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INFLUENCE OF RESIDUAL STRESSES INDUCED BY COLD CURVING ON THE RESISTANCE OF I-SECTION STEEL MEMBERS

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INTRODUCTION

Cold curving is today the cheapest and more common way to produce curved steel members by bending or curving of straight members. This is a repetitive process during which the straight member passes several times through a machine with bending rolls imposing progressively the desire curvature (see for example the curving machine of figure 1). The non-reversible deformations introduced by the process are associated with additional self-equilibrated stresses locked in the section. Inevitably, these locked-in stresses, also called residual stresses, will have an influence on the amount of stresses that the curved member can resist during service.

Actually, the presence of residual stresses is not specific to arches and such stresses also exist in hot-rolled straight members due to thermal effects as the section is cooled down when exiting the mill. It is well known that these stresses have non negligible influence on the behaviour of the member (especially on beam-columns) and this influence has been abundantly studied, quantified and incorporated in the design methodology and all standards for steel construction. Yet the way these stresses influence the member behaviour depends on their distribution in the cross section, which depends, in turn, on their physical origin. As those origins are very different in the case of curved members than in straight members, it seemed important to provide a better understanding of these residual stresses and of their influence on the overall behaviour of arches.

Figure 1: Example of cold curving machine (courtesy EMEK S.A., Greece)

1 DISTRIBUTION OF RESIDUAL STRESSES

1.1 About the curving process

Construction standards generally neglect the influence of the production process on the behaviour of curved members and the only reference on the subject is the design guide edited by the Steel Construction Institute \([1]\). Pertinent information and main consequences of the continuous cold curving process can be found, however, in journals linked with materials processing technology, like for example the articles of Bjorhovde for I-beams \([2]\) and of Paulsen and Welo for rectangular hollow sections \([3]\). The first one is relatively qualitative, it presents the simple analytical model which is generally admitted for the calculation of residual stresses (see following section and figure 2) and also provides recommendations to avoid local buckling during curving. The second is
dedicated to the curving of rectangular hollow sections and consists of a refined elasto-plastic modelling of the process. Models of local curving methods also exist [4] but they will not be developed here, as the paper focuses on members with continuous curvature.

1.2 Residual stresses in steel members

In steel members, the residual stresses, also called locked-in stresses, have different origins which depend on the production process of the member: milling, welding, (un)coiling, forming or curving. It is generally distinguished between stresses induced by hot working processes (milling, welding, induction curving) and those induced by cold working processes (almost all forming processes). Residual stresses due to milling (also called thermal-induced stresses) or welding are relatively well known and well documented, and their influence is taken into account in most available standards for steel construction. Characteristic residual stress distributions for I-beams can be found in many books and manuals like that of the European Convention for Constructional Steelwork [5] or that of the Structural Stability Research Council [6]. Residual stresses due to cold forming processes are also well known [7], but their exact quantification is still under investigation, especially thanks to the development of non-destructive measuring techniques [8]. Numerous studies of the influence of these residual stresses on the behaviour and stability of the member were conducted, so that their effect is nowadays taken into account by construction standards, for example in Eurocode 3 through different buckling curves for cold formed sections (see EC3 1.1, table 5.5.3). Concerning the residual stresses induced by the cold curving process, the experimental data are few and recent. As a matter of fact, the only values are given by Spoorenberg et al. [9] who worked on curved I-beams and used the sectioning method for their measurements. They validate the method by comparing measurements of residual stresses in the straight members with the theoretical thermal stresses and reproduce then the same method on the members after curving. Their results are compared with a simple analytical model currently found in the literature [1,2]. The agreement between the measurements and the model is satisfactory in the flanges, but it is poor in the web, as a consequence of the restricted hypotheses of the analytical model, which will be detailed in the next paragraph.

1.3 Analytical model for residual stresses due to curving

The existing analytical model for the evaluation of the residual stresses in a curved member concerns I-beams and rectangular hollow sections with an elastic perfectly plastic behaviour. The model supposes that the flanges fully yield during the curving process and that the original residual stresses of the straight member can be neglected. The first hypothesis is almost always verified for usual curvatures. The second one is automatically verified in the flanges because their yielding during the curving process implies the removing of the former residual stresses (whatever their origin). This hypothesis thus implies that the initial residual stresses in the webs can be neglected. This assumption is generally not true, as shown by the experimental results of Spoorenberg et al. [9], but, as the webs play a secondary role in the flexural behaviour of steel arches, the consequences of this assumption should not have a large influence on the results. Thus, according to these hypotheses, the bending moment imposed by the rolls might be evaluated from the difference between the total plastic moment \( M_{pl} \) and the elastic moment of the part of the web which is still elastic (see figure 2):

\[
M_e = -M_{pl} + 2 \int_0^{y_c} yf_y(1 - y/y_c)\tau_u dy
\]  

(1)

where \( t_w \) is the thickness of the web and \( y_c \) is the height of the last elastic fibre of the section, which for a symmetric cross section is given by: \( y_c = \epsilon_y R_c \), where \( \epsilon_y \) is the yield strain and \( R_c \) the curving radius. When the member goes out of the rolls, the curving moment is released in a step which is called elastic spring back. For the member, this is equivalent to the application of an elastic bending moment \( M_{sb} \), so that the total bending moment becomes equal to zero. The spring back moment \( M_{sb} \) induces, hence, an elastic deformation of the member in the opposite direction of that of the curving moment characterised by the spring back radius \( R_{sb} = -EI/eM_c \). The final radius \( R_f \) of the arch is then obtained by subtracting the deformation due to spring back from that due to curving:
\[ \varepsilon_f = \frac{y}{R_f} = \frac{y}{R_c} - \frac{y}{R_{sh}} \]  

(2)

And hence, introducing the expression (1) of \( M_c \), one gets:

\[ \frac{1}{R_f} = \frac{1}{R_c} - \varepsilon_f \left( \frac{W_{pl}}{W_c} - \frac{1}{3} \varepsilon_f^2 R_{sh}^2 \right) \]  

(3)

By inverting the above expression, the theoretical curving radius which will lead to the desired curvature of the member can now be calculated. The corresponding distribution of residual stresses is then shown in figure 2. The characteristic values of the residual stresses \( \sigma_r \) in the curved member are evaluated from:

\[ \sigma_r^f = f_y (1 - a_c), \quad \sigma_r^w = f_y (1 - \eta_c a_c) \]  

(4)

where

\[ a_c = M_c / f_y W_{el}, \quad \eta_c = \varepsilon_c R_c / y_{max} \]  

(5)

![Figure 2: Curving stresses, elastic spring-back and residual stresses](image)

The residual stresses in the section are thus defined by two non-dimensional ratios: \( \eta_c \) which characterises the deformation of the member during curving and \( a_c \), which can be viewed as a partial aspect ratio and characterises the part of the plastic capacity which was used during curving. For very small radii, \( a_c \) is equal to the aspect ratio of the section \( W_{pl} / W_{el} \), while for large radii it approaches unity. Hence, the higher the curvature, the higher the ratio \( a_c \) and the higher the magnitude of the residual stresses.

1.4 Numerical model of the curving process

In parallel to this simple analytical beam model presented above, numerical bi-dimensional models were investigated using the finite element software ADINA® dedicated to the dynamic incremental non-linear analysis of structures. The objective was to reproduce numerically the curving of a straight member, and thus to assess the analytical model by comparison of the obtained residual stresses and to study the sensitivity of the model to changes of the various assumptions. For conciseness, this study is not developed here but, generally speaking, the numerical results validate the analytical model and the residual stress distributions in both cases are very close to each other. It must however be reminded here that the member is supposed to be initially stress-free which is not exactly true and which might explain the differences observed by Spoorenberg et al. [9] when comparing the analytical model validated here with their experimental results.

2 INTERACTION DIAGRAMS

The previous section has shown that the curving process induces a non-symmetrical distribution of residual stresses through the section height and significant magnitude of residual stresses in the web (see figures 2). It is thus expected that these stresses will have an influence on the elasto-plastic behaviour of the curved member and more specifically, that the asymmetry of the distribution will lead to a different behaviour, depending on the sign of the bending moments and that the high level of stresses in the web will cause early yielding under normal forces. An in-depth study of the
influence of the residual stresses is thus conducted following the methodology of Sophianopoulos et al. [10] for their study on thermal-induced residual stresses. The study is decomposed into two parts: first the determination of the elastic domain and then that of the plastic domain. The model supposes that the material is elastic-perfectly plastic, neglecting hardening.

2.1 Determination of the elastic domain

Due to the asymmetry of residual stresses distribution, it must be distinguished between bending moments which tend to open the arch (positive moments which increase the radius of curvature and thus act in the opposite direction to the curving moment) and bending moments which tend to close the arch (negative moments which thus reduce the radius of curvature and act in the same direction as the curving moment). The elastic domain is defined by the actions (bending moments \( M_N \) and normal forces \( N \)) that cause stresses below the yield stress \( f_y \). From the expressions of the residual stresses (4), one deduces that for \( M_N > 0 \), the limit of the elastic domain is given by:

\[
\frac{M_N}{f_y W_d} + \frac{|N|}{f_y S} = 2 - a_c
\]  
(6)

and that, for \( M_N < 0 \), this limit is given by:

\[
-\frac{M_N}{f_y W_d} + \frac{1}{\eta_c} \frac{|N|}{f_y S} = a_c
\]  
(7)

Observing that the ratio \( a_c \) is always larger than one (5), it appears immediately that the residual stresses reduce the dimension of the elastic domain for opening positive moments (6) and increase it for closing negative bending moments (7). It appears also that, for closing bending moments, the effects of normal forces are amplified by the ratio \( 1/\eta_c \) which is evidence of the role of residual stresses in the web. These facts are illustrated by the interaction diagrams of figure 5a, which represent these limits for the HEB360 cross-section of an arch with 60 m curving radius made of S235 steel (\( \eta_c = 0.37 \) and \( a_c = 1.11 \)) and for the corresponding stress free cross-section.

2.2 Determination of the plastic domain

For the plastic domain, it is still distinguished between opening and closing bending moments but some more distinctions have to be introduced. Indeed, it is supposed that the central part of the section undertakes the normal forces and the external parts the bending moments. The limit between these parts is denoted by \( y_N \) and it is evaluated from the magnitude of the normal forces and the geometrical characteristic of the section. Depending on the value of \( y_N \) relatively to the height of the flanges and to the limit of the elastic region during curving \( y_c \), eight different sub-cases have to be taken into account. The simplest case corresponds to opening moments \( (M_N > 0) \) and \( y_N \) in the web:

\[
y_N = \frac{N}{2t_w f_y}
\]  
(8)

The limit of the plastic region is then given by:

\[
M_N + 2 \int_{y_N}^{y_{max}} \sigma_y y dy dz = f_y W_{pl} - f_y y^2 t_w
\]  
(9)

The usual interaction equation is then found by substituting (8) in (9) and rearranging terms:

\[
\frac{M_N}{f_y W_{pl}} + \frac{S^2}{4t_w f_y W_{pl}} \left( \frac{N}{S f_y} \right)^2 = 1 - \frac{2t_w}{f_y W_{pl}} \int_0^{y_N} \sigma_y y dy
\]  
(10)

In the same way, for closing bending moments \( (M_N < 0) \) and \( y_N \) within the web, one gets:

\[
-\frac{M_N}{f_y W_{pl}} + \frac{N^2}{4t_w f_y W_{pl} f_y} = 1 + \frac{2t_w}{f_y W_{pl}} \int_0^{y_N} \sigma_y y dy
\]  
(11)
Such expressions are hence developed for each sub-case. The various limits are then represented on interaction diagrams similar to that in figure 5b which was obtained for HEB360 section with a 60 m curving radius. It appears clearly that the influence of residual stresses on the limits of the plastic domain is much smaller than on those of the elastic domain. Closing bending moments are better resisted than opening bending moments for which the plastic resistance is slightly reduced when compared to the stress free member (a few percent). The main consequence of the residual stresses is thus a widening of the elasto-plastic domain which will be characterised by a progressive loss of stiffness of the member (see next section and figure 7).

![Interaction diagrams for a HEB360 with R=60 m, a) Elastic domain & b) Plastic domain](image)

### 2.3 Numerical determination of the arch resistance

A numerical study was then conducted to validate this analytical model. Different combinations of loads were tested corresponding to eight specific couples of bending moments and normal forces generating stresses with constant ratios $\sigma_N/\sigma_M$ along the member length. The uniform bending moment is obtained by applying opposite bending moments at the ends and the uniform normal force by a uniformly distributed radial pressure. The curved member is isostatically supported with two inclined rollers according to the tangent to the arch at the extremities and one vertical roller at crown. The analyses include material and geometrical non-linearities, considering that it has been verified that the geometric non-linearities do not affect significantly the ratio $\sigma_N/\sigma_M$. Two numerical models are investigated: one initially curved member which is stress free and one curved member which was obtained by numerical curving of a straight member.

The results of the comparison between the analytical and numerical models are presented in figure 6. They correspond to an arch with a final radius of 32 m and a HEB360 cross-section ($y_c/y_{max} = 0.2$ and $d_c = 1.12$). There is clearly very good agreement for the values of the first yield in the case of opening moments as well as in that of closing moments. For the limits of the plastic domain, a diagram similar to that of figure 6 was drown and showed also very good agreement.

In addition, those constant loading couples were used to evaluate the loss of stiffness due to residual stresses through a comparison of the strain energy in the stress free and the curved members. Figure 7 shows the variation of the strain energy (which is practically calculated from the work of external forces) with the load factor. The arches have still a HEB360 section with $R_f = 32$ m and they are submitted to bending moments and normal forces such that $(N/N_{pl}) / (M/M_{pl}) = 0.28$. The loss of stiffness and the earlier yielding of the cross-section with residual stresses are limited but obvious.

### 3 CONCLUSIONS AND FURTHER WORKS

This analytical and numerical study has shown that residual stresses due to the cold curving process cause modifications of the elasto-plastic behaviour of the member, which depend on the geometrical characteristics of the section, on the curving radius and on the type of loading (opening or closing bending moment). This influence is characterised by two dimensionless parameters $a_c$ and $\eta_c$ which
include all geometric and mechanical information of the curved member. Qualitatively: the smaller the curving radius and the higher the aspect ratio \((W_{pl}/W_{el})\) of the section, the more significant the influence of the curving residual stresses. More precisely, it was shown that the plastic limit of a curved member is reduced in some cases by approximately 5\%, so that a 5\% safety margin for curved members in addition to that of straight members with identical section seems reasonable.

This recommendation, however, relies on the conservative hypothesis that the material has no hardening. It would thus be interesting to introduce hardening in the actual model and to study its interaction with the curving process in the determination of the elastic and plastic domains. Actually the on-going complementary study incorporates hardening as well as initial thermally induced residual stresses. It will hopefully lead to stress distributions that will be closer to experimental results. In any case, there are so little available data on residual stresses due to curving that one could benefit a lot for a complementary experimental program dedicated to their measurement, in a similar way to the experimental programs which were organised in the seventies for the measurement of thermally induced and welding residual stresses.

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