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Development of shear and cross section deformable beam finite elements applied to large deformation and dynamics problems

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Abstract. *In this paper, beam finite elements based on the absolute nodal coordinate formulation (ANCF) are presented, in which the orientation of the cross section is parameterized by means of slope vectors only. Resulting, no singularities due to an angle parameter occur and the mass matrix is advantageously constant. A continuum mechanics as well as a structural mechanics based formulation for the elastic forces are investigated. Static as well as dynamic examples show accuracy and high order convergence of the presented beam finite elements.*

Keywords: Multibody System Dynamics; Beam Finite Elements; Absolute Nodal Coordinate Formulation.

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1 INTRODUCTION

The absolute nodal coordinate formulation (ANCF) has been developed for the modeling of large deformations in multibody dynamics problems by Shabana [14, 15]. In contrast to classical nonlinear beam finite elements in literature, the ANCF does not use rotational degrees of freedom and therefore does not necessarily suffer from singularities emerging from angular parameterizations. Compared to the classical formulation, in which the mass matrix is not constant with respect to the generalized coordinates, ANCF elements generally lead to a constant mass matrix, which is advantageous in dynamic analysis.

In the present approach, a linear and a quadratic ANCF beam finite elements are presented, in which the orientation of the cross section is parameterized by means of slope vectors. The elements serve as the three-dimensional generalization of existing planar shear deformable ANCF beam finite elements presented in [12]. Since the nodal vector of degrees of freedom includes the displacement vector and the two transversal slope vectors only, the elements belong to the group of so-called ”gradient deficient” elements. The latter elements are alternatives to so-called ”fully-parameterized” elements, in which the nodal coordinates are based on three slope vectors, representing the position gradient, see e. g. [5, 6]. A similar choice of nodal coordinates has been presented by Kerkkänen et al. [7] for a two-dimensional two-noded ANCF element and García-Vallejo et al. [4] presented a three-noded analogue. The deformed geometry of the proposed ANCF beam finite elements is defined by position and two slope vectors in each node, see Fig. 1 for a sketch of the elements. The three nodes are chosen at the end points and at the midpoint of the beam axis. Since the slope vectors $\mathbf{r}_{,\eta}^{(i)}$ and $\mathbf{r}_{,\zeta}^{(i)}$ are no unit vectors, a cross section deformation is not prohibited. In the two-noded linear element, the displacement along the beam axis is interpolated with linear shape functions, while the three-noded quadratic element uses quadratic shape functions for the displacement interpolation. The orientation of the cross section is interpolated linearly in both elements. The shape functions are chosen as the standard Lagrange polynomials presented by Zienkiewicz and Taylor [19, Eq.(8.18)].

The geometric description of the proposed ANCF beam finite elements, the precise definition of the degrees of freedom and a continuum mechanics as well as a structural mechanics based formulation for the elastic forces have been presented in a previous work [10]. The existing ANCF finite elements in the open literature showed significant problems regarding the formulation of elastic forces. Different approaches for the work of elastic forces are

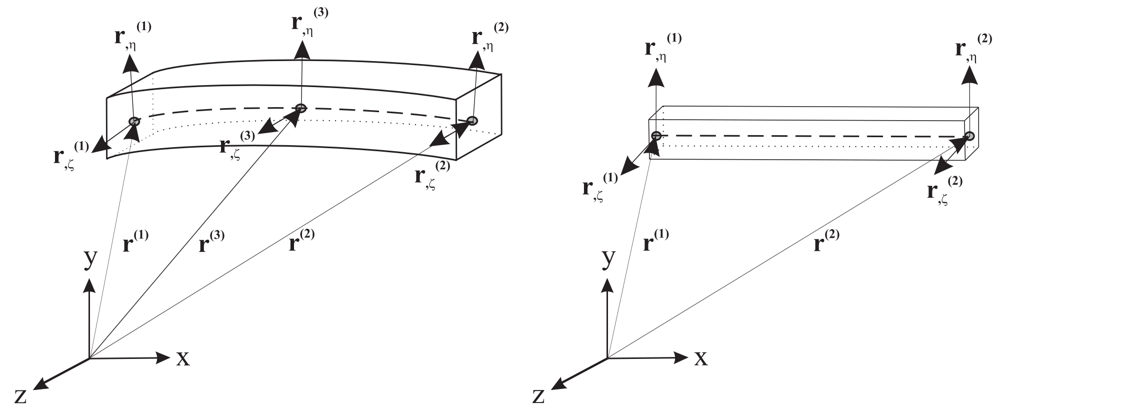

Figure 1: The geometric description of the elements is based on a position vector $\mathbf{r}^{(i)}$ and two slope vectors $\mathbf{r}_{,\eta}^{(i)}$ and $\mathbf{r}_{,\zeta}^{(i)}$ in the i-th node. These vectors are defined on a scaled and straight reference element, given in coordinates (ξ, η, ζ).

presented and compared to the formulations in literature. A continuum mechanics based formulation is discussed and extended in order to avoid Poisson locking. In addition, a structural mechanics based formulation of the elastic forces is discussed, which includes a term accounting for cross section deformation, which is not considered in the classical theory. The decision, which formulation should be used for an application, depends on the utilized material law. Concerning the elastic forces, it has been already shown, that the structural mechanics based formulation is in accordance with well established nonlinear rod elements of Simo and Vu-Quoc [17].

The ANCF elements have been implemented in the framework of the multibody and finite element research code HOTINT[1]. Several static examples and an eigenfrequency analysis have been already performed and a comparison to results provided in the literature have shown accuracy and high order convergence in static applications, see [9, 10]. In a further paper [11], the performance of the proposed beam finite element in dynamics is investigated in several stability examples. Complex buckling tests of e. g. Argyris [1] can be solved accurately and efficiently. For details on the dynamic analysis of the presented beam finite elements see [11].

The present paper shows an state-of-the-art review on different ANCF elements in literature and compare choice of degrees of freedom and the formulation for the elastic forces to the present approach, see Fig. 2 in the following section.

1.1 Overview of structural beam and continuum mechanics/solid beam finite elements

Here, a selection of different beam parameterizations is presented, see Fig. 2 for an overview of structural beam and continuum mechanics/solid beam finite elements. Figure 2(S1) shows the degrees of freedom of the classical structural beam finite element by Simo and Vu-Quoc [17].

The rotation is defined as the general mapping of local to global orientation by means of $\mathbf{\Lambda}(\xi) \in \mathrm{SO}(3)$ [17]. The parameterization of the rotation $\mathbf{\Lambda}$ could be e.g. Euler angles or Euler parameters.

The classical continuum mechanics based ANCF has been developed by Yakoub and Shabana [18] and Fig. 2(C1) shows the according nodal degrees of freedom. This so-called fully-parameterized element incorporates the full position gradient as nodal degrees of freedom.

In [3, 13], a solid beam finite element is presented which incorporates the position vectors of each of the eight nodes as degrees of freedom, see Fig. 2(C2). In order to avoid the locking phenomenon, the strain field is modified by a combination of an assumed natural strain method and an enhanced assumed strain method [3]. Figure 2(C) and (S) show the degrees of freedom of the proposed gradient deficient beam finite elements [8]. A solid continuum mechanics based formulation as well as a hybrid structural mechanics based formulation for ANCF beams are presented in Chapter 5.2.2 and 5.2.4 of [8] for the 2D and 3D beam formulations, respectively.

A hybrid formulation can also be found in the work of Betsch and Steinmann [2], which combines the modeling of [18] and the configuration space of [17], see Fig. 2(SC). The rotational parameterization with directors (12 coordinates) is reduced by a projection method and therefore only three degrees of freedom result for the rotation [2].

[1] http://www.hotint.org/

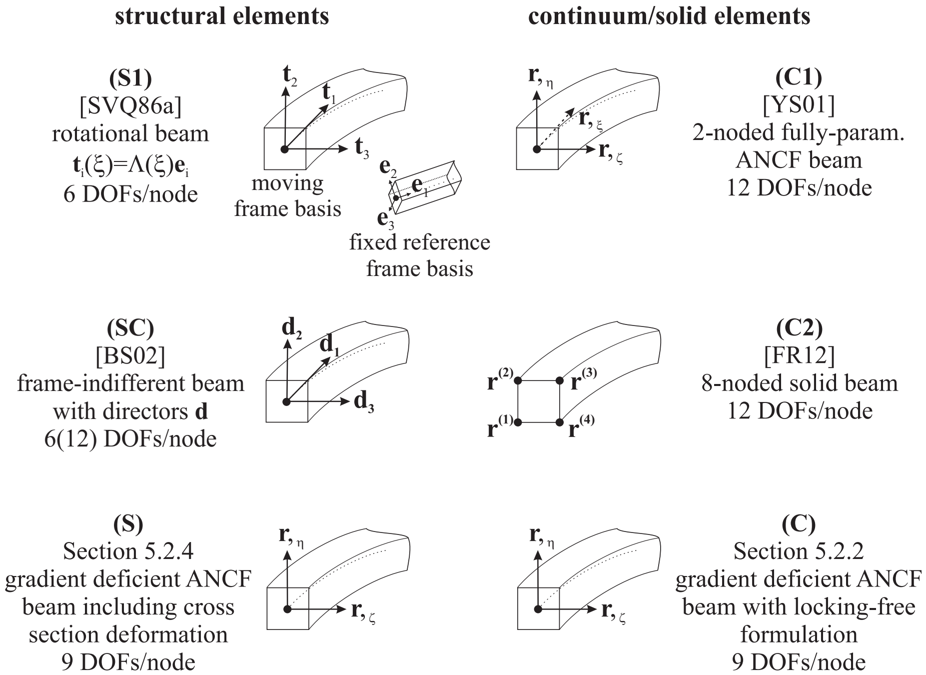

Figure 2: Overview of a selection of structural beam and continuum mechanics/solid beam finite elements.

The obtained equations constraining the so-called directors can be avoided by incorporating a null space method.

2 CONCLUSIONS

In the present paper, improved beam formulations are presented, which are mainly used for flexible multibody dynamics. A locking-free continuum mechanics based formulation as well as a structural mechanics based formulation including an correction term for shear and cross section deformation are presented. The performance of the proposed two- and three-dimensional finite beam elements is investigated by the analysis of several static and dynamic examples. In contrast to many previous investigations, a convergence analysis, coupled three-dimensional large deformation examples and buckling problems are studied here. A comparison to results provided in the literature, to analytical solutions, and to the solution found by commercial finite element software shows high accuracy and high order of convergence. Therefore the presented ANCF elements have high potential for large deformation three-dimensional structural problems. A comparison of the proposed formulation to approaches in literature is given and the advantages of the proposed elements are emphasized.

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