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Ouidad Yousfi

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Financial Capital Structure in LBO Projects Under Asymmetric Information

O. YOUSFI

MRM, University of Montpellier, ouidad.yousfi@univ-montp2.fr

Abstract:

This paper studies the relationship between the financial capital structure in LBO (Leveraged Buy Out) acquisitions and the agents’ incentives when there is a double sided moral hazard problem. The entrepreneur and the LBO fund provide complementary efforts that influence the distribution of the project’s returns which may take either a high or a low value. The former agent needs to raise capital to take a company private. Both an LBO fund and a bank have sufficient funds to finance the investment. Hence, the involvement of the bank is not needed to get the project going.

We show that there is no debt-equity contract that implements the first best efforts. Despite the fact that financing the project through a mixture of debt and equity or solely through equity lead the partners to provide the same level of efforts, the entrepreneur relies on both the LBO fund and the bank. To explain the high level of debt in LBO projects, the tax deductibility of the debt’s interests seems to be a convincing theoretical rationale for the involvement of banks in buyout acquisitions.

Key words: double moral hazard, debt, capital structure, LBO, tax advantage.

JEL classification: G23, G24, G32.
Introduction

Leveraged Buy Out commonly known as LBO\(^1\) accounts for a significant part of the corporate finance and plays a major role in structuring mergers, acquisitions and transmissions.

There are three main facts about LBO finance:

First, these projects are financed mostly with debt and a small amount of equity in many countries most notably in the United States, hence the term *leveraged*: these projects are typically financed with anywhere from 60\% to 90\% debt (Jensen, 1986, 1989 and Kaplan and Strömberg, 2008). Moreover, there are many kinds of debt with different levels of risk, such as the mezzanine debt, the subordinate debt and the convertible debt.

Second, the LBO fund (hereafter he) is an active investor and he is well connected with many industries: he is engaged in the day to day operations of the firm, helps to recruit key personnel, negotiates with the suppliers, the bank (hereafter he) and the other financial partners, and advises the entrepreneur (hereafter she) on all the strategic decisions.

Finally, the use of convertible securities becomes prevalent in LBO finance. But usually these securities are very rarely issued in the presence of banks or passive outside equity holders.

This paper provides a theory of LBO financing based on a contractual approach. The theoretical model that we present deals with the two first facts and studies the relationship between the financial structure and the agents’ incentives under asymmetric information.

The success of these acquisitions is explained not only by the use of debt but it depends also on the market conditions, the performance of the *Op Co* (the acquired company)\(^2\) and the partners’ abilities. The entrepreneur has technical skills and knows well the company\(^3\) while the LBO fund is an active financier: in addition to the funds, as explained before, he contributes to the project with his business expertise and managerial advice.

In practice, whether the entrepreneur is wealth constrained or not, she asks for financing from both the LBO fund and the bank. Notice that the former is not wealth constrained, so the two partners are able to finance the project without debt financing.

The questions raised in this paper are the following: *What is the role of banks’ involvement through debt-like outside financing? Is its purpose to increase the efficiency of contracting between the entrepreneur and the LBO fund or merely a mean to save taxes?*

The LBO funds, like the other private equity funds, are not wealth constrained and well

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\(^1\)LBO is the acquisition of a company, called the *Op Co*, using mostly debt and a small amount of equity. The debt is secured by the *Op Co* assets. The acquiring company, called the *holding* company or the *New Co*, uses these assets as collateral for the debt in hopes that the future cash-flows will cover the debt’s payments. Usually, the *Op Co* is a department or a subsidiary of a group or a large listed company, which is taken private through LBO acquisition.

\(^2\)For more details, see footnote 1.

\(^3\)Usually, the entrepreneur is a manager or an employee of the acquired company. Sometimes, she is an outside manager/employee in another company which operates in the same market.
established financiers: they finance large companies and issue high level of equity. In Venture Capital (hereafter VC), the entrepreneur has usually no wealth to finance her project; the VC fund is the only financier in the project and he is also providing advice.⁴

The questions are addressed in the context of a double-sided moral hazard problem where both the entrepreneur and the LBO provide complementary efforts that influence the distribution of the project’s returns, which may take either a high or a low value. We consider a model with three agents: the entrepreneur, the LBO fund and the bank. The entrepreneur is the manager of the Op Co and she wants to acquire it. She asks for advice and for funds first from the LBO fund. They sign a first contract: the holding contract and they establish the holding company (called also the holding or the New Co⁵). If they still need further funds, they can ask for financing from the bank. Then, they sign a second contract: the debt contract.

The optimal financial contracts have to meet three objectives: (1) each agent recovers at least the cost of his initial investment (2) to determine the payments of each agent when the project succeeds and when it fails and (3) to incite the entrepreneur and the LBO fund to provide efficient efforts.

This research is related to two strands of the literature. The first is the literature on the agency conflicts between the entrepreneur and private equity funds which highlights the importance of adequate incentive-based compensation schemes, the active role of the fund, and the role of stage financing and convertible securities in mitigating conflicts. Contrary to the VC, to our knowledge, there is no theoretical paper exploring these issues in LBO. Moreover, tax advantages of the debt are not considered in these papers.⁶

For instance, several papers focus on the relationship between capital structure in start-up projects and the incentives of efforts under double-sided moral hazard. Bergemann and Hege (1998) present a dynamic agency model with learning and moral hazard problems. They show that short-term refinancing⁷ is never optimal but long-term contract allowing for intertemporal risk-sharing such as the stage financing⁸ is optimal. The stage financing constrains the entrepreneur to provide optimal effort otherwise the project will not reach its short term targets. Consequently, the VC fund will not finance the next stage.

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⁴The venture capital is like the LBO, a private equity investment in which professional outside investors provide funds to new and high-potential-growth companies. Their aim is to take the company public through an IPO (Initial Public Offering). However, the LBO investors take the acquired company private. The VC is called the private to public while the LBO is the public to private.

⁵New Co is the abbreviation of New Company.

⁶In venture capital, the start-ups cannot usually issue debt because the entrepreneur is wealth constrained and she cannot offer any collateral to secure the VC’s investment except for the venture’s returns.

⁷The entrepreneur sign a new contract with a new VC fund, but only for one period. The next period, she is looking for another one; the VC market is supposed to be competitive.

⁸While the commitment is to fund the entire amount, in venture capital projects, the funding is contingent on the company attaining his short-term objectives. At each stage, the entrepreneur asks for additional capital from the VC fund.
Cornelli and Yosha (2003) show that the stage financing may induce a *window dressing* problem: the entrepreneur presents good short-term performance whether the project’s result of the period is good or bad to avoid the abandon or the liquidation of the project. They conclude that the use of the convertible debt solves the *"window dressing"* problem. When the project looks too profitable, the VC fund will convert his debt into equity which reduces the entrepreneur’s profit.

In the current model, we consider that the whole project’s cost is paid at the date of the signature of the contracts and ignore the learning problem. We show that under some restrictive conditions, these contracts induce them to invest efficiently.

Schmidt (1999, 2003) focuses on the incentive properties of the convertible securities and shows that there is no debt-equity contract that induces both the entrepreneur and the VC fund to invest efficiently. He argues that the use of convertible securities leads both agents to provide optimal efforts. When he considers a multi-dimensional investment\(^9\), the convertible debt contract still implements the first best investments of both parties. In the current paper, for the sake of simplicity, we consider that the LBO fund does not issue convertible securities to assess the properties of the debt financing.

My model is closely related to those of Casamatta (2003) and Repullo and Suarez (2004). They present a double-sided moral hazard model with three agents. The third part in their models is a pure financier.

Casamatta (2003) considers that the efforts are substitutes and that the incentives of the entrepreneur and the fund depend on their payments. She shows that the involvement of the third party allows the implementation of the first best efforts only if the project is not very risky: the pure financier is used as a budget breaker. In the current model, despite the fact that the efforts are complementary, we generalize the Casamatta’s results and show that there are many financial contracts that implement the first best efforts. Optimally chosen, the entrepreneur does not contribute financially to the project. However, the bank cannot be used as a budget breaker otherwise the debt’s payments should be decreasing with the project’s returns. We conclude that there is no debt-equity contract in practice leading the two parties to invest efficiently. Contrary to Casamatta, we show that under taxation, the entrepreneur prefers hiring a consultant who provides only business advice rather than contracting with an LBO fund who provides both funds and advice.

Repullo and Suarez (2004) consider a wealth constrained entrepreneur and argues that she must ask for financing only from the partner who provides both advice and funds. Consequently, the entrepreneur relies only on the fund. We show that financing the project with a mixture of debt and equity or only through equity lead the entrepreneur and the LBO fund to provide the same levels of efforts. However, the entrepreneur prefers always contracting with both the LBO fund and the bank rather than hiring only the former.

The second strand of the literature addresses the tax advantages of the debt financing. Modigliani and Miller (1963) show that the investment decisions depend on the financial capital structure when

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\(^9\) First, he assumed that entrepreneur’s investment is one-dimensional. Then, he supposed that the entrepreneur has to choose a multi-dimensional investment vector such as to invest in R&D, to spend effort in order to organize the firm, to hire the key staff and to invest in marketing, in supplier and customer relations.
there is taxation. In contrast with Modigliani and Miller (1963), Miller (1977) distinguished between the corporate and the personal income taxes and find that for individual, the effects of the two types of taxes end up cancelling each other. He suggests that there is a tax saving advantage only in macroeconomic level.

In these models, there is no asymmetric information. When we apply the corporate income tax to the revenues, despite the double sided moral hazard problem, the deductibility of the debt’s interests seems to be the appropriate explanation of the bank’s involvement in the financing of the LBO projects.10

Lowenstein (1985) argues that tax plays a major role in explaining the increase of the firm’s value. Kaplan (1989a) consider a sample of 76 LMBO11 projects exited between 1980 and 1986. He reports that the median value of tax benefits estimated at the time of going private through LBO has a lower bound of 21% and an upper bound of 143% of the premium paid to the pre-buyout shareholders. Notice that their estimated value depend on the rate of the debt of the acquired company paid and on the tax rate applied to the interest deductions. In a second paper, (Kaplan, 1989b) estimates that interest tax shield can explain from 4% to 40% of the firm’s value. However, Kaplan and Strömberg (2008) explain that "It is safe to say, therefore, that taxes create some value, but difficult to say exactly how much" because it requires restrictive assumptions of the tax advantage of debt (net of personal taxes), the expected permanence of the debt, and the riskiness of the tax shield. Jensen (1989) notices that there are other tax revenues which are not studied in these projects12 and argues that these taxes compensate the tax saving advantages of the LBO’s debt13. Most of these papers are empirical, they do not study the effects of the tax-deductibility on the agents’ incentives when there is an asymmetric information. When the debt’s interests are tax-deductible, we show that the entrepreneur relies on the debt financing to save taxes and not to improve their efforts.

The paper is structured as follows. The model and the assumptions are presented in the Section 1. The model is then solved for the optimal financing strategy in Section 2. In Section 3, we consider some variations on these contracts and compares the agents’ incentives under debt-equity contract and a pure equity contract. We introduce the tax advantage in the model and solve for the optimal financial capital in Section 4. Concluding remarks are in Section 5. All the proofs are presented in the Appendix.

10 This result is robust when we add the personal income tax which implies that the Miller’s result does not hold in our framework.

11 LMBO (Leveraged Management Buy Out) is a LBO project where the entrepreneur(s) is/are the manager(s) and/or the employees of the acquired company.

12 He counts five sources of additional tax revenues generated in these projects: capital gains taxes paid by pre-buyout shareholders, capital gains taxes paid on post-buyout asset sales, tax payments on the large increases in operating earnings generated by efficiency gains, tax payments by creditors who receive interest payments on the LBO debt and taxes generated by more efficient use of the company’s total capital.

13 As an example, see the RJR Nabisco (Jensen, 1989).
1. The model

We consider an entrepreneur \( E \) who wants to acquire a company. The amount of funds needed is denoted by \( K > 0 \) and is exogenously given. She asks first for advice and for funds from the LBO fund \( A \). If he invests money into the project, he issues the amount of equity \( i \) in exchange of an outcome’s share: \( 1 - \beta \) (\( 0 \leq \beta \leq 1 \)). The two partners may ask for further funds from the bank. Let \( I \) denotes the amount of the debt. If they still need further funds, the entrepreneur issues the residual capital \( W = K - (i + I) \).

The project is risky and generates an observable random revenue \( \theta \). It depends on the quality of the project, the entrepreneur skills, the market conditions... It may take either a high or a low value: \( \theta^u \) when the project succeeds and \( \theta^d \) when it fails (\( \theta^u > \theta^d \)). When the project fails, \( \theta^d \) is equal to its liquidation value. The probability of success is denoted \( p(e, a) \), where \( e \) and \( a \in \mathbb{R}_+ \) are the efforts provided respectively by the entrepreneur and the LBO funds. The entrepreneur’s effort depends on her technical skills and past experience. The LBO fund provides a technical/managerial effort such as the monitoring or providing business advice. We assume that the probability function \( p(e, a) \) is increasing and concave, and add the condition \( \frac{\partial^2 p(e, a)}{\partial e \partial a} > 0 \) to ensure that these efforts are complementary. Furthermore, \( p(e, 0) = p(0, a) = 0 \) which means that both agents must provide strictly positive efforts otherwise the project fails with probability 1.

There is a continuum of LBO fund and banks. All agents are risk neutral and the risk-free rate is normalized to 0. When contracts are signed, agents cannot abandon the project before the date of exit.

1.1. The sequence of events in the model

The sequence of events is summarized in the following figure:

![Sequence of Events Diagram](image)

- At the date 0, \( E \) and \( A \) negotiate and sign a first contract: the holding contract. If they still

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\(^{14}\)In the current model, the entrepreneur and the LBO fund are supposed to be not wealth-constrained.

\(^{15}\)Making efforts \((e, a) = (0, 0)\) is a Nash equilibrium but it leads also to the failure of the project.
need further funds, they can ask for financing from the bank. Then, the holding and the bank $B$ sign a second contract: the debt contract.

- At the date 1, $E$ and $A$ have to provide respectively the non contractible efforts $e$ and $a$. Both efforts are costly, let $u(e)$ and $v(a)$ denote their cost functions. Assume that these functions are increasing, convex and satisfy $u(0) = v(0) = u'(0) = v'(0) = 0$.

- At the date 2, the project is completed. If it succeeds, the bank gets the payment $D$ and the two agents share the residual amount; they get respectively $\beta(\theta^u - D)$ and $(1 - \beta)(\theta^u - D)$, otherwise the bank is paid the collateral $H$ ($H \leq \theta^d$). If $H = \theta^d$, the entrepreneur and the LBO fund get zero payoffs. If $H < \theta^d$, they obtain respectively $\beta(\theta^d - H)$ and $(1 - \beta)(\theta^d - H)$.

1.2. Financial contracts

To keep things simple, we consider that there are two financial contracts that must be specified in the model:

1. The holding contract $(i, \beta)$
   It specifies the amount of equity that must be issued by the LBO fund and his share of benefits $(1 - \beta)$. He participates in the project if his expected gain is positive. This condition is written:
   \[
   EU^A = (1 - \beta)[p(e, a)(\theta^u - D) + (1 - p(e, a))(\theta^d - H)] - v(a) - i \geq 0 \quad (PC_A)
   \]
   where $(PC_A)$ is the participation constraint of the LBO funds. Because of the competition among the LBO funds, the LBO fund $A$ proposes the best contract possible to the entrepreneur. The $(PC_A)$ is therefore binding.

2. The debt contract $(I, D, H)$
   The bank lends $I$ at the date 0. At the date 3, if the project’s result is a success, he is paid $D$, otherwise he gets $H$. The bank is willing to lend money only if he will gain the same amount of money if he makes a risk-free investment. This means that:
   \[
   EU^B = p(e, a)D + (1 - p(e, a))H - I \geq 0 \quad (PC_B)
   \]
   where $(PC_B)$ is the participation constraint of the bank. Because of the competition among the banks, this constraint is also binding.

1.3. The first best efforts

Before solving the game, let us determine the optimal levels of efforts when all inputs are contractible when there is no asymmetric information. The social value (or the net present value NPV) of the project is given by:

\[
 V(e, a) = p(e, a)\Delta \theta + \theta^d - u(e) - v(a) - K
\]
where $\Delta \theta = \theta^u - \theta^d$. The first best efforts $e^{FB}$ and $a^{FB}$ are given by the first order conditions of $V(e, a)$:

$$\frac{\partial V(e, a)}{\partial e} \Big|_{e^{FB}, a^{FB}} = p_e(e^{FB}, a^{FB}) \Delta \theta - u'(e^{FB}) = 0$$

$$\frac{\partial V(e, a)}{\partial a} \Big|_{e^{FB}, a^{FB}} = p_a(e^{FB}, a^{FB}) \Delta \theta - v'(a^{FB}) = 0$$

The equations (1) and (2) can be written:

$$\Delta \theta = \frac{u'(e^{FB})}{p_e(e^{FB}, a^{FB})} = \frac{v'(a^{FB})}{p_a(e^{FB}, a^{FB})}$$

The ratios of the marginal cost to the marginal probability of efforts are equal to the difference between the revenues of success and failure. If the agents provide the first best efforts, the probability of success is denoted by $p^{FB} = p(e^{FB}, a^{FB})$. Then, the optimal social value of the project is given by:

$$V^{FB} = p^{FB} \Delta \theta + \theta^d - u(e^{FB}) - v(a^{FB}) - K$$

$V^{FB}$ is assumed to be strictly positive.

There are many ways to implement the first best solution. If there are no transaction costs, the identity of the agent providing funds is irrelevant. The entrepreneur may ask for money from the bank ($I = K$) and for advice from the LBO fund: they sign two contracts (the holding and the debt contracts). The latter can also provide both without relying on the debt financing ($i = K$). So, they sign one contract: the holding contract. The entrepreneur can also finance the project alone ($W = K$) and relies on the advice of a consultant who provides only the effort $a$. She is indifferent whether the adviser is a consultant or an LBO fund. We show hereafter that under moral hazard, this is no longer true.

2. The optimal financial contracts

2.1. The incentive constraints

We consider now that the efforts are unobservable. The entrepreneur (respectively the LBO fund) provides the level of effort $e$ (respectively $a$) that maximizes her (respectively his) expected profit given its costs, the level of effort provided by the other agent and the signed contracts. Given the fact that the efforts $e$ and $a$ are complementary, the effort of each agent is expected to be an increasing function of the other effort.

The incentive constraints of $E$ and $A$ are solutions of:

$$e(a) \in \arg \max_{e \in \mathbb{R}_+} \beta[p(e,a)(\theta^u-D) + (1-p(e,a))(\theta^d-H)] - u(e) - W \quad (IC_E)$$

$$a(e) \in \arg \max_{a \in \mathbb{R}_+} (1-\beta)[p(e,a)(\theta^u-D) + (1-p(e,a))(\theta^d-H)] - v(a) - i \quad (IC_A)$$
which gives
\[ \beta(\Delta \theta - D + H) = \frac{u'(e)}{p_e(e, a)} \]  
(4)
\[ (1 - \beta)(\Delta \theta - D + H) = \frac{\nu'(a)}{p_a(e, a)} \]  
(5)

Differentiating the equations (4) and (5) enables us to write:

\[ e'(a) = \frac{u'(e)p_{ea}(e, a)}{u''(e)p_e(e, a) - u'(e)p_{ee}(e, a)} \geq 0 \]
\[ a'(e) = \frac{\nu'(a)p_{ea}(e, a)}{\nu''(a)p_a(e, a) - \nu'(a)p_{aa}(e, a)} \geq 0 \]

The effort of the entrepreneur is, as explained previously, an increasing function of the effort of the LBO fund, and vice-versa.

### 2.2. The optimal financial structure

Competition among the LBO funds and the banks induces \( A \) and \( B \) to propose respectively the holding and the debt contracts which maximize the expected profit of the entrepreneur. The optimal financial contracts are solution of the following program:

\[ (\beta^*, i^*, I^*, D^*, H^*) \in \arg \max_{\beta, i, I, D, H} EU^E = \beta[p(e, a)(\Delta \theta - D + H) + \theta^d - H] - u(e) - K + i + I \]
\[ \text{s.t.} \quad (PC_A), (PC_B), (IC_A) \text{ and } (IC_E) \]

with the additional conditions:

\[ 0 \leq D \leq \theta^u, \ 0 \leq H \leq \theta^d \text{ and } 0 \leq \beta \leq 1 \]  
(6)

Given the fact that the participation constraints \( (PC_A) \) and \( (PC_B) \) are binding, the amounts of equity and debt issued respectively by the LBO fund and the bank are given by:

\[ i = (1 - \beta)[p(e, a)(\Delta \theta - D + H) + \theta^d - H] - v(a) \]  
(7)
\[ I = p(e, a)(D - H) + H \]  
(8)

We replace \( i \) and \( I \) respectively with (7) and (8) in \( EU^E \). The optimal financial contracts lead the entrepreneur to maximize the social value of the project under the incentive constraints \( (IC_E) \) and \( (IC_A) \):

\[ \max_{\beta, D, H} V(e, a) = p(e, a) \Delta \theta + \theta^d - u(e) - v(a) - K \]
\[ \text{s.t.} \quad (4) \text{ and } (5) \]  

with the conditions (6).
Proposition 1 There is no debt-equity contract that induce the entrepreneur and the LBO fund that solve the double sided moral hazard problem.

The proof of this proposition is presented in the appendix A.

First, we show that the optimal financial contracts depend on the quality of the entrepreneur’s project.

If the project is not very risky, in the sense $\theta^u \leq 2\theta^d$, we show that there are many financial contracts that implement the first best efforts. Contrary to Casamatta (2003), the project is financed only through the equity issued by the LBO fund and the debt raised by the bank. Whether the entrepreneur is wealth constrained or not, she does not contribute financially to the project which means that all the projects which are not very risky may be taken private through buyout acquisition.

Under the moral hazard, the entrepreneur prefers contracting with the LBO fund rather than hiring a consultant who exerts the effort $a$ without investing money into the acquisition.

Whatever the impact of their efforts on the project’s performance, the payments of the two parties are equal despite the fact that the entrepreneur does not issue equity: they get $\theta^u - K - v(a^{FB})$ when the project succeeds and $\theta^d - K - v(a^{FB})$ when it fails. The difference between these payments is equal to the difference between the revenues of the project $\Delta \theta < \theta^d$. The more the difference $\Delta \theta$ is large, the more it is difficult to reach the first-best levels of efforts because they become very high.

In order to induce the agents to provide optimal efforts, the bank payments must be decreasing with the project’s outcome. Indeed, $\Delta \theta$ is not very large, the two parties have to exert low levels of efforts to get the highest payments; the success payments.

Without constraining the debt’s payments to be nondecreasing with the revenues, as shown in Casamatta (2003), if the project is not very risky, the involvement of a third part in the project constrains the agents to provide the first best efforts. In addition, the entrepreneur captures the whole social value of the project while the others recoup their money in expectations. So, the third part plays the role of the budget breaker. However, as explained before the bank is expecting higher payment in the good state of nature. This implies that there is no debt-equity contract that implement the first best levels of efforts when the project is not very risky.

When the project is very risky in the sense $\theta^u > 2\theta^d$, the financial contracts are unique in the specified model and the three agents must invest jointly money into the project.

Notice that again the bank’s payments should be decreasing with the revenues and the specified contract should exhibit the features of a live or die contract (Iness, 1990, Casamatta, 2003). The bank is paid only in case of failure; he gets all the failure’s revenue, and the success revenue is shared between the entrepreneur and the LBO fund.

The moral hazard problem induces the agents to make non optimal efforts. Notice that the first best levels of efforts become high ($\Delta \theta$ becomes very large), we need powerful incentive mechanisms to encourage the entrepreneur and the LBO fund to provide optimal efforts. The threat of leaving the whole failure revenue to the bank and the incentive of sharing the success revenue between them
is not enough to induce them to exert the first-best levels of efforts. Consequently, the entrepreneur cannot get the whole optimal social value.

Furthermore, whether the project is very risky or not, under the moral hazard, the entrepreneur relies on an LBO fund rather than a consultant who contributes only with business advice.

To get the highest revenues, the entrepreneur and the LBO fund could collude and claim that the result is a success in all the states of nature. If the project fails, they sell the assets and the equipment, pay $D$ to the bank and share the residual amount. To avoid such behavior, hereafter we consider that the debt’s payments are nondecreasing with the project’s revenues. We assume that the bank’s payment is the highest in the good state of nature. So, we consider the specified model and add the following condition:

$$D \geq H.$$  \hspace{1cm} (9)

to the entrepreneur’s program.

The solution shows that the financial contracts do not depend on the quality of the project and that the entrepreneur does not contribute financially to the project. Contrary to Repullo and Suarez (2004), the entrepreneur asks for financing from both the LBO fund and the bank, despite the fact that the latter contributes only with money. In our model, the entrepreneur still prefer contracting with an LBO fund than hiring a consultant\(^{16}\).

Whether the project is very risky or not, the nondecreasing condition constraint gives the bank at least as much in the good state as in the bad state. He gets equal payments in cases of success and failure: these payments are equal to the amount of debt raised by the bank. The bank interest rate is therefore null so lending money to the entrepreneur and the LBO fund is not a risky investment for the bank: he gets his money back whether the project succeeded or failed.

Adding the condition of nondecreasing payments of the bank leads the entrepreneur and the LBO fund to exert non optimal efforts. Accordingly, we conclude that there is no debt-equity contract that induces them to invest efficiently.

\section{Debt-equity contract or pure equity contract?}

In this section, we raise the following question: why the entrepreneur does not ask for advice from a consultant and for financing from the bank?

To answer the question, we consider that the entrepreneur can either hires a consultant who provides only the effort $a$, or an LBO fund who provides both money and effort.

**Lemma 1**  \hspace{1cm} \textit{Financing the project through a mixture of debt and equity or solely through equity leads to the same levels of efforts.}

The proof of this lemma is presented in the appendix $B$.

\footnote{\hspace{1cm} If the financier provides both the effort and money, he is an LBO fund. If he does not contribute with money, he is a consultant.}
This lemma states that the entrepreneur hires a consultant despite the fact that he does not contribute financially to the project. This result is in contrast with Casamatta (2003) and is due to our hypothesis of complementary efforts $e$ and $a$.

The entrepreneur invests a strict positive amount of money into the project and the bank raises a lower amount of debt than in the presence of an LBO fund. This implies that the amount of debt is an increasing function of the amount of equity issued by the LBO fund.

If the project is not very risky, the first-best levels of efforts are not very high and the threat of leaving a larger share’s outcome to the bank in case of failure, are sufficient to motivate the entrepreneur and the consultant to exert optimal efforts. However, the entrepreneur still prefer contracting with the LBO fund rather than hiring a consultant because she must concede a share of her rent to the latter to encourage his effort. Consequently, the expected gain of the entrepreneur is not optimal and the participation constraint of the consultant is not binding: the consultant is paid the difference between the optimal social value of the project and the entrepreneur’s expected gain.

In this case the levels of efforts are lower than those provided in the presence of the LBO fund. In order to get the consultant’s advice necessary to succeed the project, the entrepreneur must pay him a share of her own profit. This affects the entrepreneur’s incentives in three ways:

1. Paying the consultant decreases the entrepreneur’s profit which decreases her effort’s incentives.

2. Recall that $e$ and $a$ are complementary and costly. In order to increase the success probability, the financial contracts must motivate both. If the effect of the consultant’s effort on the project’s performance is larger than the entrepreneur’s effort, this implies that the consultant advice is more efficient which should increase his payment and simultaneously, the entrepreneur should provide higher effort; costly efforts. Otherwise, encouraging the LBO fund alone to provide high effort has no impacts on the project’s performance.

3. Increasing the levels of efforts $e$ and $a$ increases the success probability.

The effects (1) and (2) are negative, the third one is positive but not high enough to compensate the other ones which decreases the agents’ incentives. Indeed, given the fact that the efforts are complementary, the entrepreneur must hire the consultant to get the business advice she needs. This advice increases the value of the project. The entrepreneur is not able to recoup the cost of this enhancement in social value, however. The rent she would have to surrender to the consultant would be too large compared to the increase in value the consultant’s advice would induce, but it is necessary otherwise the project is not implemented. In the previous sections, when the LBO’s rent can be captured by asking him to contribute sufficiently to the project: when he invests money, he reduces the cost of getting the effort $a$.

If we constrain the debt’s payments to be non-decreasing with the revenues, the presence of the consultant leads the entrepreneur to finance her project only through debt. Financing the whole acquisition price $K$ only through the debt is not a risky task for the bank: he is paid as much when the project succeeded as when it failed. The bank interest rate is therefore null. This implies also
that all the projects may be taken private through buyout acquisition whether the entrepreneur is wealth constrained or not.

Despite the fact that both parties provide the same levels of efforts as in the previous section, the entrepreneur prefers contracting with an LBO fund than hiring a consultant.

4. The optimal financial structure with taxation

In this section, we consider that the debt’s interests are tax-deductible and study the impact of the tax saving advantage on the agents’ incentives and on the financial capital structure.

Let $\tau$ denotes the corporate income tax, $0 \leq \tau \leq 1$. Hereafter, we assume that $D = (1 + r) I$ where $r$ is the bank interest rate.

After taxation, the success revenue is $(1 - \tau) \theta^u - (1 + r) I] + (1 + r) I = (1 - \tau) \theta^u + \tau (1 + r) I$

where $(1 - \tau) \theta^u - (1 + r) I$ is shared between the entrepreneur and the LBO fund. The failure revenue is equal to $(1 - \tau) (\theta^d - H) + H = (1 - \tau) \theta^d + \tau H$ where $(1 - \tau) (\theta^d - H)$ is shared between them. If the project is liquidated $(H = \theta^d)$, they have no payments.

Lemma 2 When the debt’s interests are tax-deductible, it is optimal to finance the acquisition only through debt.

The proof is presented in the appendix $C$.

Under taxation, financing the project through debt does not improve the agent’s incentives: a debt-equity contract or a pure equity contract leads them to exert the same levels of efforts. These efforts are not optimal and are decreasing with the corporate income tax $\tau$.

The intuition is the following, whether the project is very risky or not, the bank is paid as much in the good state as in the bad state. His payment is equal to the amount of the raised debt. The partners’ payments, and consequently their incentives, do not depend on the presence of the bank. However, the tax-deductibility of the interests make this presence very useful: it is a mean to improve their payments. In other words, the tax-deductibility of the debt’s interests creates an incentive to raise the highest level possible of debt in spite of an incentive to exert effort. If the financial structure of the project in the previous sections depends on the moral hazard, the result of this lemma holds even if the efforts are observable.

Financing the investment only through debt increases the expected gain of the entrepreneur by $\tau K$ and ensures the LBO fund an expected gain strictly positive despite the fact that he does not invest money into the project. This result implies that under taxation, the entrepreneur prefers hiring a consultant rather than contracting with an LBO fund.

If she would deal with an LBO fund who would invest an amount of money $i$ into the project, her gain would decrease by $\tau i$ in expectation$^{17}$. This is very intuitive result since the equity is not

$^{17}$Notice that the expected gain of the consultant should be not null to ensure effort provision. The participation constraint of the consultant is not binding.
tax-deductible and it does not improve the incentives for efforts provision.

Notice that, under taxation, all the projects looking for financing and advice may be taken private through buyout acquisitions even if the entrepreneur has no initial wealth to invest into her project. Tax savings seem the only convincing theoretical rationale for the bank's involvement in these projects. This in line with many empirical studies, among others Kaplan (1989a) and Lehn and Poulsen (1989), which provide an evidence that many companies are taken private only to save taxes and that without taxation the debt itself is not value adding.

5. Conclusion

This paper has studied the properties of the optimal financial contracts in LBO. We studied the relationship between the agents' incentives and the financial structure under moral hazard. The entrepreneur raise funds from both the LBO fund and the bank, despite the fact that the involvement of the latter agents is not needed to get the project going.

To explain the high level of debt in buyout acquisitions, we show that the tax-deductibility of the debt's interests provides a rationale for the bank's involvement. However, it does not improve the incentives for efforts provision.

It would probably be interesting to study the use of convertible securities in the presence of a passive financier as the bank. Another topic for further research is how the cash-flow and control rights are allocated when the profitability of the project varies. To our knowledge, research in this direction is still pending.
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Appendix

More formal proofs are available upon request.

A. Proof of proposition 1

Our proof is in two steps.

1st step

The social value of the project is optimal if the agents provide the first best efforts, in other words, the efforts $e^{FB}$ and $a^{FB}$ must satisfy the incentive constraints (4) and (5).

Given the equations (3), the constraints (4) and (5) give:

$$\Delta \theta = \beta(\Delta \theta - D + H) = (1 - \beta)(\Delta \theta - D + H)$$  \hspace{1cm} (A1)

which implies that the entrepreneur and the LBO fund share equally the project’s outcome: $\beta = \frac{1}{2}$. Given the fact that the project’s revenues are fixed, if the optimal contracts assign the highest share to the entrepreneur, this encourages her effort but it reduces the incentives of the LBO fund. The optimal financial contracts must boost simultaneously the incentives of both; so they must get equal shares. We notice that this sharing rule depends neither on the capital structure, nor on the efforts.

If we replace $\beta = \frac{1}{2}$ in (A1), we obtain:

$$H = \Delta \theta + D$$

This means that $\Delta \theta + D < \theta^d \iff D < 2\theta^d - \theta^u$, but $D \geq 0$, in other words, the project must be not very risky in the sense $\theta^u \leq 2\theta^d$ and the failure payment of the bank is higher than his success payment. But, in practice, the bank is paid higher payment if the project succeeded.

Given the participation constraints of the LBO fund and the bank $(PC_A)$ and $(PC_B)$, the amounts of $i$ and $I$ are written respectively:

$$i = \frac{1}{2}[2p^{FB}(e, a)\Delta \theta + \theta^d - H] - v(a^{FB})$$  \hspace{1cm} (A2)

$$I = -p^{FB}(e, a)\Delta \theta + H$$  \hspace{1cm} (A3)
Moreover, given $K - W = i + I$, we conclude that:
\[
H^* = 2[K - W + v(a^{FB})] - \theta^d
\]
\[
D^* = 2[K - W + v(a^{FB})] - \theta^u
\]

We replace $H^*$ and $D^*$ in the participation constraints $(PC_A)$ and $(PC_B)$, we get the optimal amounts of equity and debt which must be issued respectively by the LBO fund and the bank:
\[
i^* = p^{FB} \Delta \theta + \theta^d + W - K - 2v(a^{FB})
\]
\[
= V^{FB} + u(e^{FB}) - v(a^{FB}) + W
\]
\[
I^* = 2[K - W + v(a^{FB})] - p^{FB} \Delta \theta - \theta^d
\]

These optimal solutions exist under the following conditions:
\[
H^* \geq 0 \iff \theta^d \leq 2[K - W + v(a^{FB})] \tag{A4}
\]
\[
D^* \geq 0 \iff \theta^u \leq 2[K - W + v(a^{FB})] \tag{A5}
\]
\[
i^* \geq 0 \iff \theta^d \geq K - W + 2v(a^{FB}) - p^{FB} \Delta \theta \tag{A6}
\]
\[
I^* \geq 0 \iff \theta^d \leq 2[K - W + v(a^{FB})] - p^{FB} \Delta \theta \tag{A7}
\]
\[
H^* \leq \theta^d \iff \theta^d \geq K - W + v(a^{FB}) \tag{A8}
\]
\[
D^* \leq \theta^u \iff \theta^u \geq K - W + v(a^{FB}) \tag{A9}
\]

The equation (A4) (respectively (A9)) is satisfied if (A5) (respectively (A8)) is satisfied.

On one hand, the conditions (A5) and (A8) imply that:
\[
l \leq \theta^d < \theta^u \leq 2l \tag{A10}
\]

where $l = K - W + v(a^{FB})$.

On the other hand, the conditions (A6) and (A7) give:
\[
l + v(a^{FB}) - p^{FB} \Delta \theta \leq \theta^d \leq 2l - p^{FB} \Delta \theta \tag{A11}
\]

but $l - v(a^{FB}) = K - W > 0$ which means that (A11) is always satisfied. Given that $2l - p^{FB} \Delta \theta < 2l$, (A10) and (A11), we deduce that:
\[
l \leq \theta^u \leq 2l \tag{A12}
\]
\[
l + \min\{v(a^{FB}) - p^{FB} \Delta \theta, 0\} \leq \theta^d \leq 2l - p^{FB} \Delta \theta \tag{A13}
\]

We conclude that if the project is not very risky in the sense $\theta^u \leq 2\theta^d$, under the conditions (A12) and (A13), all the following financial contracts implement the first best efforts:
\[
\beta = \frac{1}{2}
\]
\[
i = V^{FB} + W + u(e^{FB}) - v(a^{FB})
\]
\[
W \in [0, K]$

and

\[
D = 2[K - W + v(a^{FB})] - \theta^u \\
H = 2[K - W + v(a^{FB})] - \theta^d \\
I = 2[K - W + v(a^{FB})] - p^{FB} \Delta \theta - \theta^d
\]

where \( l = K - W + v(a^{FB}) \).

However, given the fact that the financial contracts are chosen to maximize the expected gain of the entrepreneur, the LBO fund and the bank must propose the best contracts possible to her. Consequently, \( EU^E \) is optimal if the entrepreneur does not issue equity; \( W^* = 0 \) which implies that \( i^* + I^* = K \). We conclude therefore that if the project is not very risky (\( \theta^u \leq 2\theta^d \)), under the conditions \( \theta^d \in [l' + \max\{v(a^{FB}) - p^{FB} \Delta \theta, 0\}, 2l' - p^{FB} \Delta \theta] \) and \( \theta^u \in [l', 2l'] \), the optimal financial contracts are given by

\[
\beta^* = \frac{1}{2} \\
i^* = V^{FB} + u(e^{FB}) - v(a^{FB}) \\
W^* = 0
\]

and

\[
D^* = 2[K + v(a^{FB})] - \theta^u \\
H^* = 2[K + v(a^{FB})] - \theta^d \\
I^* = 2[K + v(a^{FB})] - p^{FB} \Delta \theta - \theta^d
\]

where \( l' = K + v(a^{FB}) \). We conclude that if the project is not very risky, the entrepreneur and the LBO fund exert the first-best levels of efforts \( e^{FB} \) and \( a^{FB} \). Moreover, the entrepreneur maximize her expected profit by not investing money into the project.

Now we consider the case of a very risky project where \( \Delta \theta \) becomes very large (\( \theta^u > 2\theta^d \)), the first-best levels of efforts become very high and we need therefore high powerful incentive mechanisms to reach these levels.

The general model does not enable us to deduce the properties of the optimal financial contracts. This is why, we restrict ourselves to the following functional form:

\[ p(e, a) = e^{1-\alpha} a^\alpha \]

where \( \alpha \in [0,1] \) is the LBO’s effort elasticity of the success probability: it measures the \( a \)’s impact on the project’s performance. For the functions of cost, we rely on the following specifications:

\[ u(e) = \frac{e^2}{2\lambda} \text{ and } v(a) = \frac{a^2}{2\mu} \]

where \( e \) and \( a \) take values on \([0,1]\) and \( \lambda > \mu > 0 \) which implies that the LBO’s effort is more costly than the entrepreneur’s effort.
The social value of the project and the first best efforts are written:

\[ V(e, a) = e^{1-\alpha}a^\alpha \Delta \theta - \frac{e^2}{2\lambda} - \frac{a^2}{2\mu} + \theta^d - K \]

\[ e^{FB} = \phi(\alpha)\Delta \theta \quad \text{(A14)} \]

\[ a^{FB} = \left[ \frac{\mu \alpha}{\lambda(1-\alpha)} \right]^{\frac{1}{2}} \phi(\alpha) \Delta \theta \quad \text{(A15)} \]

where \( \phi(\alpha) = [\lambda(1-\alpha)]^{1-\alpha/2}[\mu \alpha]^{\alpha/2} \). These efforts are increasing with the difference between the revenues of the project. Notice that when the project is very risky, the levels of the first best efforts become very large so the agents need powerful incentives mechanisms.

These efforts lead to the success with the probability \( p^{FB} = \frac{\phi^2(\alpha)}{\lambda(1-\alpha)} \). Assume \( \Delta \theta \leq \frac{\lambda(1-\alpha)}{\phi^2(\alpha)} \) such that this probability is inferior to 1.

When \( a \)'s impact on the success probability is high, in the sense \( \alpha \geq \frac{1}{\mu+\lambda} \), despite the fact that it is the most expensive effort, \( a^{FB} \) is larger than \( e^{FB} \), otherwise the entrepreneur’s effort is more efficient \( (e^{FB} > a^{FB}) \).

The optimal social value of the project is given by:

\[ V^{FB} = \frac{[\phi(\alpha)\Delta \theta]^2}{2\lambda(1-\alpha)} + \theta^d - K \]

Notice that the NPV of the project is increasing with the \( e \)'s impact on the success probability. When \( \alpha \) is small, in the sense \( \alpha < \frac{1}{\mu+\lambda} \), \( V^{FB} \) decreases with \( \alpha \) and increases otherwise.

When the efforts are not contractible, the incentives constraints (4) and (5) give the following efforts in equilibrium:

\[ e^* = [\lambda \beta(1-\alpha) \rho(\beta)]^{1/2}(\Delta \theta - D + H) \quad \text{(A16)} \]

\[ a^* = [\mu \alpha(1-\beta) \rho(\beta)]^{1/2}(\Delta \theta - D + H) \quad \text{(A17)} \]

where \( \rho(\beta) = [\lambda \beta(1-\alpha)]^{1-\alpha}[\mu \alpha(1-\beta)]^{\alpha} \). It is easy to check that the difference between the bank’s payments in cases of success and failure must be inferior to the difference between the project’s revenues \( \theta^d \) and \( \theta^u \), i.e.

\[ D - H \leq \Delta \theta \quad \text{(A18)} \]

Otherwise, the efforts (A16) and (A17) could be negative. Besides, the agents’ efforts are increasing with the difference between the project’s revenues. The more high the bank’s payment in case of success, the less the entrepreneur and the LBO fund will be induced to provide efforts. In contrast, the increase of \( H \) (the threat of getting low payments) increases the efforts \( e \) and \( a \).

Then, the project succeeds with the following probability:

\[ p^* = \rho(\beta)(\Delta \theta - D + H). \]
Assume \((\Delta \theta - D + H) \leq \rho^{-1}(\beta)\) to ensure that this probability is inferior to 1.

If the financial contracts assign the whole revenue to the entrepreneur or to the LBO fund, the project will fail with probability equal to 1 \((\rho(0) = \rho(1) = 0)\). This enables us to conclude that: 
\(0 < \beta < 1\).

The expected gain of the entrepreneur is given by:

\[
EU^E = \frac{1}{2} (1 + \alpha) \beta \rho(\beta) (\Delta \theta - D + H)^2 + \beta \left( \theta^d - H \right) - W
\]  

(A19)

Because of the competition among the LBO funds and the bank, the participation constraints \(CP_A\) and \(CP_B\) are binding. We deduce that:

\[
i = (1 - \frac{1}{2} \alpha) (1 - \beta) \rho(\beta) (\Delta \theta - D + H)^2 + (1 - \beta) \left( \theta^d - H \right)
\]

(A20)

\[
I = \rho(\beta) (\Delta \theta - D + H) (D - H) + H
\]

(A21)

We replace (A16), (A17), (A20) and (A21) in (A19). Consequently, the entrepreneur is induced to maximize the social value of the project:

\[
\max_{\beta, D, H} V(.) = \rho(\beta) \Delta \theta (\Delta \theta - D + H) + \theta^d - K
\]

\[-\frac{1}{2} (\alpha + \beta - 2 \alpha \beta) \rho(\beta) (\Delta \theta - D + H)^2
\]

with the conditions:

\(0 < \beta < 1, 0 \leq D \leq \theta^u\) and \(0 \leq H \leq \theta^d\)  

(A22)

The straight application of the Khun-Tucker theorem gives:

\[
\hat{\beta}(\alpha, k) = \begin{cases} 
(1-\alpha)(2-\alpha)-k-\sqrt{\Delta} & \text{if } \alpha \in [0, 1/2] \\
\frac{1}{2} & \text{if } \alpha = \frac{1}{2}
\end{cases}
\]

\[
\hat{i} = (1-\frac{1}{2} \alpha) (1 - \hat{\beta}) \rho(\hat{\beta}) (\theta^u)^2
\]

and

\[
\hat{D} = 0, \quad \hat{H} = \theta^d \quad \text{and} \quad \hat{i} = \theta^d (1 - \theta^u \rho(\hat{\beta}))
\]

\[
\hat{W} = K - \hat{i} - \hat{I}
\]

where 
\[k = \frac{\theta^d}{\theta^u}, \Delta = \alpha (1 - \alpha) \theta^u \left[ (1 + \alpha)(2 - \alpha) \theta^u - 6 \theta^d \right] + (\theta^d)^2 \quad \text{and} \quad \rho(\hat{\beta}) = [\lambda \hat{\beta}(1 - \alpha)]^{1-\alpha} [\mu\alpha(1 - \hat{\beta})]^\alpha.
\]

Contrary to the general model, the optimal sharing rule \(\hat{\beta}\) depends now on the efforts; on the value of \(\alpha\). If \(\alpha > \frac{1}{2}\), the LBO’s share should be larger than the entrepreneur’s one. Despite the fact that \(\alpha\) is more costly than \(\epsilon\), it has the highest impact on the success probability, so the optimal financial contracts should provide powerful incentives to the LBO fund to motivate him to exert more effort. Consequently, they give him the highest outcome’s share whatever the project’s result.
Otherwise, the entrepreneur’s effort is more efficient. Accordingly, the optimal contracts should assign him the highest share of revenues.

When the project is highly risky, the agents make the suboptimal efforts:

$$\hat{e} = [\lambda \hat{\beta}(1 - \alpha)\rho(\hat{\beta})]^{\frac{1}{2}}\theta^u \quad \text{and} \quad \hat{a} = [\mu \alpha (1 - \hat{\beta})\rho(\hat{\beta})]^{\frac{1}{2}}\theta^u \quad \text{(A23)}$$

Replacing $\rho(\hat{\beta})$ by its expression in (A23) gives:

$$\hat{e} = \hat{\beta}^{1 - \frac{\alpha}{2}}(1 - \hat{\beta})^{\frac{\alpha}{2}}\phi(\alpha)\theta^u \quad \text{and} \quad \hat{a} = \hat{\beta}^{\frac{1 - \alpha}{2}}(1 - \hat{\beta})^{\frac{1 + \alpha}{2}}\left[\frac{\mu \alpha}{\lambda(1 - \alpha)}\right]^{\frac{1}{2}}\phi(\alpha)\theta^u \quad \text{(A24)}$$

where $\phi(\alpha) = [\lambda(1 - \alpha)]^{1 - \frac{\alpha}{2}}[\mu \alpha]^{\frac{\alpha}{2}}$.

These efforts depend also on $\alpha$. Let us focus on the particular case where the two efforts have equal impacts on the success probability; $\alpha = \frac{1}{2}$. Hence $\hat{e}$ and $\hat{a}$ are written:

$$\hat{e} = \frac{1}{4}\lambda^{\frac{1}{2}}\mu^{\frac{1}{2}}\theta^u = \frac{1}{4}\frac{\theta^u}{\Delta e^{FB}} \quad \text{and} \quad \hat{a} = \frac{1}{4}\lambda^{\frac{1}{2}}\mu^{\frac{1}{2}}\theta^u = \frac{1}{4}\frac{\theta^u}{\Delta a^{FB}} \quad \text{(A25)}$$

which are nonoptimal efforts.

Consequently, the expected gain of the entrepreneur is given by:

$$EU^E = \left(2\alpha\theta^u - \theta^d\right)\frac{\phi^2(\alpha)}{2\lambda(1 - \alpha)}\theta^u + \theta^d - K < V^{FB}$$

She now issues a strict positive amount of equity. If the entrepreneur is wealth-constrained in the sense $W < \hat{W}$, her project is not taken private through buyout acquisition. Only the projects where the entrepreneur has an initial capital $W \geq \hat{W}$ can be financed through both the LBO fund and the bank. This condition implies that the initial wealth of the entrepreneur $W$ must be larger than a minimum level $W$ for financing to be granted.

Whether the project is very risky or not, the bank should be expecting higher payment in case of failure than in case of success. However, in practice the bank’s payments are not decreasing with the project outcome.

These results suggest that the debt should be substituted for example for equity or convertible debt, and the bank for a pure financier as in the framework of Casamatta (2003). Such modifications lead to the Casamatta’s results: under specific conditions, the presence of a third party leads the entrepreneur and the LBO fund to provide optimal levels of efforts because of the threat of having lower payment in case of failure. The pure financier plays the role of the budget breaker.

2$^{nd}$ step

If we add the condition that constraints the deb’s payments to be non-decreasing to the entrepreneur’s program. We have to determine $\hat{\beta}, \hat{D}, \hat{H}$ that maximize the social value of the project:

$$\max_{\beta, D, H} V(.) = \rho(\beta)\Delta \theta(\Delta \theta - D + H) + \theta^d - K$$

$$-\frac{1}{2} (\alpha + \beta - 2\alpha \beta) \rho(\beta)(\Delta \theta - D + H)^2$$
with the following conditions: 

\((A22)\) and \((9)\).

Again, the straight application of the theorem of Kuhn-Tucker shows that there are many financial contracts that enable the entrepreneur and the LBO fund the provide the same levels of efforts; suboptimal efforts. These contracts are given by:

\[
\tilde{\beta}(\alpha) = \begin{cases} 
\frac{2(1+1/2)}{2(1-2\alpha)} & \text{if } \alpha \in [0, 1/2] \\
\frac{2(1-\alpha)(1-\alpha^2)(2-\alpha)}{2} & \text{if } \alpha = \frac{1}{2}
\end{cases}
\]

\(\bar{\rho} = (1-\tilde{\beta})(1-\tilde{\beta}_t)(\Delta \theta)^2 \)

\(W \in [0, K]\)

and

\(\hat{D} = \tilde{H} = \tilde{I} = K - W - (1-\tilde{\beta})(1-\tilde{\beta}_t)(\Delta \theta)^2\)

But because of the competition between the LBO funds and the banks, the agents \(A\) and \(B\) must offer the best contracts possible to the entrepreneur to maximize her expected gain. This implies that the entrepreneur does not invest money into the project.

Under the condition of nondecreasing debt’s payments, the financial contracts are given by:

\[
\tilde{\beta}(\alpha) = \begin{cases} 
\frac{2(1+1/2)}{2(1-2\alpha)} & \text{if } \alpha \in [0, 1/2] \\
\frac{2(1-\alpha)(1-\alpha^2)(2-\alpha)}{2} & \text{if } \alpha = \frac{1}{2}
\end{cases}
\]

\(\bar{\rho} = (1-\tilde{\beta})(1-\tilde{\beta}_t)(\Delta \theta)^2 + \theta^d - K \)

\(\bar{W} = 0\)

and

\(\hat{D} = \tilde{H} = \tilde{I} = K - (1-\tilde{\beta})(1-\tilde{\beta}_t)(\Delta \theta)^2\)

The agents provide the following efforts:

\(\varepsilon = [\lambda(1-\mu\alpha)(1-\tilde{\beta}(\alpha))]^{1/2} \Delta \theta\) and \(\bar{\alpha} = [\mu\alpha(1-\tilde{\beta}(\alpha))\rho(\tilde{\beta}(\alpha))]^{1/2} \Delta \theta\)

which are also written:

\(\varepsilon = [\tilde{\beta}(\alpha)]^{1-\bar{\alpha}}[(1-\tilde{\beta}(\alpha))^{1/2} \bar{e}^{FB} \text{ and } \bar{\alpha} = [\tilde{\beta}(\alpha)]^{1-\bar{\alpha}}[(1-\tilde{\beta}(\alpha))^{1/2} \bar{a}^{FB}\]

Then, the project succeeds with the following probability:

\(\hat{p} = \rho(\tilde{\beta}(\alpha)) \Delta \theta\)

As explained previously, if the LBO’s effort has a lower impact on the success probability, in the sense \(\alpha \in [0, 1/2]\), the LBO fund should perceive the lowest benefit’s share. In contrast, the effort
a is more efficient, so the optimal financial contracts must encourage him to exert the highest effort possible by assigning him the highest share.

If the efforts have equal impacts on the success probability \((\alpha = \frac{1}{2})\): given the fact that \(e\) and \(a\) are strict complementary and that the revenues are fixed, the optimal financial contracts must boost simultaneously the incentives of both agents by giving them equal shares.

**B. Proof of lemma 1**

The participation constraints of the consultant and the bank \((CP_A)\) and \((CP_B)\) are written:

\[
\begin{align*}
EU^A & = (1 - \beta)[p(e, a) (\Delta \theta - D + H) + \theta^d - H] - v(a) \geq 0 \\
EU^B & = p(e, a) (D - H) + H - (K - W) \geq 0
\end{align*}
\]

(B1) \hspace{1cm} (B2)

Hence, the optimal financial contracts maximize the expected gain of the entrepreneur under the participation constraints \((B1)\) and \((B2)\), and the incentive constraints \((4)\) and \((5)\):

\[
\begin{align*}
\max_{\beta, e, a, D, H} \quad & EU^E = \beta [p(e, a) (\Delta \theta - D + H) + \theta^d - H] - u(e) - W \\
\text{s.t} & \quad (4), \ (5), \ (B1) \text{ and } (B2)
\end{align*}
\]

with the conditions \((6)\).

The participation constraints of the consultant and the bank give

\[
\begin{align*}
(1 - \beta) [p(e, a) (\Delta \theta - D + H) + \theta^d - H] - v(a) \geq 0 \\
W & = K - H - p(e, a) (D - H)
\end{align*}
\]

(B3) \hspace{1cm} (B4)

In contrast with the participation constraint of the bank, the participation constraint of the consultant may not be binding.

When we replace \((B4)\) in the objective function of the entrepreneur, we get the following program:

\[
\begin{align*}
\max_{\beta, e, a, D, H} \quad & EU^E = \beta [p(e, a) (\Delta \theta - D + H) + \theta^d - H] - u(e) - K \\
& + H + p(e, a) (D - H) \\
\text{s.t} & \quad (4), \ (5) \text{ and } (B3)
\end{align*}
\]

with the conditions \((6)\).

It is easy to check that the social value of the project is optimal if the agents provide the first best efforts, in other words, when \(\beta = \frac{1}{2}\) and the bank’s payments are decreasing with the revenues’ project, i.e. \(H - D = \Delta \theta\).
The theorem of Khun-Tuker enables us to deduce that all the following contracts implement the first best efforts when the project is not very risky:

\[
\tilde{\beta} = \frac{1}{2} \\
K + p^{FB} \Delta \theta - \theta^d \leq W \leq K - (1 - p^{FB}) \Delta \theta
\]

and

\[
\hat{D} = K - W - (1 - p^{FB}) \Delta \theta \\
\hat{H} = K - W + p^{FB} \Delta \theta \\
\tilde{I} = K - W
\]

But as explained in the appendix A, because of the competition, the LBO fund and the bank offer the best contracts possible to the entrepreneur. These contracts maximize her expected profit, so she invests the lower amount of money possible into the acquisition: \(\hat{W} = K + p^{FB} \Delta \theta - \theta^d\). Accordingly, the expected gains of the entrepreneur and the consultant are given by:

\[
EU^E = \theta^d - K - u(e^{FB}) \\
EU^A = p^{FB} \Delta \theta - v(a^{FB}) = V^{FB} - EU^E
\]

They are assumed to be positive. The entrepreneur concedes a share of her outcome to the consultant to motivate him to provide business advice. Contrary to Casamatta (2003), the entrepreneur must hire a consultant because his advice is complementary to her effort.

If the project is very risky, we replace \(p(e, a) = e^{1-\alpha} a^\alpha\) and \(v(a) = \frac{a^2}{2\mu}\) in the participation constraints of the consultant and the bank (B1) and (B2) which gives:

\[
EU^A = (1 - \beta) \left\{ e^{1-\alpha} a^\alpha (\Delta \theta - D + H) + \theta^d - H \right\} - \frac{a^2}{2\mu} \geq 0 \quad (B5) \\
EU^B = e^{1-\alpha} a^\alpha (D - H) + H - (K - W) \geq 0 \quad (B6)
\]

The expected gain of the entrepreneur is therefore written:

\[
EU^E = \beta \left\{ e^{1-\alpha} a^\alpha (\Delta \theta - D + H) + \theta^d - H \right\} - \frac{e^2}{2\lambda} - W \quad (B7)
\]

The efforts of the entrepreneur and the LBO fund in the equilibrium are given respectively by (A16) and (A17).

Because of the competition among the banks, the participation constraint of the bank \((CP_B)\) is binding which implies that:

\[
W = K - H - e^{1-\alpha} a^\alpha (D - H) \quad (B8)
\]

At the opposite, the participation constraint of the consultant \((B5)\) may not be binding.
We replace (A16), (A17) and (B8) in the objective function (B7), the financial contracts induce the entrepreneur to solve the following program:

$$\max_{\beta, D, H} EU^E = \beta \left\{ \frac{1}{2} (1 + \alpha) \rho(\beta) (\Delta \theta - D + H)^2 + \theta^d - H \right\}$$

$$+ H - K + \rho(\beta)(\Delta \theta - D + H) (D - H)$$

s.t $$(1 - \beta) \left[ (1 - \frac{1}{2} \alpha) \rho(\beta) (\Delta \theta - D + H)^2 + \theta^d - H \right] \geq 0$$

with the conditions (6).

Straight application of the theorem of Khun-Tucker gives under the condition $2\theta^d < \theta^u < \frac{\alpha^2 + 2\theta^d}{\alpha}$, the optimal financial contracts:

$$\tilde{\beta} = \frac{2}{2 + \alpha}$$

$$\tilde{W} = K - \theta^d + \frac{1}{2} \alpha \left( \frac{\alpha}{2} \right)^\alpha \phi(\alpha) (\Delta \theta)^2$$

and

$$\tilde{D} = \left( \frac{1}{2} \alpha + 1 \right) \theta^d - \frac{1}{2} \alpha \theta^u \quad \text{and} \quad \tilde{H} = \theta^d$$

$$\tilde{I} = \theta^d - \left( \frac{\alpha}{2} \right)^{\alpha + 1} \phi(\alpha) (\Delta \theta)^2$$

Only the projects where the entrepreneur has an initial capital $W$ larger than a level $\tilde{W}$, are financed. The third party raise the amount of money $\tilde{I} = \theta^d - \rho F B \Delta \theta$ only if he is paid the whole revenue if the project fails and $\tilde{D} = 2\theta^d - \theta^u$ otherwise.

The first-best levels of efforts are very high, hiring a consultant has a negative impact on the agents’ incentives:

$$\tilde{e} = \left( \frac{\alpha}{2} \right)^{\frac{\alpha}{2}} \phi(\alpha) \Delta \theta < e^{FB} \quad \text{(B9)}$$

$$\tilde{a} = \left( \frac{\alpha}{2} \right)^{\frac{\alpha + 2}{2}} \left[ \frac{\mu \alpha}{\lambda (1 - \alpha)} \right]^{\frac{\lambda}{2}} \phi(\alpha) \Delta \theta < a^{FB} \quad \text{(B10)}$$

The debt’s payments are non-decreasing with the project’s outcome. Contrary to the LBO fund in the previous sections, the participation constraint of the consultant is not binding. The entrepreneur must give the consultant a positive share of her outcome to improve his incentives for effort provision:

$$EU^E = (1 - \beta) \left\{ e^{1 - \alpha} a^\alpha (\Delta \theta - D + H) + \theta^d - H \right\} - \frac{\alpha^2}{2 \mu} > 0 \quad \text{(B11)}$$

But the expected gain of the bank is still null because of the competition between banks.
The entrepreneur’s program becomes:

$$\max_{e, a, \beta, D, H} E^{UE} = \beta [p(e, a)(\Delta \theta - D + H) + \theta^d - H] - u(e) - K + I$$

$$\text{s.t.} \quad D \geq H, \quad (A21), \quad (6) \quad \text{and} \quad (B11)$$

We show that there are a set of financial contracts that lead the entrepreneur and the LBO fund to provide the same levels of efforts: suboptimal efforts. These contracts are given by:

$$\tilde{\beta}(\alpha) = \left\{ \begin{array}{ll}
\frac{(2-\alpha)(1-\alpha)-\sqrt{\alpha(1-\alpha^2)(2-\alpha)}}{2(1-\alpha)} & \text{if } \alpha \in [0, 1[ \setminus \left\{ \frac{1}{2} \right\} \\
\frac{1}{2} & \text{if } \alpha = \frac{1}{2}
\end{array} \right.$$  

$$D' = H' = I = K - W$$

$$W \in [0, K]$$

But the entrepreneur is better off if she does not finance the project $W' = 0$ which implies that $D' = H' = I = K$.

We recall that the expected gain of the consultant is not binding, otherwise he would not provide effort:

$$EU^A = (1-\tilde{\beta}(\alpha)) \left\{ \theta^d - K + \left[ 1 - \frac{1}{2} \alpha \left(1-\tilde{\beta}(\alpha)\right) \right] \rho \left( \tilde{\beta}(\alpha) \right) (\Delta \theta)^2 \right\}$$

which reduces the entrepreneur profit. Her expected gain is:

$$EU^E = \tilde{\beta}(\alpha) \left\{ \theta^d - K + \left[ 1 - \frac{1}{2} (1-\alpha) \right] \rho \left( \tilde{\beta}(\alpha) \right) (\Delta \theta)^2 \right\}$$

which completes the proof of the lemma 1.

\section*{C. Proof of lemma 2}

Under the condition of the interest deductibility, when the efforts $e$ and $a$ are observable, the expected gain of the entrepreneur becomes:

$$EU^E = \beta (1-\tau) \left\{ p(e, a)\theta^u + [1 - p(e, a)] \theta^d - p(e, a) (1 + r) I \right\} - [1 - p(e, a)] H \} - u(e) - W$$

$$E^{(1)}$$

Recall that the LBO fund and the bank have to offer the best contracts possible to the entrepreneur because of the competition, which enable us to write:

$$i = (1 - \beta) (1-\tau) \left\{ p(e, a)\theta^u + [1 - p(e, a)] \theta^d - p(e, a) (1 + r) I \right\} - v(a)$$

$$I = p(e, a) [(1 + r) I] - H$$

$$E^{(2)}$$

$$i = (1 - \beta) (1-\tau) \left\{ p(e, a)\theta^u + [1 - p(e, a)] \theta^d - p(e, a) (1 + r) I \right\} - [1 - p(e, a)] H \} - v(a)$$

$$I = p(e, a) [(1 + r) I - H] + H$$

$$E^{(3)}$$
We replace the second term of the equation (C3) in (C1) and (C2) which gives

\[
EU^E = \beta (1 - \tau) \left\{ p(e, a) \theta^u + [1 - p(e, a)] \theta^d - I \right\} - u(e) - W \quad (C4)
\]

\[
i = (1 - \beta) (1 - \tau) \left\{ p(e, a) \theta^u + [1 - p(e, a)] \theta^d - I \right\} - v(a) \quad (C5)
\]

Given \( K = W + i + I \), we conclude that:

\[
EU^E = (1 - \tau) \left[ p(e, a) \Delta \theta + \theta^d \right] - u(e) - v(a) - K + \tau I \quad (C6)
\]

Notice that \( EU^E \) is strictly increasing with the level of debt. This means that the entrepreneur is better off if the acquisition is financed only through the bank:

\[
I^{PI} = K \\
i^{PI} = W^{PI} = 0
\]

This contrasts sharply with the first best solution derived in Section 1 without taxation, the entrepreneur asks for advice from a consultant and for funds from the bank. The identities of the agents providing advice and financing is important.

The levels of efforts \( e^{PI} \) and \( a^{PI} \) that maximize the expected gain of the entrepreneur are given by:

\[
(1 - \tau) \Delta \theta = \frac{u'(e)}{p_e(e, a)} = \frac{v'(a)}{p_a(e, a)} \quad (C7)
\]

It is easy to check that the tax deductibility of the debt’s interests leads the two parties to provide non optimal efforts. These efforts are decreasing with the corporate income tax: if \( \tau \) converges to 0, they converge to the first-best levels of efforts.

Relying on the specified model gives the following results:

\[
e^{PI} = (1 - \tau) e^{FB} \quad (C8)
\]

\[
a^{PI} = (1 - \tau) a^{FB} \quad (C9)
\]

and

\[
p^{PI} = (1 - \tau) p^{FB} \quad (C10)
\]

The bank is paid \( D^{PI} = (1 + r) K \) in case of success and \( H^{PI} = \frac{1-(1+r)p^{PI}}{1-p^{PI}}K \) in case of failure. It is easy to check that the debt’s payments are nondecreasing with the project’s revenues

\[
D^{PI} - H^{PI} = \frac{rK}{1 - p^{PI}} > 0
\]

We replace (C8), (C9), (C10) and \( i = 0 \) in the participation constraint of the LBO fund. The optimal sharing rule is therefore given by:

\[
\beta^{PI} = 1 - \frac{v(a^{PI})}{(1 - \tau) (p^{PI} \Delta \theta + \theta^d - K)}
\]
When the information is perfect, the expected gain of the entrepreneur is given by:

\[ EU^E = (1 - \tau)^2 \left\{ p^{FB} \Delta \theta - u(e^{FB}) - v(a^{FB}) \right\} + (1 - \tau) \left( \theta^d - K \right) \leq V^{FB} \]

if \( \tau \) converges to 0, \( EU^E \) converges to \( V^{FB} \).

*The financial structure under moral hazard*

1. *The pure equity contract*

The expected gain of the entrepreneur and the LBO fund are written:

\[ EU^E = (1 - \tau) \beta \left[ p(e, a) \Delta \theta + \theta^d \right] - u(e) - W \geq 0 \]

\[ EU^A = (1 - \tau) (1 - \beta) \left[ p(e, a) \Delta \theta + \theta^d \right] - v(a) - i = 0 \]

The efforts in equilibrium are given by the first order conditions of \( EU^E \) and \( EU^A \). They are written:

\[ e_T^\alpha = \left[ \lambda \beta (1 - \alpha) / \rho(\beta) \right]^{1/2} (1 - \tau) \Delta \theta \]  \hspace{1cm} (C11)

\[ a_T^\alpha = \left[ \mu \alpha (1 - \beta) / \rho(\beta) \right]^{1/2} (1 - \tau) \Delta \theta \]  \hspace{1cm} (C12)

Given \( 0 < \beta < 1 \), these efforts are strictly lower than the first best efforts.

The entrepreneur’s program is therefore written:

\[ \max_{\beta} EU^E = \frac{1}{2} [2 - \alpha - (1 - 2\alpha) \beta] \rho(\beta) (1 - \tau)^2 \Delta \theta^2 + (1 - \tau) \theta^d - K \]

s.t. \hspace{1cm} 0 \leq \beta \leq 1

The first order condition of \( EU^E \) gives the following results:

\[ \tilde{\beta}(\alpha) = \left\{ \begin{array}{ll}
\frac{(2-\alpha)(1-\alpha)-\sqrt{\alpha(1-\alpha^2)(2-\alpha)}}{2(1-\alpha^2)} & \text{if } \alpha \in ]0, 1[ / \{ \frac{1}{2} \}
\frac{1}{2} & \text{if } \alpha = \frac{1}{2}
\end{array} \right. \]

\[ i_T^* = (1 - \frac{1}{2}\alpha) \left( 1 - \tilde{\beta}(\alpha) \right) \rho \left( \tilde{\beta}(\alpha) \right) (1 - \tau)^2 (\Delta \theta)^2 + (1 - \tau) \left( 1 - \tilde{\beta}(\alpha) \right) \theta^d \]

\[ W_T^* = K - i_T^* \]

The holding contract is unique. The financial contribution of the LBO fund is decreasing with the corporate income tax \( \tau \). Consequently, if \( \tau \) increases, the amount of equity issued by the entrepreneur should increase. If the project is financed only through the equity issued by the LBO fund, the agents provide the following efforts:

\[ e_T^\alpha = \left[ \lambda \tilde{\beta}(\alpha)(1 - \alpha) / \rho(\tilde{\beta}(\alpha)) \right]^{1/2} (1 - \tau) \Delta \theta \]  \hspace{1cm} (C13)

\[ a_T^\alpha = \left[ \mu \alpha (1 - \beta) / \rho(\tilde{\beta}(\alpha)) \right]^{1/2} (1 - \tau) \Delta \theta \]  \hspace{1cm} (C14)
It is straightforward to see that the efforts (C13) and (C14) are inferior to (A28). There are no tax advantages when the entrepreneur and the LBO fund issue only equity to fund the acquisition which decreases their payments and then their incentives to exert high levels of efforts. Given \( 0 < \beta(\alpha) < 1 \), it is easy to check that the entrepreneur and the LBO fund provide lower efforts than those provided under perfect information.

2. The debt-equity contract

The efforts of equilibrium \( e_T^* \) and \( a_T^* \) are now given by the first order conditions of \( EU^E \) and \( EU^A \) ((C4) and (C5)):

\[
\beta (1 - \tau) \Delta \theta = \frac{u'(e)}{p_e(e, a)} \quad (C15)
\]

\[
(1 - \beta) (1 - \tau) \Delta \theta = \frac{v'(a)}{p_a(e, a)} \quad (C16)
\]

It straightforward that these efforts are not optimal when there is tax deductibility of the debt’s interests. In the specified model, the conditions (C15) and (C16) enable us to deduce the following efforts:

\[
e_T^* = [\lambda \beta (1 - \alpha) \rho(\beta)]^{1/2} (1 - \tau) \Delta \theta \quad (C17)
\]

\[
a_T^* = [\mu \alpha (1 - \beta) \rho(\beta)]^{1/2} (1 - \tau) \Delta \theta \quad (C18)
\]

Then, the project succeeds with the probability \( p_T^* = \rho(\beta) (1 - \tau) \Delta \theta \). Assume \( \Delta \theta \leq \frac{1}{\rho(\beta)(1 - \tau)} \), otherwise the success probability is larger than 1. The entrepreneur’s program is written:

\[
\arg \max_{\beta, I} EU^E = \frac{1}{2} [2 - \alpha - (1 - 2\alpha) \beta(\beta) [(1 - \tau) \Delta \theta]^2 + (1 - \tau) \theta^d - K + \tau I
\]

s.t. \( 0 < \beta < 1 \) and \( 0 \leq I \leq K - W \)

The first order conditions of \( EU^E \) and the participation constraints give the following results:

\[
\beta(\alpha) = \begin{cases} 
\frac{(2-\alpha)(1-\alpha) - \sqrt{\alpha(1-\alpha^2)(2-\alpha)}}{2(1-2\alpha)} & \text{if } \alpha \in \left] 0, 1 \right[ \setminus \left\{ \frac{1}{2} \right\} \\
\frac{1}{2} & \text{if } \alpha = \frac{1}{2}
\end{cases}
\]

\[
i_T^* = W_T^* = 0
\]

and

\[
I_T^* = K
\]

\[
H_T^* = \frac{1 - \rho \left( \beta(\alpha) \right) (1 - \tau) (1 + r) \Delta \theta}{1 - \rho \left( \beta(\alpha) \right) (1 - \tau) \Delta \theta} K
\]

Under taxation, the entrepreneur prefers hiring a consultant rather than contracting with the LBO and she asks for funds only from the bank. This implies that the high level of debt in buyout
acquisitions is a way not to enhance the agents’ incentives but to profit from the tax-deductible interests.

Financing the acquisition only through debt increases the expected gain of the entrepreneur by $\tau K$ and the consultant is paid:

$$EU^A = (1 - \tau) \left( 1 - \beta(\alpha) \right) \left\{ (1 - \frac{1}{2} \alpha) \rho \left( \beta(\alpha) \right) (1 - \tau) \left( \Delta \theta \right)^2 + \theta^d - K \right\} > 0$$

which completes the proof of the lemma 2.
REFERENCES


LEHN K. et POULSEN A., (1989), "Free cash flow and stockholde gains in going private transac-


This preprint was prepared with the AAS \LaTeX\ macros v5.2.