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Application of Postponement Strategy in Distribution Period for Treating Uncertain Demand in Supply Chain Management

Yahong ZHENG, Khaled MESGHOUNI

Abstract—Uncertain demand is one of the most important uncertainties in supply chain management. Different from the traditional methods, such as dynamic inventory control, estimation of demand, we propose to apply postponement strategy to treat uncertain demand in the process of distribution in supply chain. Postponement has been appreciated in recent decades, mainly in manufacturing period of realizing mass customization and decoupled system. With postponement strategy, we could address demand with very different quantity. Based on assumption of ideal cooperation among all the enterprises in the same supply chain, a linear programming model is generated, with bi-objective of minimizing the inherent costs of goods and transportation cost. Through tested in an example numeric, it is demonstrated feasible to reschedule supply chain regardless of quantity of demand and to supply different replenishment strategy for decision maker.

Index Terms—uncertain demand, postponement, demand-supply subsystem, replenishment strategy, supply chain rescheduling.

I. INTRODUCTION

Uncertainties in supply chain occur mainly in demand, in the process of distribution and fabrication. Because of large influence scope of uncertain demand, it has attracted attention of many researchers. Although uncertainties in demand have been concerned much recently, there is still not a general approach to solve it.

A. Uncertainties in demands

Uncertainties in demand are mainly the oscillations and surges of demand. Because the market is dynamic, uncertainty is an essential character of demand. The primary factor causing uncertainty in demand is the customers. Their necessities, desire and anticipation of consummation, value of consuming, tendency, belief in the production, as well as the degree of infection between consumers can all influence the quantity of consummation. Another factor influencing demand is the outer environment, such as the policy, assurance, advertisement, accuracy of searching information, production and its life cycle and so on.

Uncertainties in demand influence easily the inventory level of the upstream enterprises in supply chain, the suppliers of raw materials, the manufacturers, the retailers, etc. Some researches verify that the influence of uncertainty of demand to the retailer is larger than that to the manufacturer. Because of demand uncertainty and inaccurate and asymmetric information, there is a very universal phenomenon called “Bullwhip Effect”, which provokes hard measurable consequences of poor customer service level. Meanwhile, this phenomenon deteriorates in the process of broadcasting. The distance of the broadcasting is longer, the augment of the uncertainty increases. Between the two ends of a supply chain, the material and the consumer, the deviation is the largest.

As dealing with uncertainties in demand is so urgent, many researchers have considered it in the supply chain management. In the immense literatures, the traditional methods of demand estimation and inventory control are discussed most (L.W.G. Strijbosch and J.J.A. Moors, 2005, 2006; J. CHOI et al. 2005; Gerald Reiner and Johannes Fichtinger, 2000; Ilias S. Kevork, 2010, Francisco Campuzano, 2010). Nevertheless, the main disadvantage of the traditional methods is that they are always concentrated in a small scope of optimization. In recent years, the strategies of robustness and flexibility have become hot. Some researchers have tried to form a robust supply chain to make it immune to the uncertainties of demand. However, most of them is just a super idea and is hard to realize. The flexible or agile system have also been much tried to improve the ability to cope with uncertainties in demand. M. Barad et al. considered the flexibility as the ability of the manufacturing system to cope with internal and external variation with high competitive competency and high economic profitability. Patricia M. Swafford et al. present an approach to achieve supply chain agility through IT integration and flexibility. In their opinion, supply chain flexibility represents operational abilities within the supply chain functions and supply chain agility represents the speed of the aggregate supply chain to adapt in a more customer-responsive manner. Christopher S. Tang considers in his research robust strategies for mitigating operational and disruption risks and enhancing the efficiency and resiliency of supply chain management.

As we known, the approaches above are not universal to all the supply chain management, they are only applied in a given case and environment. What we want to do is to develop a general approach that can be applied to most of the practical cases of supply chain management. And the most obvious detect, sometimes we called the key technical issue of the robust system, is the defining of previous contingent. The robustness of the system depends on the number of the previous contingent and the method treating it. However, the large scale of the prediction will bring expensive budget and cost to the system. As to this point, the strategy of postponement can avoid this problem.

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B. Strategy of postponement

Originally, postponement is known as late customization or delayed product differentiation, which was first discussed by W. Alderson (1957). Since then, postponement strategy has been applied in many industries, including the high-tech industry, the food industry, the garments industry, etc. It is necessary to recognize that the postponement strategy is a double-sided sword. It can bring benefits to the enterprise, such as the reduced inventory, the pooling risk, the accurate forecast. However, the disadvantages also exist, such as the high cost of the designing and manufacturing of the common components, the cost of reconfiguration of the supply chain structure. Therefore, the postponement is not suitable to all situations. It depends on the conditions. There is a trade-off between the additional costs and the benefits. Whether the postponement strategy matches the situation or not depends on the practical condition. Yu-Ying Huang and Shyh-Jane Li (2008) suggest firms to choose the suitable postponement decision according to their business environment, the static decision and the dynamic decision, to cope with external environmental changes. Although the postponement is more used by the suppliers, it is also used by the demanders. Chunyang Tong (2010) implies the order postponement in a supply chain. He shows that both the manufacturer and the retailer gain when the order is places later under some conditions.

Many factors can influence the effects of the postponement strategy, the products price, the cost of each stage of the supply chain, the packaging, the assembling, the inventory cost, the service-level, etc. Shihua Ma et al. (2002) found that a key factor for commonality and postponement decisions is the interactions between processing time and the component procurement lead times.

The approach of postponement is concreted according to the practical problem by different authors, to achieve the optimal postponement strategy coping with the problem. Gregory A. Graman (2010) proposed a partial-postponement decision cost model and demonstrated its application in determining the levels of finished-goods inventory and postponement capacity. It is solved using a non-linear programming formulation. He has also illustrated the relation and impacts between the related factors and the expected costs and the postponement capacity. Q.L.Zen et al. (2006) developed a systematic approach to determine the optimal timing for staged order commitment, with categorizing attributes and aggregation of processes to reduce the complexity.

Postponement has been considered as an effective method to treat demand uncertainty. Viswanath Cvs and Stephen M. Gilberti (2002) proposed a two-tier supply chain model to demonstrate that below a threshold level of demand uncertainty the supplier as well as the buyers can benefit from providing early purchasing opportunities versus postponement. As defined by Gregory A. Graman (2010), while the certain parts of demands are solved through make-to-stock strategy, a combination of made-to-stock and postponement is called "partial" or "tailored" postponement. The partial postponement strategy is a flexible method to respond to the demand uncertainty. In the scenario that the demand is independent of time and stochastic, Aviv Y and Federgruen A (1998) indicate that the benefits of postponement are confined into two factors, statistic economies of scale and risk pooling via common buffers.

Although postponement is similar to delay, it is necessary to distinguish the difference between them. In fact, the term "delay" is always regarded detrimental, especially, delay in the production and delay in the transportation. It is a type of uncertainty in supply chain. Rifat Sipahi and Ismail Ilker Delice (2010) proposed a differential equation to analyze impact of three delays on the inventory behavior and to obtain an ordering policy to make inventory variation insensitive to the detrimental effects of the delays. Here, postponement is a subjective strategy, rather than an objective delay phenomenon in supply chain process. In literatures viewed, postponement used in managing demands in the period of supplying is much fewer than that in the manufacturing process. The work best similar to ours is Ananth V. Iyer et al. (2003). They analyzed demand postponement as a strategy to handle demand surges and showed that postponement strategy may lead to reduced investment in initial capacity. But it limited the model in a single period of postponement demands. In our work, based on the model of Ananth V. Iyer, we considered the practical condition that including both the regular demand and the postponed demand.

II. Problem statement

In our work, we propose an intuitive and rather simple approach to cope with the uncertainties in demand, the postponement in supplying process. It disregards to the uncertain demand in our strategy and maintaining the status quo of the capacity of inventory and fabrication. In a word, our technique is under the environment where the hardware in the supply chain does not change. We respond to the demand surges after the demands occur. What the supplier should do before the demand unfolds is keeping the normal safe inventory level and negotiating a good cooperation contracts between them. Our approach is feasible in an absolute cooperative environment, from the standpoint of the whole supply chain.

Our esprit is that we do not concern the demand distribution or the prior planning process. We focus only on the rescheduling of the supply chain in a simplified supply-demand subsystem described in fig 2, to make clear the flow of products and make sequent computation easier.

Through the process of structure simplification, the objective of research of the complicated supply chain network is turning to the single sub-system of supply chain. There is only one level of demand-supply relation in the sub-system. Our main idea is when the demand occurs, according to the total inventory of all the suppliers, we distribute the equal quantity of demand to certain suppliers. When the
uncertainty has realized, the demand $D_i$ is deterministic. And the question is how to decide the quantity of each demander. $\beta$ is given to represent the ratio of the demand postponed, thereby $1 - \beta$ is the part of demands satisfied in regular period, $\alpha_{ij}$ is set to describe the ratio of each demand $i$ satisfied by supplier $j$ in regular period, with stocks. This step is called scheduling process. The next step is to complete the unsatisfied demand, i.e. the postponed part $\beta$, with $\beta_{ij}$ to describe the ratio of demand $i$ satisfied by supplier $j$ in postponement period. We call this step a rescheduling process of the sub supply chain system. At last, we reschedule the supply chain hierarchically, from the resource of the demand, i.e. the final market to the end of materials, iterating the following program, as in Fig 2, using three iterations.

The iterating procedure of scheduling is described as in Fig 3.

As described by Ananth V. Iyer et al. [9] (2003), the specific manner in which demand postponement occurs can follow the following two possible schemes: (a) Postpone a fraction of demand for each customer: every unit of demand is split with delivered in the regular period and 1- delivered in the postponement period. In this scheme, every customer is affected and has a fraction of his demand postponed; (b) Postpone all demands for a fraction of customers: a fraction $1 - \beta$ of the demand is postponed and thus delivered entirely in the postponement period, and then the remaining fraction $\beta$ of the demand is delivered in the regular period. Note that in this case, any given customer may see his demand delivered entirely in the regular period or entirely in the postponement period depending on whether his demand was postponed or not. In our work, different from Ananth V. Iyer, we consider that the postponement is planned after the demand unfolds.

Resumptively, our strategy of postponement is executed in two stages:

i). determine the optimal fraction of total postponed demand $\beta$;

ii). determine the optimal fraction $\alpha_{ij}$ for each supply-demand relation in the regular period.

iii). determine the optimal fraction $\beta_{ij}$ for each supply-demand relation in the postponement period.

Here, we also consider that the supplier reimburses the demander a predetermined unit postponement cost $c_3$. And we assume that the compensation is equivalent to all demanders. The nomenclature is given in Appendix A.

III. OPTIMAL FRACTION OF TOTAL POSTPONED DEMAND $\beta$

The fraction of demand to satisfy in the regular period is $(1 - \beta)$. The capacity of the suppliers (signifying mainly the inventory) must be able to satisfy the demand in the regular period, the $(1 - \beta)$ part demand. The capacity of the suppliers is mainly the inventory level. We have assumed that information of suppliers is already known. The inventory level is constant. So we can get an inventory constraint as follows:
\[(1 - \beta) \sum_{i=1}^{n} D_i \leq \sum_{j=1}^{m} k_j \tag{1}\]

And in the period of postponement, the demand is satisfied by manufacturing. According to the manufacturing capacity (supply capacity) of the suppliers, we can get the manufacturing constraint:

\[\beta \sum_{i=1}^{n} D_i \leq \sum_{j=1}^{m} s_j t_j \tag{2}\]

In the inequality above, \(t_j\) must be controlled in the allowable postponing time \(T\), which includes manufacturing time \(t_j\) and transporting time \(p_j\),

\[K = \sum_{j=1}^{m} k_j \tag{3}\]

\[T = t_j + p_j \tag{4}\]

\[D = \sum_{i=1}^{n} D_i \tag{5}\]

Although \(t_j\) is a variable waiting to design, with the purpose of simplifying the definition of \(\beta\), in reality, according to the allowable postponement time, we can choose an expected value constant \(t\), we note,

\[S = t \sum_{j=1}^{m} s_j \tag{6}\]

Then we obtain that

\[1 - \frac{K}{D} \leq \beta \leq \frac{S}{D} \tag{7}\]

In order to reduce the complexity of calculating, we assume that the unit cost of conversation \(c_1\), the unit cost of manufacturing \(c_2\) and the unit compensation for postponing to demanders are identical to every supplier. And the allowable lead time of delivery \(T\) is also equivalent.

The expected cost of the supplier in the supplying process is

\[V_1(\beta) = c_1 \sum_{j=1}^{m} k_j + c_2 \beta \sum_{i=1}^{n} D_i + c_3 \beta \sum_{i=1}^{n} D_i + c_4 \left( \sum_{j=1}^{m} k_j - (1 - \beta) \sum_{i=1}^{n} D_i \right) = c_1 K + \beta D (c_2 + c_3 + c_4) + c_4 (K - D) \tag{8}\]

The first term of the above formulation is the cost of the production of inventory; the second term is the new manufacturing cost for satisfying the postponed demands, including all the costs of manufacturing, cost of material, processing, assembly, etc.; the third part is compensation for postponing paid to demanders; the last part is conservation costs. As the transportation cost is related to the single amount of delivery and the calculation is rather complicated, in this period of calculation of \(\beta\), we do not consider the transporting cost.

From the function of cost, we find that the cost is proportional to the postponement fraction \(\beta\).

So the optimal value of \(\beta\) is:

\[\beta^* = 1 - \frac{K}{D} \tag{9}\]

And the optimal expected cost is

\[V_1^*(\beta) = c_1 K + (D - K)(c_2 + c_3 + c_4) + c_4 (K - D) = c_1 K + (D - K)(c_2 + c_3) \tag{10}\]

IV. Optimal fraction of supplying to each supplier

After the total postponement fraction has been determined, the optimal fraction to each supplier in the regular period \(\alpha_{ij}\) and that in the postponement period \(\beta_{ij}\) can be calculated. Firstly, we will discuss separately the two parts of calculations, and then we integrate them to execute the calculation of optimization.

A. Optimal fraction of supplying to each supplier in regular period, \(\alpha_{ij}\)

In the relationship of present enterprise, the long-term cooperation is appreciated. So in reality, most of the time, the demand is send to the familiar customers. In our work, we have supposed a complete ideal situation of the cooperation among the enterprizes in the same supply chain (SC). The demand is allocated just according to the objective of minimization of cost and the maximization of service level.

As the cost of production, conservation, compensation for postponing paid to demanders is concerned with total postponement fraction, here we only have to consider the distance and cost of transportation with \(\alpha_{ij}\). We should also consider the practical distance and cost from the supplier to the relative demander. Let \(r_{ij}\) denote the distance between the supplier and the demander.

The cost of transporting concerns mainly with the distance of delivery:

\[\min V_2(\alpha_{ij}) = \min [c_5 \sum_{i=1}^{n} \sum_{j=1}^{m} r_{ij} \cdot D_i \cdot \alpha_{ij}] \tag{11}\]

The unit cost of transporting \(c_5\) is probably inversely proportional to the amount of delivery. So, for the sake of low cost, the suppliers deliver production only when they have a reasonable amount and transporting cost. For example, the threshold of the amount is \(m\), when the quantity of delivery \(q_m\), they do not want to deliver. So there is a delivery amount constraint:

\[D_i \cdot \alpha_{ij} \geq m \tag{12}\]

The service level refers mainly to the satisfaction of the demand, which is already included in the following demand constraint:

\[\sum_{i=1}^{n} \sum_{j=1}^{m} D_i \cdot \alpha_{ij} = (1 - \beta) \sum_{i=1}^{n} D_i \tag{13}\]

Demand satisfied in the regular period is by the inventory production, that is

\[\sum_{i=1}^{n} D_i \cdot \alpha_{ij} \leq k_j \tag{14}\]
The natural attribution of the rate of distribution is:

\[ 0 \leq \alpha_{ij} \leq 1 \]  

From the formulations above, we can get the fraction of demand postponement allocated to each supplier.

**B. Optimal fraction of postponement to each supplier in the postponement period, \( \beta_{ij} \)**

After the total postponement fraction and the optimal fraction of supplying of each supplier in the regular period have been determined, the optimal fraction of each supplier in the postponement period can be calculated. Different from calculating the total cost of postponement, the allocation of postponement is more complicated. Here, like the allocation of demand in regular period, the cost of transporting concerns mainly with the distance of delivery:

\[ \min V_3(\beta_{ij}) = \min\{c_5 \sum_{i=1}^{n} \sum_{j=1}^{m} r_{ij} \cdot D_i \cdot \beta_{ij}\} \]  

Transferring amount constraint:

\[ D_i \beta_{ij} \geq m \]  

The total postponed demand distributed to each supplier, we get:

\[ \sum_{i=1}^{n} \sum_{j=1}^{m} D_i \beta_{ij} = \beta^* \sum_{i=1}^{n} D_i \]  

The postponed demand is satisfied by the new supply capacity (from the nodes upstream or manufacturing itself):

\[ \sum_{i=1}^{n} D_i \beta_{ij} \leq t \cdot s_j \]  

Demand constraints are:

\[ \sum_{i=1}^{n} \beta_{ij} = 1 - \sum_{i=1}^{n} \alpha_{ij} \]  

Natural attribution of the rate of distribution is:

\[ 0 \leq \beta_{ij} \leq 1 \]  

From the calculation of the formulations above, we can get the fraction of demand postponement allocated to each supplier.

**C. Integrated calculation**

If we use the separated calculation, the constraints are not considered simultaneously, and then we may get some solutions infeasible. Therefore, we integrate the formulations in the two periods and solve them simultaneously.

We get the optimizations to be solved as follows:

\[ \min V_2(\alpha_{ij}, \beta_{ij}) = \min\{c_5 \sum_{i=1}^{n} \sum_{j=1}^{m} r_{ij} \cdot D_i \cdot \alpha_{ij} + c_6 \sum_{i=1}^{n} \sum_{j=1}^{m} r_{ij} \cdot D_i \cdot \beta_{ij}\} \]  

All the constraints (12)-(15), (17)-(21) in the two periods must be satisfied here.

![Fig. 4. SC network consisting of factories, warehouses and distributors](image)

![Fig. 5. a) Sub-system1 b) Sub-system 2](image)

**V. NUMERICAL RESULTS**

To demonstrate the feasibility and effectiveness of our approach dealing with uncertainties in demand, we apply it to the example of Alev Taskin Gumus et al. (2009).

**A. The model description from the original SC network**

The case used in [3] is a SC network design presented for a reputable multinational company in alcohol free beverage sector. The existing SC, the cost and capacity data from existing SC network refer to [3].

In this model (shown as Fig 4), 2 factories (\( F_1, F_2 \)), 3 warehouses (\( W_1, W_2, W_3 \)) and 6 distributors (\( D_2, D_3, D_4, D_5, D_6 \)) are selected from the company’s system in order to explain the existing design of the network, and considering that the product flow is followed by only one product of the company. The question to solve is to decide and design the best SC network to satisfy the demand, simultaneously to minimize the supply cost.

Firstly, we simplify the SC network as two sub-systems as in Fig 5.

Besides applied in the case where the inventories of suppliers do not satisfy current demands, our approach is also an effective scheduling method in allocation of demand in the case where the stock is enough to satisfy demands. To demonstrate this point, we will apply our method in two kinds of demands, the first one is the case where demands can be satisfied by inventory, and the other one is where demands can not be satisfied by inventory.

The data in Table 1 to Table 5 are the original data. In order to use our methodology, we adjust the parameters:

The concept of transportation distances are replaced by the different unit transportation costs. Therefore, \( c_5 = 1, r_{ij} \) refers to the transportation costs.
As we do not know exactly the physical inventory, we assume that the inventory is equal to the warehouse capacity. Thus, corresponding the data of the capacities of the factories and warehouses to our model, in sub-system 1, \( k_1 = 3,785,630, k_2 = 1,564,479, k_3 = 346,094, K = \sum_{j=1}^{3} k_j = 5696203 \); In sub-system 2, \( k_1 = 3,011,970, k_2 = 1,298,716, \sum_{j=3}^{2} k_j = 4310686 \)

Different from the work in [3], we do not need to estimate demand. We use the estimation of demands as the real occurring demands. That is, in sub-system 1, \( D = \sum_{i=3}^{6} D_i = 394915 \); after we finished the calculation of sub-system 1, using the result, we can calculate for sub-system 2. In this case, \( D < K \), therefore, we need not use the postponement strategy for rescheduling the SC network, that is, \( \beta = 0 \). The optimization method is enough for the scheduling in this case.

### B. Results of calculation for the case with estimated demand

The problem waiting to be solved is a linear programming problem. We use LINDO 6.1 to resolve it.

In order to simplify the execution of calculation, we ignore the constraint of transporting amount, which is assumed to satisfy the transportation principle, using some method such as out-sourcing, carpooling and so on.

1) **Calculation results for sub-system 1**: In this case, \( D_1K \), therefore, we need not use the postponement strategy for scheduling the SC network, that is, \( \beta = 0 \). The optimization method is enough for the scheduling in this case.

Calculation results for sub-system 1 as referred to Table VI. Allocation of the demands of distributors to each warehouse in the case with estimated demand is specified in Table VII.

Calculation results for sub-system 2: As calculated from the results in sub-system 1, we get the demands in sub-system 2 as in Table VIII.

Distributors demand

1. D1 116,803
2. D2 55,425
3. D3 74,668
4. D4 9,660
5. D5 81,539
6. D6 56,820

---

**TABLE I**

The transportation costs from the factories to the warehouses (cent/case)

<table>
<thead>
<tr>
<th>Factories</th>
<th>Warehouses</th>
<th>D1</th>
<th>D2</th>
<th>D3</th>
<th>D4</th>
<th>D5</th>
</tr>
</thead>
<tbody>
<tr>
<td>W1</td>
<td>W2</td>
<td>0.48</td>
<td>0.65</td>
<td>0.63</td>
<td>0.71</td>
<td>0.45</td>
</tr>
<tr>
<td>F1</td>
<td>W3</td>
<td>0.60</td>
<td>0.36</td>
<td>0.55</td>
<td>0.52</td>
<td>0.72</td>
</tr>
<tr>
<td>W2</td>
<td>W3</td>
<td>0.25</td>
<td>0.17</td>
<td>0.39</td>
<td>0.06</td>
<td>0.05</td>
</tr>
</tbody>
</table>

**TABLE II**

The transportation costs from the warehouses to the distributors (cent/case)

<table>
<thead>
<tr>
<th>Warehouses</th>
<th>Distributors</th>
</tr>
</thead>
<tbody>
<tr>
<td>W1</td>
<td>D1</td>
</tr>
<tr>
<td>F3</td>
<td>W2</td>
</tr>
</tbody>
</table>

**TABLE III**

The capacities of the factories and warehouses

<table>
<thead>
<tr>
<th>Factories/Warehouses</th>
<th>Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1</td>
<td>301970</td>
</tr>
<tr>
<td>F2</td>
<td>1,298,716</td>
</tr>
<tr>
<td>W1</td>
<td>3,785,630</td>
</tr>
<tr>
<td>W2</td>
<td>1,564,479</td>
</tr>
<tr>
<td>W3</td>
<td>346,094</td>
</tr>
</tbody>
</table>

**TABLE IV**

The estimated demand of distributors (case)

<table>
<thead>
<tr>
<th>Distributors</th>
<th>Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>D1</td>
<td>116,803</td>
</tr>
<tr>
<td>D2</td>
<td>55,425</td>
</tr>
<tr>
<td>D3</td>
<td>74,668</td>
</tr>
<tr>
<td>D4</td>
<td>9,660</td>
</tr>
<tr>
<td>D5</td>
<td>81,539</td>
</tr>
<tr>
<td>D6</td>
<td>56,820</td>
</tr>
</tbody>
</table>

**TABLE V**

Random demands of distributors (case)

<table>
<thead>
<tr>
<th>Distributors</th>
<th>Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>D1</td>
<td>1,116,803</td>
</tr>
<tr>
<td>D2</td>
<td>955,425</td>
</tr>
<tr>
<td>D3</td>
<td>1,774,668</td>
</tr>
<tr>
<td>D4</td>
<td>1,509,660</td>
</tr>
<tr>
<td>D5</td>
<td>1,581,539</td>
</tr>
<tr>
<td>D6</td>
<td>856,820</td>
</tr>
</tbody>
</table>

**TABLE VI**

Calculation results for sub-system 1 in the case with estimated demand

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_{11} )</td>
<td>0.000000</td>
</tr>
<tr>
<td>( \alpha_{12} )</td>
<td>0.000000</td>
</tr>
<tr>
<td>( \alpha_{13} )</td>
<td>0.000000</td>
</tr>
<tr>
<td>( \alpha_{21} )</td>
<td>0.000000</td>
</tr>
<tr>
<td>( \alpha_{22} )</td>
<td>0.889884</td>
</tr>
<tr>
<td>( \alpha_{23} )</td>
<td>0.119152</td>
</tr>
<tr>
<td>( \alpha_{31} )</td>
<td>0.000000</td>
</tr>
<tr>
<td>( \alpha_{32} )</td>
<td>0.000000</td>
</tr>
<tr>
<td>( \alpha_{33} )</td>
<td>1.000000</td>
</tr>
</tbody>
</table>

Objective value: 58605.25

**TABLE VII**

Allocation of the demands of distributors to each warehouse in the case with estimated demand

<table>
<thead>
<tr>
<th>Warehouses</th>
<th>Distributors</th>
</tr>
</thead>
<tbody>
<tr>
<td>W1</td>
<td>D1</td>
</tr>
<tr>
<td>W2</td>
<td>D1</td>
</tr>
<tr>
<td>W3</td>
<td>D1</td>
</tr>
</tbody>
</table>

**TABLE VIII**

Allocation of the demands of warehouses to each factory in the case with estimated demand

<table>
<thead>
<tr>
<th>Distributors</th>
<th>Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>D1</td>
<td>0</td>
</tr>
<tr>
<td>D2</td>
<td>0</td>
</tr>
<tr>
<td>D3</td>
<td>0</td>
</tr>
</tbody>
</table>

(Advance online publication: 24 August 2011)
value

\begin{table}[h]
\centering
\begin{tabular}{|c|c|}
\hline
Variable & Value  \\
\hline
$\alpha_{11}$ & 0.000000 \\
$\alpha_{12}$ & 0.000000 \\
$\alpha_{21}$ & 0.034962 \\
$\alpha_{22}$ & 0.965038 \\
$\alpha_{31}$ & 1.000000 \\
$\alpha_{32}$ & 0.000000 \\
Objective value & 138228.2 \\
\hline
\end{tabular}
\caption{TABLE IX
Calculation results for sub-system 2 in the case with estimated demand}
\end{table}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
Factories & Warehouses & $W_1$ & $W_2$ & $W_3$ \\
\hline
$F_1$ & 0.000000 & 0.034962 & 1.000000 \\
$F_2$ & 0.000000 & 0.965038 & 0.000000 \\
\hline
\end{tabular}
\caption{TABLE X
Allocation of the demands of warehouses to each factory in the case with estimated demand}
\end{table}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|}
\hline
Variable & Value & Variable & Value & Variable & Value  \\
\hline
$\alpha_{11}$ & 1.000000 & $\alpha_{12}$ & 0.000000 & $\alpha_{13}$ & 0.000000 \\
$\alpha_{21}$ & 0.000000 & $\alpha_{22}$ & 0.057377 & $\alpha_{23}$ & 0.000000 \\
$\alpha_{31}$ & 0.129865 & $\alpha_{32}$ & 1.000000 & $\alpha_{33}$ & 0.000000 \\
$\alpha_{41}$ & 0.000000 & $\alpha_{42}$ & 0.000000 & $\alpha_{43}$ & 0.000000 \\
$\alpha_{51}$ & 1.000000 & $\alpha_{52}$ & 0.000000 & $\alpha_{53}$ & 0.000000 \\
$\beta_{11}$ & 0.000000 & $\beta_{12}$ & 0.000000 & $\beta_{13}$ & 0.000000 \\
$\beta_{21}$ & 0.000000 & $\beta_{22}$ & 0.480097 & $\beta_{23}$ & 0.000000 \\
$\beta_{31}$ & 0.000000 & $\beta_{32}$ & 0.000000 & $\beta_{33}$ & 0.000000 \\
$\beta_{41}$ & 0.000000 & $\beta_{42}$ & 0.000000 & $\beta_{43}$ & 0.000000 \\
$\beta_{51}$ & 0.000000 & $\beta_{52}$ & 0.000000 & $\beta_{53}$ & 0.000000 \\
$\beta_{61}$ & 0.000000 & $\beta_{62}$ & 0.000000 & $\beta_{63}$ & 0.000000 \\
Objective value & 1421900 &  \\
\hline
\end{tabular}
\caption{TABLE XI
Results for sub-system 1 in the case with unexpected demand}
\end{table}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
W & D1 & D2 & D3 \\
\hline
W1 & 1.000000 & 0.000000 & 0.129865 \\
W2 & 0.000000 & 0.000000 & 0.195019 \\
W3 & 0.000000 & 0.000000 & 0.195019 \\
\hline
\end{tabular}
\caption{TABLE XII
Allocation of the demands of distributors to each warehouse for sub-system 1 in the case with unexpected demands in regular period}
\end{table}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|}
\hline
W & D1 & D2 & D3 & D4 & D5 \\
\hline
W1 & 0.000000 & 0.000000 & 0.129865 & 0.195019 & 0.000000 \\
W2 & 0.000000 & 0.000000 & 0.195019 & 0.000000 & 0.000000 \\
W3 & 0.000000 & 0.000000 & 0.195019 & 0.000000 & 0.000000 \\
\hline
\end{tabular}
\caption{TABLE XIII
Allocation of the demands of distributors to each warehouse for sub-system 1 in the case with unexpected demands in regular period}
\end{table}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
Factories & Warehouses & $W_1$ & $W_2$ & $W_3$ \\
\hline
F1 & 0.000000 & 0.258978 & 1.000000 \\
F2 & 0.000000 & 0.741022 & 0.000000 \\
\hline
\end{tabular}
\caption{TABLE XIV
Optimal allocation of the demands of warehouses to each factory in case 1}
\end{table}

C. Results of calculation for the case with unexpected demand

In the uncertain demand environment, where $D > K$, the postponement strategy is just appropriate to cope with the unexpected demand. We can get similar calculation results as the ones above.

1) Calculation results for sub-system 1: The superiority of our algorithm is that we can deal with unexpected demands. The quantity of demands in this section we will deal with is beyond the inventory level and capacity of the suppliers.

In sub-system 1, $\beta = \beta^* = 1 - \frac{K}{D} = 0.1$ The calculation results is in Table XI.

The ratio of supplying for each warehouse in the regular period is as in Table XII, and in the postponement is as in Table XIII.

2) Calculation results for sub-system 2: We use the results of calculation in sub-system 1 as the input data for sub-system 2. The upstream supply for the factory is assumed to be enough, so the supply capacity is the manufacturing capacity. In our example, the total products needed by the warehouses are much less than the stocks. Therefore, if the factories only need to supply the need in the postponement period of the warehouses, in this case, in sub-system 2, we need not use the postponement strategy. However, we still use the optimization model to get the optimal allocation of the supply. The warehouse empty should surely be replenished. We can calculate the cost of replenishing the inventory. There exit two other cases: the second case is the factories only need to send its stocks to the warehouse, and the third case is the factories need to fulfill the need of warehouses in the postponement period, and simultaneously need to replenish the inventory of the warehouses. We will execute the calculations in the three cases separately. Sometimes, the manager must make the decision of choosing the optimal manner. Then we compare the calculations of the three cases above, we can find which cost least. The one with the lowest cost is the optimal proposal.

Case 1: the factories only need to supply the need in the postponement period of the warehouses. In this case, the postponement strategy is not needed. We use the calculation of the regular period. We get the results as follows: $\alpha_{11} = 0.0000, \alpha_{12} = 0.0000, \alpha_{21} = 0.258978, \alpha_{22} = 0.741022, \alpha_{31} = 1.0000, \alpha_{32} = 0.0000.$

\[ \min V_2(\alpha_{ij}) = 392580 \]

The optimal allocation of demand is as Table XIV. For replenishing the inventory, here, $D = 5696203, K = 4310686$. $K < D$, postponement strategy is needed. We get: $\alpha_{11} = 0.795632, \alpha_{12} = 0.0000, \alpha_{21} = 0.0000, \alpha_{22} = 0.830127, \alpha_{31} = 0.0000, \alpha_{32} = 0.0000, \beta_{11} = 0.204368, \beta_{12} = 0.0000, \beta_{21} = 0.0000, \beta_{22} = 0.169873, \beta_{31} = 1.0000, \beta_{32} = 0.0000.$

\[ \min V_2(\beta_{ij}) = 113330 \]

The optimal allocation of demand for the regular period as is in Table XV, for the postponement period is as Table XVI.

The sum of separate supplying for the demands of ware-
TABLE XV
OPTIMAL ALLOCATION OF THE DEMANDS OF WAREHOUSES TO EACH FACTORY IN REGULAR PERIOD

<table>
<thead>
<tr>
<th>Factories</th>
<th>Warehouses</th>
<th>W1</th>
<th>W2</th>
<th>W3</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1</td>
<td>0.795632</td>
<td>0.000000</td>
<td>0.000000</td>
<td></td>
</tr>
<tr>
<td>F2</td>
<td>0.000000</td>
<td>0.830127</td>
<td>0.000000</td>
<td></td>
</tr>
</tbody>
</table>

TABLE XVI
OPTIMAL ALLOCATION OF THE DEMANDS OF WAREHOUSES TO EACH FACTORY IN POSTPONEMENT PERIOD

<table>
<thead>
<tr>
<th>Factories</th>
<th>Warehouses</th>
<th>W1</th>
<th>W2</th>
<th>W3</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1</td>
<td>0.204368</td>
<td>0.000000</td>
<td>1.000000</td>
<td></td>
</tr>
<tr>
<td>F2</td>
<td>0.000000</td>
<td>0.169873</td>
<td>0.000000</td>
<td></td>
</tr>
</tbody>
</table>

TABLE XVII
OPTIMAL ALLOCATION OF THE DEMANDS OF WAREHOUSES TO EACH FACTORY IN CASE 2

<table>
<thead>
<tr>
<th>Factories</th>
<th>Warehouses</th>
<th>W1</th>
<th>W2</th>
<th>W3</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1</td>
<td>0.000000</td>
<td>0.150693</td>
<td>0.849307</td>
<td></td>
</tr>
<tr>
<td>F2</td>
<td>0.000000</td>
<td>1.000000</td>
<td>0.000000</td>
<td></td>
</tr>
</tbody>
</table>

TABLE XVIII
OPTIMAL ALLOCATION OF THE DEMANDS OF WAREHOUSES TO EACH FACTORY IN REGULAR PERIOD IN CASE 3

<table>
<thead>
<tr>
<th>Factories</th>
<th>Warehouses</th>
<th>W1</th>
<th>W2</th>
<th>W3</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1</td>
<td>0.422688</td>
<td>0.216952</td>
<td>1.000000</td>
<td></td>
</tr>
<tr>
<td>F2</td>
<td>0.000000</td>
<td>0.391524</td>
<td>0.000000</td>
<td></td>
</tr>
</tbody>
</table>

D. Discussions

According to the result of ANN simulation in [3], the first and second factories, and the first and second warehouses are open, but the third warehouse is closed. On the other hand analytical method gives a solution in which all the factories and the warehouses are open. While ANN simulation finds 182,021 dollars for the minimum cost, analytical method’s result is 167,231 dollars. Dealing with the same quantity of demands, our results show the optimal scheduling is that, the first and second factories, the second and third warehouses are open, while the first warehouse is not uses. The minimum cost is 196833.45 dollars. It is not as good as compared to the results in [3], however, the limit of their methods is the constraint that the capacity of the warehouses should be equal or more than the demand of the distributor. This is just what we want to deal with, the case where the demand is beyond the capacity of the inventory capacity.

Moreover, another advantage of our method is that it can quickly get the optimal replenishment strategy after the emptying of stocks which is used to satisfy the demands in regular period. There are usually several strategies to choose, we can execute our postponement strategy for different cases separately, and then we compare them to find the one with least cost as the optimal proposal.

VI. CONCLUSIONS AND FUTURE RESEARCH

In this paper, we have proposed a postponement strategy in the scheduling of supply chain network to cope with uncertainties in demand, based on the hierarchal sub-system of the supply chain network and the ideal cooperation of the agents in supply chain. It is factually a method transforming the uncertainty to certainty. The simplification of supply chain network to demand-supply subsystems is to simplify computation of large scale of variables. A linear programming model is employed to get the optimal allocation of the supplier to the demand, with the minimization of the supply cost as the objective. It is demonstrated feasible and powerful in the scheduling of supply chain in practical example. We found that, even in the cases where the inventory is enough to satisfy the demand, our optimization model is also appropriate. The postponement strategy is only needed when the total inventory cannot satisfy the total demand. We have compared the results of treating the same demands to the results in [3]. It is completely reasonable.

Inevitably, some drawbacks and limits exist in our research. In fact, we did not consider the physical position relationship of the members of supply chain. In that case, we need more data of the distribution of all the agents of the supply chain, which will make the scheduling more complicated. In our future study, we can take it into account to complete the scheduling process, to avoid the phenomenon of roundabout. As to the products in the logistics, we only

houses and for replenishing the inventory of warehouses is 505910.

Case 2: the factories only need to send its stocks to the warehouse to satisfy the need of warehouses in the postponement period.

In this case, we use the contrary allocation of the supply to de demand. We get:

\[ \alpha_{11} = 0.0000, \alpha_{12} = 0.0000, \alpha_{21} = 0.150693, \alpha_{22} = 1.000000, \alpha_{31} = 0.849307, \alpha_{32} = 0.0000. \]

\[ \min \ V_2(\alpha_{ij}) = 256220. \]

The allocation of supply of factories to each warehouse is as in Table XVII.

Case 3: the factories need to fulfill the need of warehouses in the postponement period, and simultaneously need to replenish the inventory of the warehouses.

In this case, \( D = 7794893, K = 4310686, K < D \), postponement strategy is needed.

\[ \alpha_{11} = 0.422688, \alpha_{12} = 0.0000, \alpha_{21} = 0.216952, \alpha_{22} = 0.391524, \alpha_{31} = 1.0000, \alpha_{32} = 0.0000, \beta_{11} = 0.577312, \beta_{12} = 0.0000, \beta_{21} = 0.0000, \beta_{22} = 0.391524, \beta_{31} = 0.0000, \beta_{32} = 0.0000. \]

\[ \min \ V_2(\alpha_{ij}, \beta_{ij}) = 681630. \]

The optimal allocation of demand for the regular period is as in Table XVIII, for the postponement period is as Table XIX.
have treated the flow of finished products. The treatment of materials and parts will be more interesting and complex. Another drawback is in the calculation. In fact, in order to get the parameters needed in calculation of linear programming in LINDO 6.1, we have done a lot of preparing work by hands, using programming in software will bring much convenience. As well, the usage of the linear programming is limited in small large of calculation. For larger scale of calculation, a heuristic algorithm will be more appropriate. If all the calculations are integrated in software or a tool box, it will be much more convenient and simple for the managers to use our method. We will try to realize a visual process of all the design and calculation procedures.

APPENDIX A

List of notation

- $\beta_i$ - Optimal fraction of total postponed demand
- $\alpha_{ij}$ - ratio of demand $i$ satisfied by supplier $j$ in regular period
- $\beta_{ij}$ - ratio of demand satisfied by supplier $j$ in postponement period
- $D_i$ - demand of $i^{th}$ demander
- $s_j$ - supply capacity of $j^{th}$ supplier in the postponement period
- $t_j$ - manufacturing time for supply of $j^{th}$ supplier in the postponement period
- $T$ - the expected manufacturing time constant
- $p_j$ - transportation time of $j^{th}$ supplier in the postponement period
- $S_i$ - Total supply capacity of supplier in the postponement period
- $l_j$ - inventory level of the $j^{th}$ supplier
- $k_j$ - the inventory of $j^{th}$ supplier
- $I$ - total inventory
- $c_1$ - the unit cost of the production of inventory
- $c_2$ - unit cost of new manufacturing cost for satisfying the postponed demand
- $c_3$ - unit cost of compensation paid by the suppliers to demanders for postponement
- $c_4$ - unit cost of conservation between the two delivery times
- $c_5$ - unit cost of transporting

REFERENCES