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NON RESONANT CONTRIBUTIONS IN MODAL ENERGY METHODS

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ABSTRACT

In many cases, the non resonant contributions of the vibro-acoustic systems is not negligible. In Statistical Energy Analysis (SEA), this contribution is often taken into account introducing indirect coupling between subsystems. In modal energy methods like Statistical modal Energy distribution Analysis (SmEdA), only resonant modes could be taken into account.
Recent developments of modal energy methods allow now to overcome the problem of non-resonant contribution. Two complementary approaches have been developed. The first one, in the framework of SmEdA, is based on the definition of indirect coupling between modes of two non-directly coupled subsystems. The second one considers modal energy exchanges at pure tone and defines a pure tone modal coupling loss factor between all modes of subsystems.
These two new approaches will be validated and compared in a cavity/structure/cavity test case.

1 INTRODUCTION

In modal energy methods like Statistical modal Energy distribution Analysis (SmEdA), the power balance is set on modes rather than subsystems like in SEA [1]. As in SEA, modal energy methods are based on the analysis of two coupled oscillators excited by random uncorrelated forces.

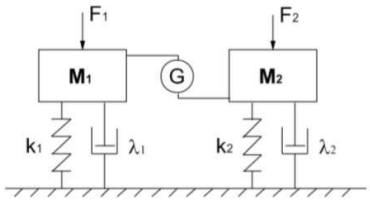

Figure 1 : Gyroscopic couplings between two oscillators

However, the subsystems are represented by their uncoupled modes and sets of oscillators (modes) are coupled through gyroscopic coupling. In SmEdA [2,3], the unkonwns are then the modal energies of each subsystems and modal coupling loss factors are defined.

2 INDIRECT COUPLING IN SMEDA APPROACH

Let's take the example of the three subsystems test case of a Cavity/Plate/Cavity. The basic idea of this approach is to represent the non resonant and non negligible contribution of one subsystems (that is to say the plate in the present example) by an indirect coupling between cavities.

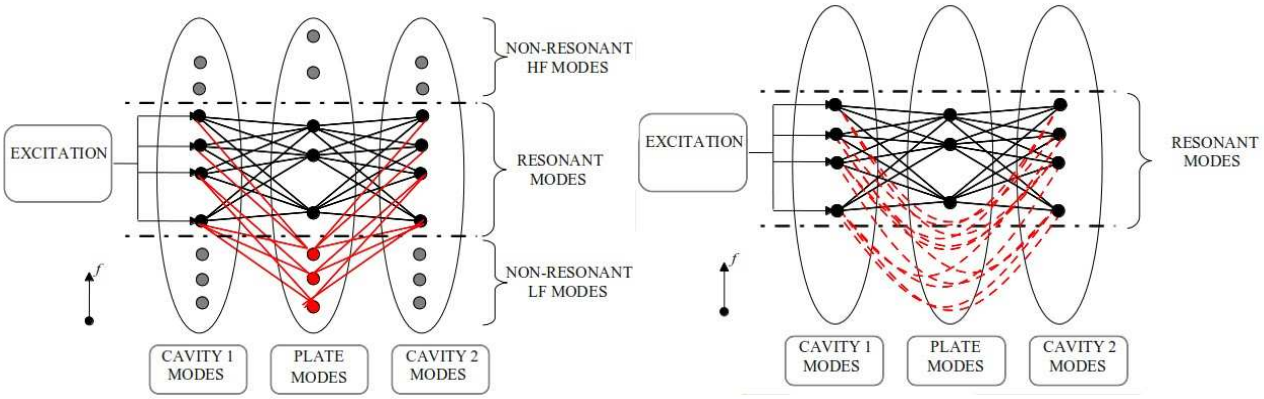

Figure 2 : (a) principle of SmEdA without non resonant modes (b) principle of SmEdA with indirect coupling to introduce non resonant modes of the plate.

Figure 2 presents the difference between SmEdA with or without non resonant contributions. In figure 2(a), a source excites modes of cavity 1 in a frequency band. With classical SmEdA, only couplings between resonant modes (that is to say in the frequency band of excitation) are taken into account. However, non resonant contribution of low frequency modes (in red) can't be neglected in some cases. To overcome this issue, indirect couplings between cavities 1 and 2 are introduced as presented in figure 2(b). As can be demonstrated using a condensation method and a dual modal formulation, the power exchanged from mode p of cavity 1 to mode r of cavity 2 due to non resonant modes of the plate writes

$$\Pi_{pr} = \beta_{pr}\left(E_p - E_r\right) \tag{1}$$

where E_p and E_r are respectively the energy of modes p and r and β_{pr} is the coupling loss factor between mode p of cavity 1 and mode r of cavity 2 due to non resonant modes of the plate which writes

$$\beta_{pr} = \left(\sum_{q \in Q_{nr}} W_{pq} W_{rq} \right)^2 \left\{ \frac{\left(\omega_p \eta_p + \omega_r \eta_r \right)}{\left[\left(\omega_p \right)^2 - \left(\omega_r \right)^2 \right]^2 + \left(\omega_p \eta_p + \omega_r \eta_r \right) \left[\omega_p \eta_p \left(\omega_r \right)^2 + \omega_r \eta_r \left(\omega_p \right)^2 \right]} \right\}. \quad (2)$$

W_{pq} and W_{rq} are the modal works between non resonant modes q of the plate and resonant modes p and r of cavities. ω_p and η_p are the natural angular frequency and the damping of mode p. Thus, taking into account this indirect coupling between cavities, the power balance of mode p of cavity 1 writes

$$\Pi_{inj}^{p} = \omega_p \eta_p E_p + \sum_{q \in \hat{Q}_r} \beta_{pq} \left(E_p - E_q \right) + \sum_{r \in \hat{R}} \beta_{pr} \left(E_p - E_r \right), \qquad \forall p \in \hat{P} \quad (3)$$

where the injected power is equal to the power dissipated by the mode plus the direct exchange between mode p and resonant modes of the plate plus the indirect exchange between mode p and modes of cavity 2 due to non resonant modes of the plate. Finally, modal energies of each subsystems can be computed solving a system of linear equations.

3 MODAL ENERGY ANALYSIS AT PURE TONE

Another way to solve this problem is to express the power balance between two modes p and q between two subsystems excited by random forces at pure tone. One can demonstrate that the injected power between modes at pure tone writes

$$\Pi_p^{\mathrm{inj}}(\omega) = \Pi_p^{\mathrm{diss}}(\omega) + \sum_q \alpha_{pq}(\omega) E_t^p(\omega) - \sum_q \alpha_{qp}(\omega) E_t^q(\omega). \quad (4)$$

where the power injected into mode p is equal to the power dissipated by mode p plus the net exchanged power between modes p and q. The coupling factor $\alpha_{pq}(\omega)$ depends on frequency and writes

$$\alpha_{pq}(\omega) = \frac{2\gamma_{pq}^2}{\left(1 + \frac{\omega_p^2}{\omega^2}\right)} \frac{\Delta_q^{\mathrm{visc}} \omega^2 \left((\omega_p^2 - \omega^2)^2 + \omega^2 (\Delta_p^{\mathrm{visc}})^2 \right) + \omega^4 \gamma_{pq}^2 \Delta_p^{\mathrm{visc}}}{\left((\omega_p^2 - \omega^2)^2 + \omega^2 (\Delta_p^{\mathrm{visc}})^2 \right) \left((\omega_q^2 - \omega^2)^2 + \omega^2 (\Delta_q^{\mathrm{visc}})^2 \right) - \omega^4 \gamma_{pq}^4}. \quad (5)$$

where ω is the angular frequency of the excitation, Δ_p^{visc} is the damping bandwith of the mode p and γ_{pq} is the gyroscopic coupling which can be related to modal works between modes as

$$\gamma_{pq} = \frac{1}{\sqrt{M_p M_q}} \int_{S_c} \Phi_p^{\mathrm{plate}}(Q) \Psi_q^{\mathrm{cav}}(Q) dQ. \quad (6)$$

where Φ_p^{plate} and Ψ_q^{cav} are respectively the displacement mode shape of modes of the plate and the pressure mode shape of modes of the cavity.Solving the system of linear equations as Eq. (4) at pure tone permits to take intrinsically into account non resonant contribution of modes (of each subsystem). No distinction of in-band or out-of-band modes is necessary and all the modes are necessary to solve the system.

4 CAVITY/STRUCTURE/CAVITY TEST CASE

The test case presented in figure 3 consists in a plate (Lx=0.9m, Ly=0.7m, ρ=7800kg/m^3, E=2e11Pa, ν=0.3, hp=1mm) in between two cavities filled of air (LzA=1m, LzB=1.2m, ρ0=1.2kg/m^3, c0=340m/s). A monopole is located in cavity A at point (0.1,0.1,0.1)

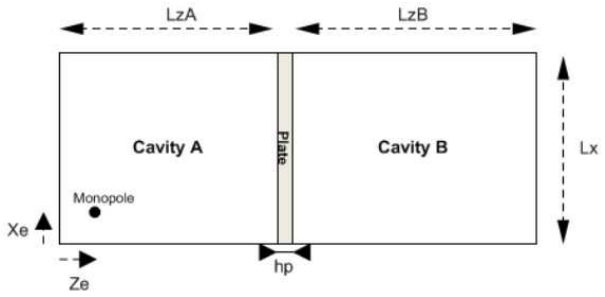

Figure 3 : Cavity/Plate/Cavity test case. A monopole is located in cavity A

The energy ratio between cavities in presented in figure 4 and a comparison between modal energy methods is done. As can be seen, without taking into account non resonant modes of the plate, SmEdA leads to an overestimation of the energy ratio. However, both presented approaches well compare with reference calculation. In figure 4, modal energy at pure tone is frequency averaged.

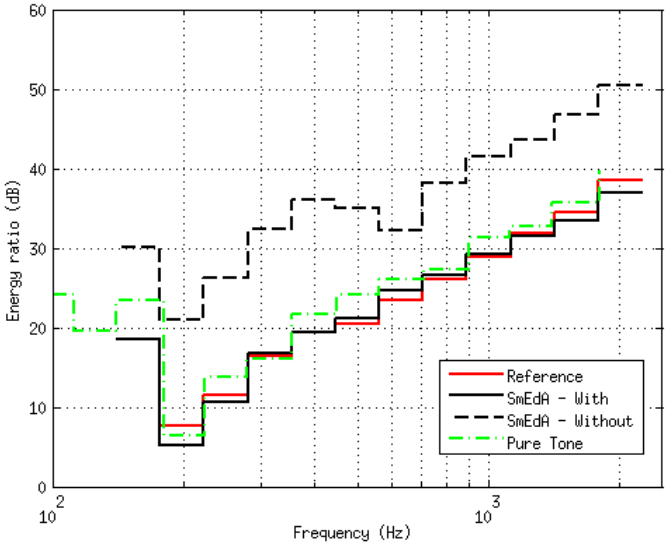

Figure 4 : Energy ratio between cavities. Comparison of modal energy approaches

5 CONCLUSION

Two modal energy methods has been validated and compared in the Cavity/plate/Cavity test case to take into account non resonant modes.

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