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# ACOUSTICS 2012

**Experimental determination of the diffusion constant for ultrasonic waves in 2-D multiple scattering media with focused beamforming**

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Experimental measurements of the diffusion constant for ultrasonic waves (around 3 MHz) propagating in water through a scattering slab (parallel metallic rods) are presented. Sample thickness is around ten times the transport mean free path. Several hundreds of transmitting/receiving positions, 40 mm off the sample surfaces, are used. Focused beamforming is achieved in emission and reception in order to mimic a set of virtual sources and receivers located at the sample surface. The ensemble average of the transmitted intensity  $\langle I(x, t) \rangle$  is estimated by averaging over all possible couples of sources/receivers apart by the same off-axis distance  $x$ . Under the diffusion approximation,  $\langle I(x, t) \rangle$  shows a gaussian dependence on  $x$ , which makes it possible to measure a diffusion constant  $D$  and thereby characterize the scattering medium. We discuss the experimental results and pinpoint the difficulties of measuring a reliable value for  $D$  on a real sample. As it was observed in previous works on the elastic mean free path, the diffusion constant  $D$  strongly depends on frequency, due to the resonant nature of the scatterers.

## 1 Introduction

Setting aside absorption effects, multiple scattering of ultrasonic waves in random distribution of scatterers immersed in a fluid is essentially described by two key-parameters: the elastic mean-free path  $l_e$ , and the diffusion constant  $D$  [1, 2]. The importance of  $l_e$  arises when one is interested in the ensemble-averaged wave field, the so-called "coherent wave". In the case of an incoming plane wave with amplitude unity traveling along  $z$ , it can be shown that the average wave field  $\langle \psi \rangle$  transmitted through a random medium can be written as  $\langle \psi \rangle = \exp(j(k_{eff}z - \omega t))$ , where  $k_{eff}$  is an effective wave number. Therefore, the intensity of the coherent wave  $|\langle \psi \rangle|^2$  decays exponentially with distance, and the elastic mean-free path  $l_e = 1/2\Im\{k_{eff}\}$  is the typical decay length for the coherent intensity. On the contrary, the diffusion constant  $D$  arises when one is interested in the average intensity  $\langle |\psi|^2 \rangle$ , rather than in the intensity of the average field. To make a long story short, in 3D it can be shown that as long as  $z \gg l_e \gg \lambda$  with  $\lambda$  the wavelength, the average energy density  $U$  transported by a wave undergoing multiple scattering is reasonably described by a simple diffusion equation [3, 4] with  $D$  as the essential parameter. In this work, we utilize controllable samples of metallic rods for which the coherent wave has been already studied, and  $l_e$  precisely measured and confronted to various theoretical models [5]. But the actual value of  $D$  in these synthetic samples is still unknown, and contradictory values have been reported [6, 7, 8], at least in backscattering. The objective of this work is to test an experimental set-up that should be suitable for transmission measurements of the diffusion constant through such forests of rods, and to present the first experimental results.

## 2 Experimental procedure

Experimental set-up is shown in Figure 1. A short ultrasonic pulse (with central frequency around 3.25 MHz, corresponding to a wavelength of 0.46 mm in water) is emitted by means of a single 0.39-mm large transducer, and recorded, after it propagated through the sample, with an array of 64 0.39-mm large transducer elements. Both the single transducer and the array are placed at a distance of 40 mm from the sample in order to avoid reflections between the surfaces of the sample and the transducers. The sample is a random set of 0.8-mm diameter steel rods with density 29 rods/cm<sup>2</sup>. A previous study [5] measured the value of the elastic mean free path,  $l_e$ , for these samples. Integrated over the frequency band 2-4 MHz,  $l_e$  was found equal to 3.15 mm while its maximum value in the same frequency band is 4.69 mm for fre-

quency 2.8 MHz. As a reminder, the density of steel  $\rho_{steel}$  is 7800 kg/m<sup>3</sup>, longitudinal velocity  $c_L$  is 5.7 mm/ $\mu$ s and transversal velocity  $c_T$  is 3 mm/ $\mu$ s. Sample thickness is around 10 times the elastic mean free path, whereas sample width - in the lateral dimension - is 28 cm. Such width enables transducers to be translated over 14 cm in the lateral dimension in order to measure the transmission impulse response matrix  $h_{ij}(t)$ , with  $j$  the source position index and  $i$  the receiver position index, without any disturbance from the edges of the sample. Distance between two contiguous positions is 0.5-mm in emission, and 0.417-mm in reception (pitch of the array). In such a configuration, one has  $1 \leq i \leq 320$  and  $1 \leq j \leq 280$ .

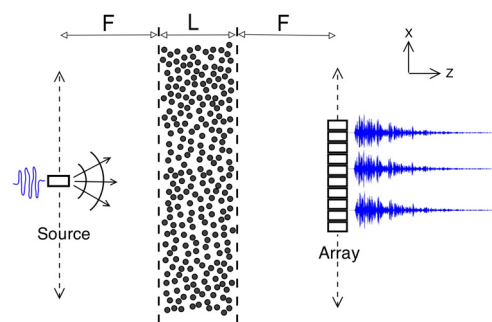


Figure 1: experimental set-up. A short ultrasonic pulse propagates through a random set of steel rods immersed in water. Scattered waves are recorded on a 64-element array. The source and array can be translated parallel to the sample for ensemble averaging.

The general principle of the experiment is to acquire a set of impulse responses  $h_{ij}(t)$ , with actual sources and receivers away from the sample. Then, by means of classical focussed beamforming in emission as well as in reception, we create an other set of virtual emitters and receivers, located at the front and back faces of the sample. Of course, the virtual sources and receivers have a finite extent, which is determined by diffraction laws. The new signals are arranged in a matrix given by :

$$k_{RE}(t) = \sum_{i=1}^{320} \sum_{j=1}^{280} \alpha_{jE} \beta_{iR} \delta(t - \tau_{jE} - \tau_{iR}) * h_{ij}(t) \quad (1)$$

$k_{RE}$  is the acoustic field at a point R (position  $x_R$  of the virtual receiver) on the rear surface of the sample, when the virtual source is located on the point E (position  $x_E$ ) on the front surface of the sample.  $\tau_{jE}$  and  $\tau_{iR}$  are the delays applied in order to focus in emission and in reception respectively. Equation (2) gives the delays applied in emission :

$$\tau_{jE} = \frac{X - \sqrt{(x_j - x_E)^2 + F^2}}{c} \quad (2)$$

$\mathcal{X}$  is the distance between the virtual source and the farthest considered real sources.  $c$  is the velocity of sound in water ( $c = 1.497 \text{ mm}/\mu\text{s}$ ). Delays applied in reception follow the same equation but with indices  $i$  and  $R$  instead of  $j$  and  $E$ . The  $\alpha$  and  $\beta$  matrices contain apodization coefficients. They are mostly used in order to control the lateral shape of the virtual sources and receivers. The apodization coefficients  $\alpha$  and  $\beta$  were chosen so that the virtual sources and receivers had a gaussian profile in  $\exp(-x^2/(2w_0^2))$ , with a typical transverse beam width  $w_0$ . Apodization was also used to switch off real sources or receivers if their positions  $x_j$  or  $x_i$  were considered too far from a chosen focalization point ( $x_E$  for virtual source or  $x_R$  for virtual receiver). Applying the time delays and the apodization coefficients, both in emission ( $\tau_{jE}, \alpha$ ) and in reception ( $\tau_{iR}, \beta$ ), we could recreate an array of 120 gaussian sources and receivers with pitch  $p = 0.8 \text{ mm}$  and a typical width  $w_0 = p/\sqrt{2 \ln 2} \approx 0.68 \text{ mm}$ .

Calculation of the transmitted intensity is done by integrating the square of the signals on a small translating time window, following equation (3) :

$$I_{RE}(t) = \frac{1}{\delta t} \int_{t-\delta t/2}^{t+\delta t/2} k_{RE}^2(t) dt \quad (3)$$

For the width of the window, we chose  $\delta t = 2 \mu\text{s}$  (roughly 6 periods). This value is small enough relatively to the typical decay time of the diffused signals, in order to ensure a proper resolution in time, and large enough compared to the central period of the wave. After calculation of the transmitted intensity, the ensemble average of the transmitted intensity is estimated as follows. The intensity  $I_{RE}(t)$  is averaged over all source-receiver couples apart by the same distance  $x = |x_E - x_R| = np$ . The number of such couples is 120 for  $x = 0$  (the receiver faces the transmitter), and  $2 \cdot (120 - n)$  for  $1 \leq n < 120$ . The resulting averaged intensity is denoted  $\bar{I}(x, t)$ . Figure 2 shows the different steps we described until now. One realization of the transmitted signal when virtual source and receiver are aligned (on-axis) is plotted (top) with its corresponding intensity (middle). On the bottom of Figure 2, the average transmitted intensity is plotted for the on-axis configuration ( $x = 0$ ). Comparison between the middle curve and the bottom curve shows the effect of averaging.

By studying the average transmission of the intensity between two points across a scattering slab, our configuration is similar to that proposed by Page et al [3]. Thus, the same approach is followed to determine the diffusion constant. The slab is supposed to be thick enough for the diffusion approximation to hold. The transmitted acoustic energy density can be obtained by solving the diffusion equation in a slab of thickness  $L$  with infinite transverse size [9]. The Green's function of the problem  $G(x, t)$  is determined by assuming that the intensity source is a Dirac pulse in time, which begins to diffuse at a distance  $z_0$  inside the sample (typically,  $z_0$  is of the order of a transport mean-free path [10, 3]). Taking into account the appropriate boundary conditions, the Green's function for the intensity  $G(x, t)$  can be calculated [3], but its expressions depends on many parameters : the sample thickness  $L$ , the penetration length  $z_0$ , the transport speed, the diffusion constant  $D$ , and if absorption must be taken into account, an attenuation time  $\tau_a$ . Therefore a direct fit of the experimental data in Figure 2 is not the best way to determine  $D$ , given the numbers of parameters involved. Instead, we use the same idea as in [3]. At a given time  $t$ , we study the ratio of the intensities detected off-axis

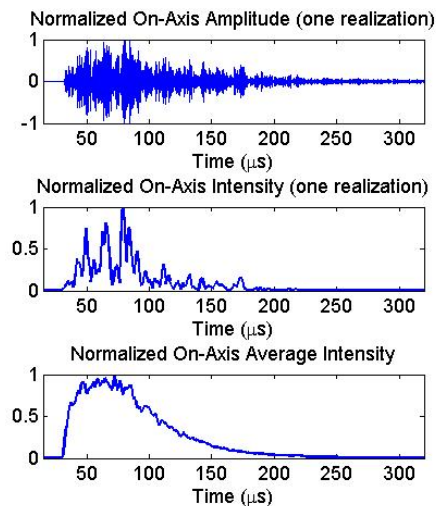


Figure 2: transmitted on-axis signal (virtual source and receiver aligned) (top); on-axis transmitted intensity calculated from previous signal (middle); on-axis averaged transmitted intensity on 120 realizations (bottom).

and on-axis, i.e.  $g(x, t) = G(x, t)/G(0, t)$ . The advantage is that, provided that the transverse size of the sample is infinite, the problem is invariant under translation along  $x$ , and  $g(x, t)$  reduces to  $g(x, t) = \exp(-x^2/(4Dt))$ , so that the effect of absorption, slab thickness and boundary conditions vanish. Since our sources are not really point-like, in order to take into account the finite extent  $w_0$  of the virtual sources or detectors, the experimental ratio  $\bar{R}(x, t) = \bar{I}(x, t)/\bar{I}(0, t)$  will be compared to  $g(x, t) * \exp(-x^2/(2w_0^2)) * \exp(-x^2/(2w_0^2))$ . The choice of a Gaussian beam simplifies the result, since the resulting intensity profile, at a given time  $t$ , is also a Gaussian  $\exp(-x^2/(2W))$ , with  $W(t) = 2Dt + 2w_0^2$ .  $W(t)$  is the typical area of the diffuse halo of intensity transmitted through the slab at time  $t$ . At each time  $t$  an estimate of  $W(t)$  is found by a linear fit of  $\ln \bar{R}(x, t)$  as a function of  $x^2$ . Finally, a second linear fit of  $W$  versus  $t$  yields an estimate of the diffusion constant  $D$ .

### 3 Experimental results and interpretations

A typical result is shown in Figure 3 where the average transmitted intensity  $\bar{I}(x, t)$  is represented as a function of lateral position  $x$  and time  $t$ . Sample is a 46-mm thick slab of density 29 rods/cm<sup>2</sup> for which the elastic mean free path  $l_e$  was previously found to be equal to 3.15 mm over the frequency interval of 2-4 MHz [5]. On the bottom part of Figure 3, each column, corresponding to a time, is normalized by its own on-axis value  $\bar{I}(x = 0, t)$ . The representation of  $\bar{R}(x, t)$  emphasizes the expansion of the diffusion halo -warm color area- with time.

The typical lateral extension of the diffusion halo,  $W$ , is estimated on the same data set, following the procedure explained in the end of section 2. It is plotted in Figure 4. The curve shows a remarkable linear behaviour over almost all the measurement time interval. This result is interesting because, as shown in [11], in a free 2d-space, the condition  $ct \gg l_e$  is not sufficient for the solution of the Boltzmann

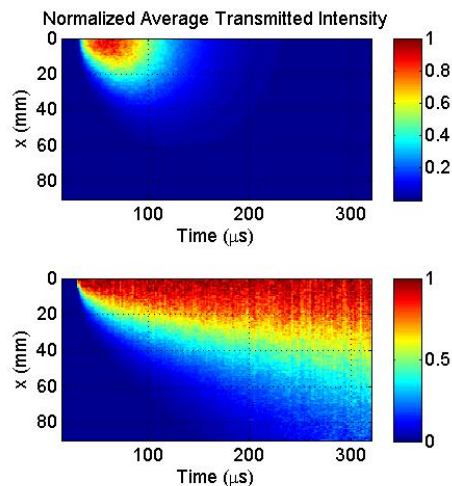


Figure 3: average transmitted intensity versus time and position. On top,  $\bar{I}(x, t)$  normalized by its maximum value. On bottom,  $\bar{R}(x, t)$ . Sample is a 46-mm thick slab of density 29 rods/cm<sup>2</sup>.

equation to converge on the solution of the diffusion equation, instead, a sufficient condition is  $ct \gg r$ , where  $r$  is the distance between the source and the observation point. In our case,  $r$  goes from 46 to 64 mm ( $z$  equals 46 mm for all the positions of receivers and  $x$  goes from 0 to 48 mm) whereas  $t$  goes from 50 to 320  $\mu$ s, and condition  $ct \gg r$  is not fulfilled at early times. Moreover, steel rods are not isotropic scatterers so that the mean free path we should consider is rather the transport mean free path  $l^* = l_e / (1 - \langle \cos \rangle)$  which takes into account the anisotropic behaviour of our scatterers.  $\langle \cos \rangle$  is the average cosine of the wave scattered by one rod. For instance, at the resonant frequency of the rods -2.75 MHz-,  $\langle \cos \rangle \approx 0.2$  and  $l^* = 5.8$  mm. The time required for the establishment of the diffusive regime is therefore lengthened. At last, it is remarkable to see that the curve does not saturate even at the end of the measurement time interval. This seems to indicate that the edges of the sample do not affect the halo expansion over the entire measurement time interval.

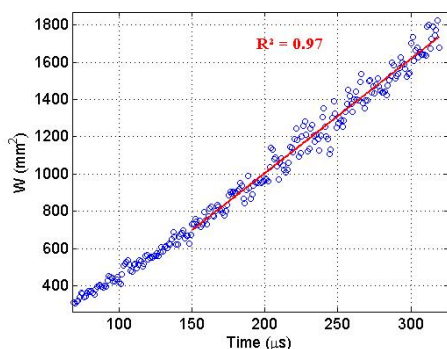


Figure 4: measured lateral extension of the diffusion halo after transmission through a 46-mm thick slab of density 29 rods/cm<sup>2</sup>. Time interval for the fit is 150-320  $\mu$ s. Corresponding D is  $3.1 \pm 0.3$  mm<sup>2</sup>/ $\mu$ s

To estimate D,  $W$  is approached as a linear function of  $t$ , over a time interval with limits  $t_{min}$  and  $t_{max}$ . Naturally, the choice of  $t_{min}$  and  $t_{max}$  can seriously influence the resulting estimate of D. Physically, two criteria must be met :  $t_{min}$

must be large enough for the diffusion approximation to be valid, and  $t_{max}$  must be small enough so that the diffuse halo of intensity has not yet reached the upper and lower edges of the sample. In 3d-space, the first condition is met as long as  $ct \gg l^*$  [11, 3]. However in 2d-space, the convergence to the diffuse regime is slower. In order to determine  $t_{min}$ , we compared the exact solution of the Boltzmann equation in free-space [11] to the solution of the diffusion equation, and chose for  $t_{min}$  the time such that the difference between the two estimates of D was below 2%. This yielded  $t_{min} = 150$   $\mu$ s. This value may seem surprisingly high, given that Figure 4 seems quite linear even at earlier times : indeed, to solve the Boltzmann equation we took for  $l^*$  the highest value in the 2-4 MHz frequency band. As to  $t_{max}$ , the criterion we applied was the following : since the halo is supposed to spread transversely from the source as a gaussian with typical width  $\sqrt{2Dt} = \sqrt{l^* c_{tr} t}$ , we imposed that  $t$  should be smaller than the time  $t_{max}$  for which 5% of the intensity halo has reached the edge of the sample, even for the outermost sources/receivers.  $c_{tr}$  is the transport velocity, which has been assumed  $\approx c$ . Again, by security, we overestimated the value of  $l^*$  by retaining its highest value at 2.75 MHz. Actually,  $t_{max}$  was found to be larger than the maximum time (320  $\mu$ s) of our recordings. Finally, the fit of  $W$  on time interval 150-320  $\mu$ s gives D equal to 3.1 mm<sup>2</sup>/ $\mu$ s. Error on this experimental value originates both from the spatial fit and from the time fit. It was evaluated to  $\pm 0.3$  mm<sup>2</sup>/ $\mu$ s. This experimental value can be compared with a theoretical value. Formula  $D = l^* c_{tr} / 2$  gives values of the diffusion constant for different frequencies over the interval 2-4 MHz, provided that the elastic mean free path has been measured and resolved in the same frequency band. Using frequency-resolved values of  $l_e(f)$  that have been reported in [5], as well as calculated theoretical values of  $\langle \cos \rangle(f)$ , we could calculate theoretical values of the diffusion constant for different frequencies in the interval 2-4 MHz, assuming  $c_{tr} \approx c$ . Averaging the latter and weighting them with the power spectrum of the transmitted signals of matrice  $k_{RE}$ , one found  $D_{th} = 2.8$  mm<sup>2</sup>/ $\mu$ s, in reasonable agreement with the measured value.

So far, the experimental value of D is, in fact integrated over the frequency spectrum of the signal, approximately 2-4 MHz, corresponding to the limited bandwidth of our transducers. However, frequency-resolved measurements of the elastic mean free path  $l_e$  [5] show that  $l_e$  strongly depends on frequency. For instance,  $l_e$  shows a resonant pic at frequency 2.75 MHz. This suggests that D will also depend on frequency. After the focusing process, every signal from matrix  $k_{RE}$  is filtered on small frequency windows of width  $\delta f = 0.15$  MHz. The lateral extension of the diffusion halo,  $W$ , is then estimated for each narrow frequency bands so as to obtain frequency-resolved measurements of D. Experimental values are compared with theoretical values (formula  $D = l^* c_{tr} / 2$ ) in Figure 5. The agreement of experimental values with theory is rather satisfactory at the resonance frequency and upper right frequencies. These frequencies are in the center of the bandwidth of our sensors and signals (3.25 MHz). In Figure 6, the lateral extension of the average transmitted intensity is plotted for three different frequency bands. On top, the central frequency corresponds to the resonance frequency of the rods -2.75 MHz- whereas middle and bottom curves correspond to frequencies 3.25 -central frequency of emitted signals- and 3.8 MHz respectively. The latter frequency is far from the resonance frequency. These curves

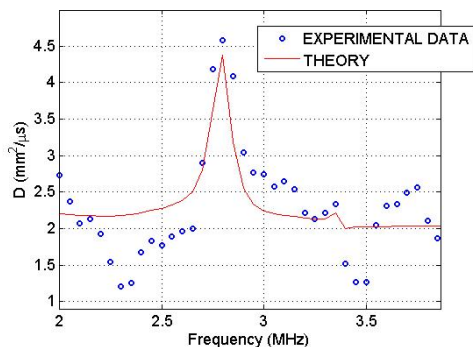


Figure 5: experimental values of  $D$  versus frequency for the 46-mm thick sample of density 29 rods/cm<sup>2</sup>

essentially show that by filtering the initial signals, the measurements of the extension  $W$  of the diffuse halo with time become more noisy. Due to frequency filtering, the time resolution is poorer; the number of independent information in a given time interval is reduced (here, roughly by a factor of 3 since  $1/0.15 \text{ MHz} \sim 6 \mu\text{s} = 3 \delta t$ ), and as a consequence the statistical error bar is enhanced by  $\approx \sqrt{3}$  so that typical error on  $D$  is of the order of  $\pm 17\%$  instead of  $\pm 10\%$ . It is therefore important to find a good compromise between frequency resolution and good level of signal to noise ratio.

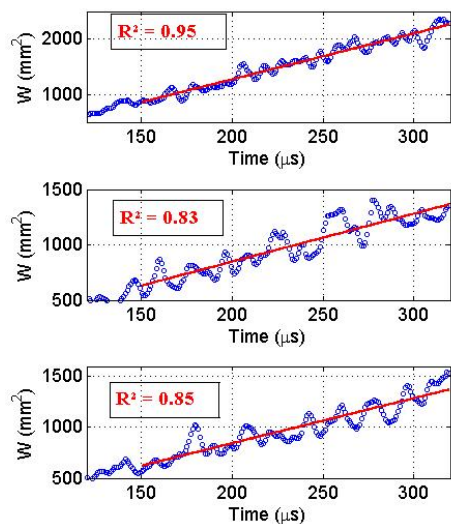


Figure 6: lateral extensions of the average transmitted intensity as a function of time for three different narrow frequency bands centered at frequencies 2.75, 3.25 and 3.8 MHz from top to bottom. Sample is a 46-mm thick slab of density 29 rods/cm<sup>2</sup>.

## 4 Conclusions and perspectives

We presented a method, for the first time used on our samples, to measure the diffusion constant of ultrasonic waves in heterogeneous medium. It relies on the diffusion approximation and it is based on the measurement of the lateral expansion of the diffusion halo behind the sample after the recording of the transmission impulse response matrix. Estimation of the lateral extension of the halo,  $W$ , at everytime,

allows to estimate the diffusion constant  $D$  of the sample. Considering broadband signals, the curve of  $W$  showed a really linear behaviour over almost the entire time interval of the measurement, which demonstrates the consistency of the diffusion approximation and therefore of the method. The estimated value for  $D$  ( $3.1 \pm 0.3 \text{ mm}^2/\mu\text{s}$ ) was found in reasonable agreement with the theoretical prediction  $D = l^* c_{tr}/2$ , assuming that  $c_{tr} = c$ . As we could expect a frequency behaviour of the diffusion constant, measurements of the diffusion constant in narrow frequency bands were presented. These measurements show that  $D$  actually depends on the frequency. However, it is too early to draw conclusions from these measurements, especially because of the uncertainty on  $D$  in narrow-band experiments. Ideally, since  $l_e$  (hence  $l^*$ ) are known at each frequency, if  $D$  could also be measured as a function of frequency with a good precision, then it would be possible to deduce the transport velocity  $c_{tr} = 2D/l^*$ , and see if it differs significantly from  $c$ , the velocity of sound in the embedding medium (water, here). It was already shown in similar samples [5] that at the resonance, the group velocity  $c_g$  of the coherent wave is significantly reduced at the resonance, dropping at  $1.25 \text{ mm}/\mu\text{s}$ . Here, the error bars are so large that we cannot conclude yet that  $c_{tr}$  is significantly different from  $c$  or  $c_g$ . In addition, the spectral study of the transmitted signals shows that their frequency contents change over time -widening over the resonance frequency- which does not simplify the measurement of  $D$ . Future work should especially focus on the determination of  $D$  in narrow frequency bands. Measurements should be refined by finding the right compromise between noise and bandwidth, to obtain values of  $D$  both reliable and well resolved in frequency. Consistent values of  $D$  could be compared with experimental values of the transport mean free path  $l^*$ , to determine the transport velocity of ultrasonic waves in our samples. Finally, the ultimate aim of this research would be to develop characterization technics that could be applied to real environments. Of course, the transition from prototype medium to real environments may be made only after understanding the phenomenon of transition from radiative regime to the diffusive regime.

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