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Temporal dynamics of incoherent waves in
noninstantaneous response nonlinear Kerr media

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Nonlinear optics with incoherent waves is attracting a renewed interest since the first experimental observation of incoherent spatial solitons and incoherent modulational instability (MI) in photorefractive crystals [1–3]. Statistical nonlinear optics is now studied in various different circumstances, including wave propagation in homogeneous [4–8] or periodic media [9] or cavity systems [10–13]. In particular, the wave turbulence (WT) theory [14,15] proved efficient in describing important phenomena, such as the irreversible process of optical wave thermalization toward the thermodynamic equilibrium state [16]. As a remarkable consequence, a classical optical wave can exhibit a process of condensation [17,18], whose thermodynamic properties are analogous to those of quantum Bose–Einstein condensation, a feature which generated much interest [8,10,11,19]. The WT theory also provides a detailed description of the self-organization of incoherent optical waves into nonequilibrium stable structures, such as spectral incoherent solitons in noninstantaneous response nonlinear media [20,21], or incoherent solitons in highly nonlocal nonlinear media [22].

In this Letter we revisit the problem of the propagation of a temporally incoherent wave in a Kerr medium characterized by a noninstantaneous response time (e.g., optical fiber). Because of the causality property inherent to the nonlinear response function, the dynamics of the incoherent wave is known to be ruled by an antisymmetric gain curve—a well-known property in the example of the Raman effect. Then the low-frequency components of the incoherent field are amplified to the detriment of its high-frequency components, thus leading to a global spectral redshift of the incoherent wave.

Our aim in this Letter is to show that this general physical picture changes in a substantial way in the presence of a highly noninstantaneous nonlinear response of the material. In contrast with the expected spectral redshift, we show here that a highly noninstantaneous response leads to a genuine process of incoherent MI of the wave, which is characterized by the growth of two symmetric MI bands within the spectrum of the incoherent wave. Using the WT theory, we derive a Vlasov-like equation that provides a detailed description of this process of incoherent modulational instability in the temporal domain. © 2012 Optical Society of America

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optical fiber systems [20,21]. By using the WT theory, these incoherent structures can be described in detail by a kinetic equation analogous to the weak Langmuir turbulence equation in plasma physics [20,21].

The system enters into a new regime when $\tau_R \gg \tau_0$, as illustrated in the numerical simulations of the NLSE [Eq. (1)] reported in Fig. 1. The initial condition is a partially coherent wave with a Lorentzian spectrum and random spectral phases, i.e., $A(z=0,t)$ is of zero mean and obeys a stationary Gaussian statistics. Note, however, that the statistics does not need to be Gaussian initially. Thanks to the highly noninstantaneous response, the field relaxes to a state of Gaussian statistics during the propagation [7]. We considered in this example an exponential-like response function, $R(t) = H(t)\tau^2 \exp(-t/\tau_R)/(2\tau_R^2)$, where the prefactor $\tau^2$ avoids discontinuities in the derivative of $R(t)$ at $t=0$. However, we underline that the particular choice of the response function $R(t)$ and of the initial spectrum do not affect the generality of our results. We observe in Fig. 1 that, in marked contrast with the expected spectral redshift, the incoherent wave exhibits a genuine process of incoherent MI, as illustrated by the snake behavior of the spectrogram [see Figs. 1(a) and 1(b), while 1(c) will be discussed later]. Note that we considered here a high value of $\tau_R/\tau_0 = 100$ so as to guarantee a good agreement with the Vlasov theory; however, similar results are obtained for smaller response times (typically $\tau_R/\tau_0 > 10$).

We stress the fundamental different nature of this incoherent MI with respect to the incoherent MI considered in instantaneous response Kerr media [4]. Indeed, in the limit $\tau_R \rightarrow 0$, incoherent MI can only take place if the spectral width of the incoherent wave is smaller than the MI frequency, $\Delta \omega \ll \omega_{MI}$ [4]. This means that temporal modulations associated to MI are more rapid than the time correlation, $t_c > \tau_0$, i.e., MI modulations take place within each individual fluctuation of the incoherent wave. This is in contrast with Fig. 1, where incoherent MI manifests itself by a slow modulation of the whole random wave profile, i.e., the modulation frequency is smaller than the spectral bandwidth, $\omega_{MI} \ll \Delta \omega$.

Vlasov approach. To describe this novel process of incoherent MI, we derive a Vlasov-like kinetic equation that governs the evolution of the averaged local spectrum of the wave, $\bar{n}_w(t,z) = \int B(t,z) \exp(-i\omega t) d\sigma$, where $B(t,z) = (A(t+\tau/2,z)\epsilon^* t - \tau/2,z)$ is the correlation function and $\langle \cdot \rangle$ denotes an average over the realizations. Because of the highly noninstantaneous response $\epsilon = \tau_0/\tau_R \ll 1$, we perform a multiscale series expansion with $B(t,z) = B^{(0)}(t,\tau,t,\epsilon z)$ and $R(t) = \epsilon R^{(0)}(t)$ (details will be reported elsewhere), which gives

$$\partial_t \bar{n}_w(t,z) = 2\beta \epsilon \partial_\omega \bar{n}_w(t,z) + \partial_\omega V(t,z) \partial_\omega \bar{n}_w(t,z),$$

with the self-consistent potential $V(t,z) = \gamma \int_\omega \bar{R}(t,t') N(t',z) d\sigma'$, where $N(t,z) = B(t,t,0,z) = (2\pi)^{-1} \int \bar{n}_w(t,z) d\omega$ denotes the intensity profile.

The Vlasov equation [Eq. (2)] is of a different form than the conventional Vlasov equations used to describe nonlinear incoherent waves (see [3,5,7,15,24]). Its structure is in fact analogous to the Vlasov equation recently derived to describe highly nonlocal spatial effects [22]. In particular, the Vlasov equation [Eq. (2)] provides an “exact” statistical description of the incoherent wave (see [22]). Note, however, that contrary to the spatial case [22], in the temporal domain the self-consistent potential $V(t,z)$ is constrained by the causality property of $R(t)$, a feature which breaks the Hamiltonian structure of the Vlasov equation.

The MI analysis is performed by following the usual procedure (see, e.g., [3,5,7,24]). We perturb the statistical stationary solution of the Vlasov equation, $\bar{n}_w(t,z) = \bar{n}_w^0 + \epsilon_w(t,z)$, with $\epsilon_w(t,z) \ll \bar{n}_w^0$, and linearize it to get $\partial_t \epsilon_w(t,z) - 2\beta \epsilon \partial_\omega \epsilon_w(t,z) - \partial_\omega^2 \bar{n}_w^0 \int d\sigma' \partial_\omega R(t,t') \int d\omega \epsilon_w(t',z) = 0$. This equation is solved by a Fourier–Laplace transform, $\tilde{\epsilon}_w(\omega,p) = \int_{-\infty}^{+\infty} d\sigma \int_{-\infty}^{+\infty} d\sigma' \exp(-ip\sigma - i\Omega t) \epsilon_w(t,z)$, where $\mathcal{R}(p)$ denotes the MI growth rate. The dispersion relation reads

$$-1 = 2\gamma \beta \Omega^2 \tilde{\mathcal{R}}(\Omega) \int_{-\infty}^{+\infty} \frac{\bar{n}_w^0}{(2\beta \Delta \omega + ip)^2} d\omega. \quad (3)$$

Assuming that the initial spectrum is Lorentzian shaped, $\bar{n}_w^0 = P \Delta \omega/[\pi (\omega^2 + \Delta \omega^2)]$, Eq. (3) gives

$$\mathcal{P}(\Omega) = |\Omega| \left( \sqrt{2\beta \gamma \mathcal{P}(\Omega)} - 2\beta \Delta \omega \right). \quad (4)$$

where the incoherent MI gain reads $g_{MI}(\Omega) = 2\mathcal{R}[\mathcal{P}(\Omega)]$. Typical MI gain profiles are reported in Fig. 2. We remark in Fig. 2(a) that, as for the conventional incoherent MI [2–5,24], the finite bandwidth of the initial spectrum ($\Delta \omega = 0$) yields a (Landau) damping effect [3], which introduces a threshold for the incoherent MI, and significantly reduces the MI gain ($g_{MI}$) and
the MI frequency (ΩMI). Also note that ΩMI < Δω, which confirms the fundamental difference with incoherent MI in instantaneous response nonlinear media [4]. We remark in Fig. 2(b) that gMI and ΩMI decrease as the response time τR increases, a feature that simply results from the scaling property of the Fourier transform R(Ω) of the response function. We recall that the validity of the Vlasov equation [Eq. (2)] is constrained by the multiscale expansion, ε = τ0/τR ≪ 1. As discussed above, in the regime τR ≪ τ0, the Vlasov equation breaks down and the appropriate kinetic description is provided by the weak Langmuir turbulence equation [20,21]. This equation does not describe any MI process, so that the incoherent MI discussed here is fundamentally related to the highly noninstantaneous regime, τ0/τR ≪ 1. We remark in this respect that, in the coherent case, MI takes place in both the weakly or highly noninstantaneous regimes, although the MI gain bands can exhibit some asymmetry [25]. Conversely, recalling that the functions Ω[R(Ω)] and ℑ[R(Ω)] are, respectively, even and odd, the MI gain in Eq. (4) is always even, i.e., incoherent MI is characterized by the growth of two symmetric sidebands.

We report in Fig. 1 (right column) the numerical simulation of the Vlasov equation [Eq. (2)]. The initial condition is a stationary spectrum, n0, periodically perturbed to seed the incoherent MI with the maximum growth rate. We stress the good agreement with the unseeded MI simulation of the NLSE.

We can expect the observation of this incoherent MI considering a PCF filled with, e.g., carbon disulfide, where τR is of a few picoseconds [23]. In this case, the simulation in Fig. 1 would correspond to a power of few watts and a correlation time of the initial wave slightly below 1 ps.

In the spatial domain, incoherent MI was shown to eventually lead to the formation of stable incoherent soliton states [22]. Here, because of the causality of R(t), the spectrum exhibits a redshift in the nonlinear regime of propagation [z > 400λ0, Figs. 1(e) and 1(f)], i.e., beyond the linear regime described by the MI. This seems to prevent the formation of stable incoherent soliton structures. The long-term evolution of the novel process of incoherent MI reported here will be the subject of future investigations.

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