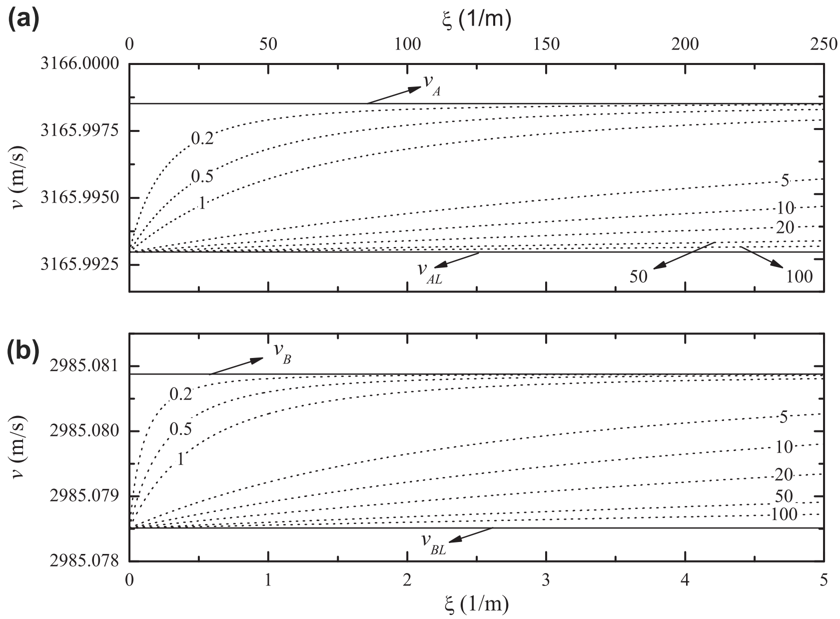

Figure 1. Variation of the phase velocity (v) vs. wave number (ξ) for the elastic imperfect parameter equal to zero and different values K_φ/ε_A (1/m) $= 0.2, 0.5, 1, 5, 10, 20, 50, 100$ (a) and K_ψ/μ_A (1/m) $= 0.2, 0.5, 1, 5, 10, 20, 50, 100$ (b).

Now, our intention is to study numerically the solution of Equation (14) for perfect mechanic with electric and magnetic imperfect contacts (Case vii). The condition given by Equation (16) fails for the above used materials and therefore there are not interfacial waves (see Table 1). Since the presence of interfacial shear horizontal waves between the interface of piezoelectric and piezomagnetic half-spaces for perfect mechanic with electric and magnetic imperfect contacts is not manifested with the conventional materials ($BaTiO_3$ and $CoFe_2O_4$), we analyse numerically the interfacial shear horizontal waves between two half-spaces with speeds very near (where the left-hand side of condition (16) is very small). For that purpose, we consider the medium A as $BaTiO_3$ and we choose a medium B (fictitious medium) under the following condition: the speeds are $v_B = \gamma v_A$, which can be obtained from the relations $c_B = \gamma^2 c_A$, $f_B^2/\mu_B = \gamma^2 e_A^2/\varepsilon_A$ and $\rho_B = \rho_A$, where γ is a number close to 1. Under the aforementioned conditions, the material parameters of the medium B are $\varepsilon_B = \varepsilon_A$, $\mu_B = \mu_A$, $\rho_B = \rho_A$, $c_B = \gamma^2 c_A$ and $f_B = \gamma e_A \sqrt{\mu_B/\varepsilon_A}$. In the calculations, we use $\gamma = 0.999$. The dispersion curves $v = v(\xi)$ obtained from Equation (14) are shown in Figure 2.

Firstly, the value K_ψ/μ_A (1/m) $= 10^4$ is fixed (magnetic perfect contact) and the electric parameter is varied for different values K_φ/ε_A (1/m) $= 0.5, 1, 5, 10, 20, 30, 50, 10^2, 10^3, 10^4$. Secondly, the electric imperfection is selected as K_φ/ε_A (1/m) $= 10^4$ (electric perfect contact) and the magnetic imperfection takes different values, for instance, K_ψ/μ_A (1/m) $= 0.5, 1, 5, 10, 20, 30, 50, 10^2, 10^3, 10^4$. The dispersion curves are very close each other for the particular case $K_\varphi/\varepsilon_A = K_\psi/\mu_A$ and they are monotonically

Table 1. Numerical proof of the condition given in Equation (16) for $BaTiO_3/CoFe_2O_4$ half-spaces.

ξ (1/m)	$\frac{K_\varphi}{\varepsilon_A}$ (1/m)	$\frac{K_\psi}{\mu_A}$ (1/m)	$\bar{C}_A\sqrt{1-\frac{v_B^2}{v_A^2}}$ (N/m^2)	$\frac{e_A^2}{\varepsilon_A^2}\Upsilon_\varphi + \frac{f_B^2}{\mu_B^2}\Upsilon_\psi$ (N/m^2)	$\bar{C}_A + \bar{C}_B$ (N/m^2)
10	0.5	10^4	1.9037×10^{10}	1.5152×10^8	1.0436×10^{11}
10	1.0	10^4	1.9037×10^{10}	1.5841×10^8	1.0436×10^{11}
10	5	10^4	1.9037×10^{10}	1.6470×10^8	1.0436×10^{11}
10	10	10^4	1.9037×10^{10}	1.6555×10^8	1.0436×10^{11}
10	20	10^4	1.9037×10^{10}	1.6597×10^8	1.0436×10^{11}
10	30	10^4	1.9037×10^{10}	1.6612×10^8	1.0436×10^{11}
10	50	10^4	1.9037×10^{10}	1.6623×10^8	1.0436×10^{11}
10	10^2	10^4	1.9037×10^{10}	1.6632×10^8	1.0436×10^{11}
10	10^3	10^4	1.9037×10^{10}	1.6640×10^8	1.0436×10^{11}
10	10^4	10^4	1.9037×10^{10}	1.6641×10^8	1.0436×10^{11}

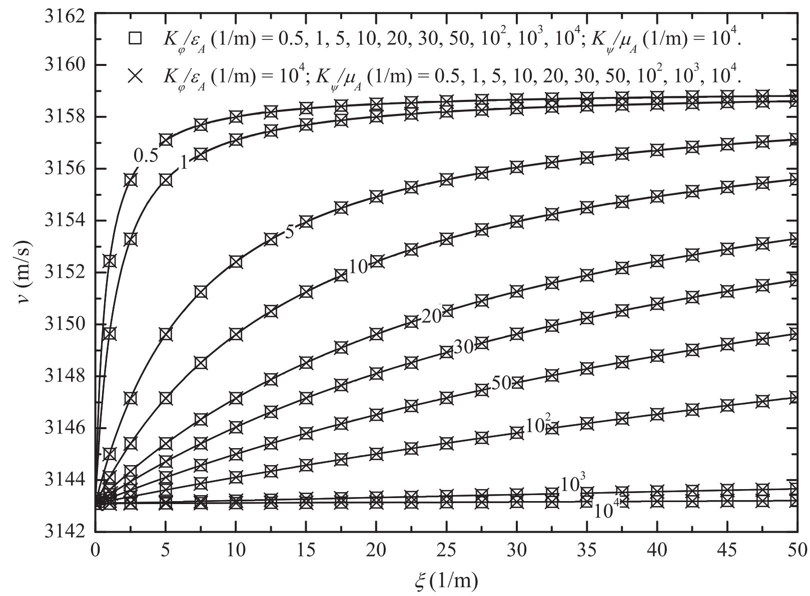

Figure 2. Variation of the phase velocity (v) vs. wave number (ξ) for the elastic imperfect parameter equal to $K_u/c_A \to \infty$ and different values of electric and magnetic imperfect parameters.

increasing functions. These curves describe interfacial waves since they are inferior to the velocity of the medium B.

Finally, a strongly influence of the mechanic, electric and magnetic imperfection adherences is observed on the presence of interfacial waves between piezoelectric and piezomagnetic half-spaces.

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