Virtual charts for shape optimization of structures

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Abstract. *The paper presents the concept of "Virtual Charts" applied to computational structural mechanics. Based on the Proper Generalized Decomposition (PGD), a model reduction method, the idea is to create a numerical tool to help engineers to perform shape optimization, where quantities of interest, computed for all the geometrical configurations, will be stored. Two different approaches have been developed in order to introduce geometry variations as parameters in the PGD. Promising results were obtained for academic examples.*

Keywords: pgd; virtual chart; shape optimization; model reduction.

1 INTRODUCTION

Despite constant progress in computing power over the past years, experimental testing is still essential during design stage of industrial structures because numerical resolution of high sized complex models often remains out of reach. And even when it is possible, each new structure is considered as a new problem, and treated independently of cases previously studied. Therefore, a very large number of simulations have to be computed. The use of experimental testing combined to the calculation of many simulations increases time and financial costs. Hence, the reduction of these costs is a crucial industrial issue.

The idea, developed here, consists in putting together similar structures, which differ from each other for the values given to some parameters, in "families" and to compute offline the general parametrized solution for each family. The quantities of interest, used for structural design, are stored in "Virtual Charts", which will be used online by the engineer during the conception step. Hence the conception time will be highly reduced due to the fact that all is remaining for the engineer is to particularize the general solution with the values of the considered parameters.

The construction of these charts is based on the model reduction method called Proper Generalized Decomposition (PGD) introduced for the first time in 1985 by P. Ladevèze under the name "Radial Approximation" [1] in order to deal with time-dependent non-linear problems. It allows to generate the solution of a problem for a whole set of parameters and its efficiency was proven for time-space [2], multiscale [3], multiphysics [4] and parametrized problems [5]. In the framework of this study in partnership with ASTRIUM-ST, the virtual charts are created to take into account geometry variations, which are a key point of the conception process. The geometry variations are considered as parameters in the PGD method and the quantities of interest are computed from the resolution of adjoint problems setting up some error estimators that allow to master the simulations.

First of all, after a short review on the PGD method, we present two different strategies developed to adapt the Proper Generalized Decompostion to geometry variations and then the capabilities of these two methods are discussed through a bidimensional academic example.

2 PROPER GENERALIZED DECOMPOSITION AND GEOMETRY VARIATIONS

2.1 Proper Generalized Decomposition

An extended presentation of the PGD method can be read in [6] and a detailed bibliography in [7]. The PGD method is based on the idea that the solution can be approximated with a good precision thanks to a reduced number of modes. One can notice the analogy with dynamics where the major part of the information is contained in the first modes and while new modes are added, the solution is enriched.
For this purpose, a separated variables representation of an unknow field f is sought. An example of a PGD approximation is given below for a time-space problem:

$$\forall(\underline{M},t) \in \Omega \times [0,T], f(\underline{M},t) \approx \hat{f}_n(\underline{M},t) = \sum_{i=1}^{n} \lambda_i(t)\Lambda_i(\underline{M}) \tag{1}$$

where Λ_i are the spatial modes and λ_i are the time functions.

In practical cases, n happens to be very small so the computation and storage costs are highly reduced. Furthermore, the functions λ_i and Λ_i are not given *a priori*. They will be computed through an iterative process.

This method can be easily extend to parametric and multidimensional problems as shown in Equation (2).

$$f(\underline{M},t,\alpha_1,\ldots,\alpha_k) \approx \hat{f}_n(\underline{M},t,\alpha_1,\ldots,\alpha_k) = \sum_{i=1}^{n} f_i^1(\alpha_1)\ldots f_i^k(\alpha_k)\lambda_i(t)\Lambda_i(\underline{M}) \tag{2}$$

where $\alpha_1,\ldots,\alpha_k$ are the parameters defining the geometry.
The aim is to build the solution for all the geometrical configurations, i.e. for all the values of $\alpha_1,\ldots,\alpha_k$. A quantity of interest can be, then, determined and stored in a Virtual Chart.

2.2 Example

Figure 1 presents an example of structure to optimize with respect to two geometrical dimensions and where the Von Mises stress maximum was considered as a quantity of interest. α and β denote the geometrical parameters associated, respectively to a length and a radius of curvature to take into account in the PGD method. The structure is clamped and a constant load is applied on the opposite extremity. It remains to adapt the PGD method in order to introduce geometry variations.

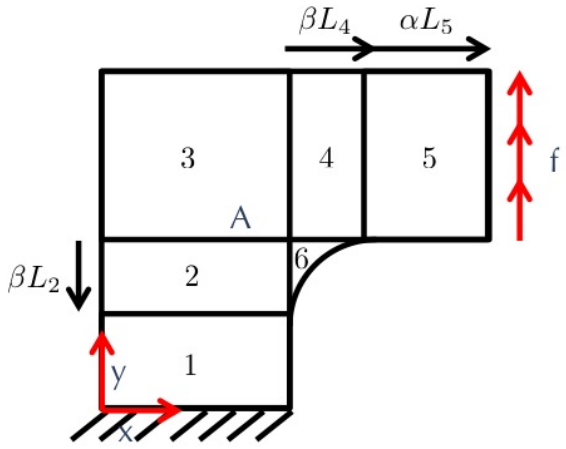


Figure 1: Test structure

2.3 Method 1

A reference structure Ω_0 is defined and meshed. Then, for each configuration $\Omega_{\boldsymbol{\alpha}}$, corresponding to a geometrical configuration parametrized by $\boldsymbol{\alpha} = \{\alpha_1,\ldots,\alpha_k\}$, it is possible to go back to the reference structure thanks to geometric transformations [8], as shown in Equation (4). For this purpose, one writes the variational formulation of the parametric problem:

$$\forall \delta \underline{u} \in \mathcal{U}^\star, \int\limits_{\mathcal{A}} \int\limits_{\Omega_{\boldsymbol{\alpha}}} \boldsymbol{\varepsilon}\left(\underline{u}(\underline{M}, \boldsymbol{\alpha})\right) : \boldsymbol{\kappa}\, \boldsymbol{\varepsilon}\left(\delta \underline{u}(\underline{M}, \boldsymbol{\alpha})\right) \, \mathrm{d}\boldsymbol{\alpha}\mathrm{d}\Omega = \int\limits_{\mathcal{A}} \int\limits_{\partial_f \Omega_{\boldsymbol{\alpha}}} \underline{f}^T \delta \underline{u}(\underline{M}, \boldsymbol{\alpha}) \, \mathrm{d}\boldsymbol{\alpha}\mathrm{d}\Gamma \tag{3}$$

- $\mathcal{A}$: space of geometrical parameters where $\boldsymbol{\alpha}$ belongs
- $\boldsymbol{\varepsilon}$: strain field
- $\boldsymbol{\kappa}$: hooke's tensor
- $\underline{u}$: displacement vector
- $\underline{f}$: load vector

It is, then, possible to study the problem with respect to the reference structure thanks to a change of variables.

$$\forall \delta \underline{u} \in \mathcal{U}^\star, \int\limits_{\mathcal{A}} \int\limits_{\Omega_0} \boldsymbol{\varepsilon}(\underline{u}(\underline{M}_0, \boldsymbol{\alpha})) : \boldsymbol{\kappa}\, \boldsymbol{\varepsilon}(\delta \underline{u}(\underline{M}_0, \boldsymbol{\alpha}))|\det \underline{\underline{J}}_{\boldsymbol{\alpha}}| \, \mathrm{d}\boldsymbol{\alpha}\mathrm{d}\Omega = \int\limits_{\mathcal{A}} \int\limits_{\partial_f \Omega_0} \underline{f}^T \delta \underline{u}(\underline{M}_0, \boldsymbol{\alpha}) \, \mathrm{d}\boldsymbol{\alpha}\mathrm{d}\Gamma \tag{4}$$

where $\underline{\underline{J}}_{\alpha}$ is the Jacobian matrix of the geometric transformation which transforms Ω_0 in $\Omega_{\boldsymbol{\alpha}}$:

$$\forall \underline{M}_0 \in \Omega_0, \, \underline{M} = T(\underline{M}_0) \in \Omega_{\boldsymbol{\alpha}} \tag{5}$$

The unknown field, here the displacement $\underline{u}$, is sought in the form of a separated variables representation:

$$\underline{u}(\underline{M}_0, \boldsymbol{\alpha}) \approx \sum_{i=1}^{n} g_i(\boldsymbol{\alpha}) \underline{U}_i(\underline{M}_0) \tag{6}$$

2.4 Method 2

In this method, the reference structure Ω_0 is defined as the union of all the different geometric configurations $\Omega_{\boldsymbol{\alpha}}$. For each geometric configuration, the structure is like "sunk" in a very soft material. A new constitutive has to be defined on Ω_0:

$$\tilde{\boldsymbol{\kappa}} = \boldsymbol{\kappa}\psi_{\boldsymbol{\alpha}} + a\boldsymbol{\kappa}(1 - \psi_{\boldsymbol{\alpha}}) \text{ with } a \ll 1 \tag{7}$$

where

$$\begin{cases} \psi_{\boldsymbol{\alpha}}(\underline{M}) & = & 1 \text{ if } \underline{M} \in \Omega_{\boldsymbol{\alpha}} \\ & = & 0 \text{ otherwise} \end{cases} \tag{8}$$

To compute the stiffness matrix of a finite element, one made the assumption that the reference structure is very finely meshed and so the stiffness matrix becomes:

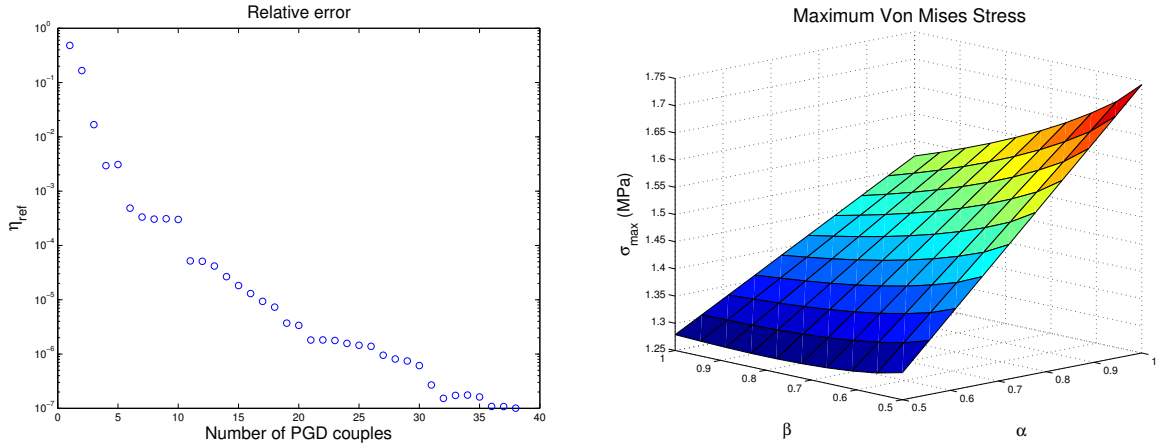
$$\underline{\underline{\tilde{K}}}_{el} = \begin{cases} \underline{\underline{K}}_{el} \text{ if the whole element belongs to } \Omega_{\boldsymbol{\alpha}} \\ a\underline{\underline{K}}_{el} \text{ otherwise} \end{cases} \tag{9}$$

where $\underline{\underline{K}}_{el}$ is the classic stiffness matrix of a finite element computed with the constitutive law $\boldsymbol{\kappa}$.

Then the displacement $\underline{u}$ is written in the form of a PGD approximation and the problem is solved on the reference structure.

2.5 Results

Both methods were tested on the example presented in Section 2.2. Figure 2a depicts the convergence of method 1 with respect to the number of PGD couples computed. It can be noticed that, with only four couples, a error inferior to 1% is reached. The error was calculated with respect to a reference solution computed thanks to a EF resolution for each geometrical configuration. An example of virtual chart built with this method is also presented in Figure 2b where the quantity of interest considered is the maximum of the Von Mises stress.



(a) Congervence

(b) Example of possible virtual chart

Figure 2: Results of a simulation using method 1

3 CONCLUSIONS

Two different approaches were introduced within the Proper Generalized Decomposition framework and used in academic examples where geometry variations were introduced as parameters. It was, then, possible to build "Virtual Charts" for quantities of interest, used for structural design. The next step is to apply these methods for more complex problems in order to approach the industrial issues of ASTRIUM-ST. In the example presented in this article, the quantity of interest (maximum of the Von Mises stress) was computed from the direct problem. The next step will be to determine the quantities of interest from the resolution of adjoint problems with error estimators.

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