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An acoustic model to control an experimental slide flute

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Abstract In this paper, we consider the problem of modeling and control of a slide flute: a kind of recorder without finger holes but ended by a piston mechanism to modify the length of the resonator. From a physical point of view, stopped pipes have not been studied so widely as open pipes, and moving boundary conditions introduce interesting mathematical problems. To control dynamical systems, it is important to elaborate a realistic model, so that control laws can be tested efficiently before they are implemented on real size prototypes. The dynamical model we have elaborated takes into account the coupling effects between the jet and the pipe which is a linear acoustic resonator. The jet is obtained by blowing through a flue channel and formed by flow separation at the flue exit, and finally directed towards a sharp edge, called the labium. A modal analysis is then performed using the linearized boundary conditions to compute the suitable blowing pressure and the suitable pipe length to obtain a desired pitch. This will constitute the “feedforward” part of our control algorithm. The feedback term is then elaborated to regulate the system to the desired set point, using online measurements on the system: the length of the piston through an encoder and the blowing pressure through a pressure sensor. First experimental results, obtained on a “mechatronic” prototype developed at Mines ParisTech will be presented.

1 Introduction

Slide flutes are mostly used for jazz and popular music, even if they sometimes appear in the classical orchestra in works like the opera by Maurice Ravel, L'Enfant et les Sorcillèges. We are interested in this paper to control this kind of instrument made of a cylindrical stopped resonator similar to a stopped organ pipe and of a blowing mouthpiece analogous to that of a recorder. Contrary to flutes, organ pipes and recorders, the variation of the pitch is here obtained through a piston mechanism. The control of the system is realized in two steps: the first step concerns the feedforward part of the controller to compute the reference blowing pressure and the reference piston’s length to obtain a desired frequency, or equivalently a desired pitch; the second step concerns the closed-loop part of the control law made of a suitable PID-controller. Let us point out that the on line measurements are the length of the resonator (equivalent to the length of the piston) which can be modified through an electric DC motor, and the input blowing pressure regulated through electric servo-valves. To compute the feedforward control part, it is important to elaborate a realistic physical model of the system. This simulator will also be useful to test and validate the whole control process before implementation on the real prototype.

In the case of open pipes, physical models for the excitation mechanism have been developed to produce quite realistic sound synthesis (see e.g. [8, 15]). In the present work, we use the same model of the excitator and couple it to a pipe model with moving boundary conditions. Even if the resulting dynamical model can be used for sound synthesis purposes, our main goal is to develop control algorithms to automatically control such physical instruments. An important application for such device is then to produce musical dynamics (piano e forte) at constant pitch by adjusting both the blowing pressure and the slide position. The obtention of physical models for analysis and synthesis for flue musical instrument such as organs or recorders has been an important research subject for a few decades. We will not be exhaustive, but we can mention the paper by Cremer and Ising [7] giving a first quasi-stationary model of the jet drive, which has been later improved by many authors (see e.g. [6, 11]). The works of Howe [13] pointed out the importance of vortex shedding at the labium. In fact, in steady blowing conditions, models not taking into account this effect (e.g. in [11]) led to an overestimation of the amplitude of the pressure oscillation in the pipe. Therefore, as in [16],

we have taken into account these interactions jet/labium, but as already mentioned, the system we are studying is different: the resonator’s length is time-varying, controlled through the piston mechanism and there is no finger hole. The whole structure can then be described by two linear Partial Differential Equations coupled with nonlinear Ordinary Differential Equations describing the boundary conditions (see [1, 2]):

- for the mouth, taking into account the jet dynamics,
- and for the piston.

In section 2 we briefly describe the slide flute prototype in development at Mines ParisTech. In section 3 we recall our pipe model and the physical models of the jet channel and the mouth. In section 4 we compute the boundary condition at the end of the resonator and at its entrance, taking into account the jet dynamics. In section 5 we present the modal analysis we have developed from the linearized boundary conditions, and we compute the different frequencies which are functions of the resonator’s length and of the steady blowing pressure. It can be noticed that the frequencies are odd multiples of the fundamental one, as expected for open-closed pipes instruments like the pan flute or the clarinet. Conversely, if we want to obtain a reference fundamental frequency, we can use those linearized boundary conditions, where we fix the desired frequency value, to compute the suitable blowing pressure and the suitable length. This will constitute the feedforward part of our control algorithm which will be presented in section 6, together with first experimental results on the prototype.

2 The slide flute prototype

The prototype is described at Figure 1. The piston of the slide flute can be translated by an electric DC motor through a slipper guide. An incremental coder allows to measure the rotation of the motor and therefore to know at each time the resonator’s length. A PWM (pulse-width modulation) numerical/analogue interface is used to convert the numerical control computed by the micro-controller. It must be noticed that the incremental coder returns a relative position; therefore, when the program starts, the reference position must be known and this is done by two stop thrusts. The micro-controller card is sampled at 10 ms.

Concerning the excitation pressure, the system has been equipped with an air compressor supplying three electric servo-valves which allow the air-regulation of the artificial mouth.
which has been connected to the flute via a joint in silicone to avoid air leaks. The pressure is measured by a pressure sensor set in the mouth and the valves apertures are controlled through the electronic card.

![Figure 1: The slide flute prototype](image1.jpg)

A graphical interface has been developed, using the software CVI, as shown in Figure 2. It allows to define the series of notes the flute has to play and their duration, as well as the value of the excitation pressure.

![Figure 2: The graphical interface](image2.jpg)

### 3 Physical models of the pipe, the jet channel and the mouth

Let us denote $\rho_0$ the air density at rest, $S_p$ the constant section of the pipe which is to be cylindrical. Assuming that the flow rate $u(x, t)$ at time $t$ and point $x$ in the pipe and the relative pressure $p(x, t) = P - P_{ atm}$ $(P_{ atm}$ denoting the atmospheric pressure) are uniform on a section, and that the transformation is adiabatic, the Euler equation giving the fluid dynamical properties together with the mass conservation law lead to the classical d’Alembert equation:

$$\frac{\partial^2 p}{\partial t^2} - c^2 \frac{\partial^2 p}{\partial x^2} = 0. \quad (1)$$

It is well known that the system can be described through ingoing and outgoing waves $\alpha(x, t)$ and $\beta(x, t)$, with respective velocities $c > 0$ and $-c < 0$:

$$\frac{\partial \alpha}{\partial t} + c \frac{\partial \alpha}{\partial x} = 0 \quad \text{and} \quad \frac{\partial \beta}{\partial t} - c \frac{\partial \beta}{\partial x} = 0. \quad (3)$$

where $\alpha(x, t)$ and $\beta(x, t)$ are related to the physical variables $p(x, t)$ and $u(x, t)$ as follows (see e.g. [1, 2]):

$$\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} u + \frac{S_p}{\rho_0 c} p \\ u - \frac{S_p}{\rho_0 c} p \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} u \\ p \end{pmatrix} = \begin{pmatrix} \alpha + \beta \\ 2 \rho_0 c (\alpha - \beta) \end{pmatrix}. \quad (4)$$

As it has been done in [4] in the case of an overhead crane with a variable length flexible cable, it is interesting to apply the following change of variable

$$x = L \sigma \quad (5)$$

to transform the system into one having a fixed spatial domain for $\sigma$, i.e. $\sigma \in [0, 1]$. According to (5), if we denote:

$$\begin{pmatrix} \tilde{\alpha}(\sigma, t) \\ \tilde{\beta}(\sigma, t) \end{pmatrix} = \begin{pmatrix} \alpha(L(t)\sigma, t) \\ \beta(L(t)\sigma, t) \end{pmatrix} \quad (6)$$

equations (2) and (3) become:

$$\begin{pmatrix} \frac{\partial \tilde{\alpha}}{\partial \tau} + \frac{c - L \sigma}{L} \frac{\partial \tilde{\alpha}}{\partial \sigma} = 0 \\ \frac{\partial \tilde{\beta}}{\partial \tau} + \frac{c + L \sigma}{L} \frac{\partial \tilde{\beta}}{\partial \sigma} = 0 \end{pmatrix} \quad (7)$$

We still have two wave equations, but with time variable velocities depending on $L$, and on the control variable $\sigma$, where $L$ denotes the time derivative of $L$. These equations will constitute our pipe model.

A simplified description of the slide flute together with some useful notations is given in Figure 3.

![Figure 3: The slide flute](image3.jpg)

Concerning the jet channel and the mouth, in [16, 15, 9] the two-dimensional geometry of the mouth is modeled in a low frequency plane wave approximation by a one-dimensional representation, by an equivalent pipe segment of length $\delta_m$ (see Figure 4) taking into account the constriction of the pipe at the blowing end.

![Figure 4: The 1D model of the mouth](image4.jpg)
In this one-dimensional representation, the flue exit, when the jet is formed, is located at an acoustic distance $\delta_{out}$ from the outside and $\delta_m$ from the entrance of the resonator. At the flue exit, because the region is compact, one can apply the mass conservation law:

$$Q_j + Q_{out} = Q_m$$

(8)

where $Q_j$, $Q_{out}$ and $Q_m$ are respectively the jet flow, the flow in the portion $\delta_{out}$ and $\delta_m$ respectively, expressed in $m^3/s$.

The pressure $p_m$ in the mouth at the flue exit can be related to the flow $Q_{out}$ by the radiation impedance, which leads in the time domain to the following linear differential equation:

$$\begin{aligned}
    p_m &= c_3 Q_{out} - c_3 Q_{out} \\
    c_1 &= \frac{\rho_0 r_m^2}{4 c S_m} \quad \text{and} \quad c_3 = \frac{\rho_0 \delta_{out}}{S_m}
\end{aligned}$$

(9)

where $S_m$ is the mouth cross section at the flue exit, and $r_m$ is the radius of a circle having the same mouth cross section, i.e. such that $\pi r_m^2 = S_m$.

Neglecting friction, the jet at the flue exit is governed by the Bernoulli equation:

$$\rho_0 l \frac{dU_j}{dt} + \frac{1}{2} \rho_0 U_j^2 = p_f - p_m$$

(10)

where $U_j$ denotes the jet velocity in the flue channel, $l$ the length of the channel, $p_f$ denotes the excitation pressure at the entrance of the channel, generated by the player and $p_m$ denotes the pressure in the mouth of the instrument (see Figure 3). Noticing that the flow continuity is assumed at the entrance of the resonator, that is: $Q_{in}(0) = u(x = 0, t) = u_0(t)$, the pressure $p(x = 0, t) = p_0(t)$ can also be related to the pressure $p_m$ through momentum conservation:

$$\begin{aligned}
    p_m - p_0 &= c_1 u_0 - \Delta p \\
    c_1 &= \frac{\rho_0 \delta_m}{S_m}
\end{aligned}$$

(11)

where $\Delta p$ represents the pressure jump across the pipe of length $\delta_{out}$. This pressure jump, responsible for the sound production, can be mainly decomposed in two terms:

$$\Delta p = \Delta p_{jd} + \Delta p_a$$

(12)

$\Delta p_{jd}$ denoting the pressure jump due to the jet drive mechanism and $\Delta p_a$ the vortex shedding when the flow separates at the edge of the labium which appears to be determinant in limiting the amplitude of the oscillation during steady-state, but also to be important to describe nonlinear behavior in the transient attack.

The term $\Delta p_{jd}$

As explained for example in [16, 15], the pressure due to the jet-drive is determined by the time derivative of the flow source $Q_1$ corresponding to the portion of the jet flow entering the pipe at the labium:

$$\Delta p_{jd} = -\frac{\rho_0 \delta_d}{S_m} \frac{dQ_1}{dt}$$

(13)

where $\delta_d$ is the acoustic distance between the dipole sources $Q_1$ and $Q_2$ generated at the labium, in the one-dimensional representation of the instrument. Assuming the jet has a Bickley velocity profile (see e.g. [14]), denoting $H$ the jet width, $y_0$ the labium position with respect to the flue exit axis (y being positive towards the interior of the pipe) the following expression is obtained:

$$Q_1 = b H \tilde{U}_j \left[ 1 + \tanh \left( \frac{\eta - y_0}{b} \right) \right]$$

(14)

where $b$ is a positive jet parameter characterizing the velocity profile at the flue exit, $\eta$ denotes the jet position which will be detailed in the next paragraph and $\tilde{U}_j$ denotes the asymptotic value of the jet velocity depending on the excitation pressure $p_f$ (see equation (10)), i.e.:

$$\tilde{U}_j = \sqrt{2 p_f / \rho_0}.$$  

(15)

The term $\Delta p_a$

Using e.g. [10, 16, 15], one can express the vortex shedding term induced at the labium by the transverse acoustic flow of the pipe by the following expression:

$$\Delta p_a = -\frac{1}{2} \rho_0 \left( \frac{u_0}{w S_m} \right)^2 \text{sign}(u_0)$$

(16)

where $\sigma_v$ is the vena-contracuta factor of the flow. It can be seen that this term is dissipative, corresponding to the kinetic energy dissipation by turbulence of the jet, formed by separation of the acoustic flow at the labium.

Physical model of the jet position

Let us now give an expression of the jet position $\eta$ in the mouth obtained from recent works e.g. [8], denoting $h$ the jet height:

$$\eta(t) = 2 \frac{u_0(t - \tau_t) h}{\pi S_m U_j} e^{\mu W}$$

(17)

where $\mu$ denotes the spatial amplification of the jet, $W$ the distance between the flue exit and the labium and the delay $\tau_t$ is given by:

$$\tau_t = \frac{W}{0.3U_j}.$$  

(18)

We can see that the delay $\tau_t$ is time varying since it depends on $U_j$. But, in the transient regime of the jet velocity, $U_j$ takes values near the origin, so that in numerical simulations, we have to wait $U_j \geq \gamma$ for a small positive value $\gamma$ to consider equation (18). Before that, we take $\eta = 0$.

4 Boundary conditions

Let us now complete the pipe model (7) with the boundary conditions at $\sigma = 0$ (i.e. $x = 0$) and $\sigma = 1$ (i.e. $x = L$).

4.1 Boundary condition at the entrance of the resonator

Let us first consider the boundary condition at the entrance of the resonator. It can be obtained replacing $p_m$ from equation (11) in equation (10), which leads to:

$$p_0(t) = p_f - \rho_0 l \frac{dU_j}{dt} - \frac{1}{2} \rho_0 U_j^2 - c_1 u_0(t) + \Delta p.$$  

(19)
This boundary condition can be rewritten in the $\alpha$ and $\beta$ variables using (4) and in the $\tilde{\alpha}$ and $\tilde{\beta}$ variables, using (6) which gives finally:

$$\tilde{\alpha}(0, t) = \tilde{\beta}(0, t) + \frac{2S_p}{\rho_0 c} \bigg[ \rho_1 - \rho_0 \frac{dU_j}{dt} - \frac{1}{2} \rho_0 U_j^2 - \frac{c_1}{2} (\tilde{\alpha} + \tilde{\beta})(0, t) + \Delta p \bigg] \quad (20)$$

**Remark 1** It can be noticed that $p_0(t)$ now depends on $u_0(t)$ but using (12), (13), (14), (16) and (17) also on $\tilde{u}_0(t)$ and $\tilde{u}_0(t - \tau)$. Moreover, we need the value of $U_j$ and its time-derivative to bring up to date the boundary condition (20). So we have to solve at each time instant, the ordinary differential equation describing the dynamical evolution of $U_j$. This equation is obtained from (10) where we replace $p_m$ by its expression (9) and using equation (8) which becomes at $x = 0$:

$$Q_{out} = Q_m - Q_j = u_0(t) - S_e U_j \quad (21)$$

$S_e$ denoting the cross section of the channel at the flue exit. Finally, the equation giving the value of $U_j(t)$ can be written:

$$c_2 S_e U_j = (\rho_0 c_3 + \rho_0 S_e) \dot{U}_j + c_3 u_0 - c_2 \dot{u}_0 = \frac{1}{2} \rho_0 U_j^2 - p_0. \quad (22)$$

When taking realistic numerical values of the constants involved in (22) it can be seen that $c_2S_e \approx 10^{-9}$, which is negligible with respect to the multiplying factor of $U_j$. Therefore, using singular perturbation arguments, one can neglect the terms in $\dot{U}_j$ in (22) and we can consider the following equation which will be used to evaluate $U_j$ and its time derivative:

$$(\rho_0 c_3 + \rho_0 S_e) \dot{U}_j = -\frac{1}{2} \rho_0 U_j^2 + c_3 u_0 - c_2 \dot{u}_0. \quad (23)$$

Finally, the boundary condition at the entrance of the resonator consists in the two equations (20) and (23) at $x = 0$.

### 4.2 Boundary condition at the end of the resonator

Considering the piston mechanism which allows the translation of the slide flute, the boundary condition at the end of the flute, can be written as a first approximation neglecting friction terms:

$$S_p \dot{p}(L, t) + F = m \ddot{L} \quad (24)$$

$F$ being the force exerted by the motor on the slide and $m$ the piston mass. In a first step, one can consider that the control variable is the piston velocity $L$, linked to the physical control $F$ homogeneous to $L$, via the integrator (or cascade) system given by (24). Then if $L$ is known, one can then compute the physical control $F$ to apply, using e.g. “backstepping” techniques (see [3]). One can therefore consider, without loss of generality, the following boundary condition at $x = L$:

$$u(L, t) = S_p \dot{L} \quad (25)$$

which can be rewritten in the $\tilde{\alpha}$ and $\tilde{\beta}$ variables, using (4) and (6):

$$\tilde{\alpha}(1, t) + \tilde{\beta}(1, t) = 2S_p \dot{L} \quad (26)$$

### 5 Modal analysis from the linearized boundary conditions

To compute the natural modes of the system, we have to keep only linear terms in the boundary conditions obtained in the previous section. More precisely, we approximate $\tanh$ by its argument in (14) for the expression of $\Delta p_m$ and we neglect the nonlinear term $\Delta p_m$ given by (16). Writing $U_j$, $\tilde{\alpha}$ and $\tilde{\beta}$ on the following form, where $\dot{U}_j$ is the steady state value of the jet velocity (see equation (15)):

$$\begin{align*}
U_j &= \dot{U}_j + U e^{j\omega t} \\
\tilde{\alpha}(x, t) &= a e^{j\omega t} e^{j\omega x/c} \\
\tilde{\beta}(x, t) &= b e^{j\omega t} e^{j\omega x/c}
\end{align*} \quad (27)$$

replacing $U_j$, $\tilde{\alpha}$ and $\tilde{\beta}$ in (20) and (23) at $x = 0$ and (26) at $x = L$ and keeping only linear terms, we have to solve an homogeneous linear system of 3 equations with 3 complex unknowns $(U, a, b)$ of the form, where denoting $\kappa(\omega) = (c_1 + K e^{-\kappa(\omega)}) u_0$:

$$A(\omega) \begin{pmatrix} a \\ b \\ U \end{pmatrix} = 0, \quad \text{with} \quad A(\omega) = \begin{pmatrix} c_1 i \omega + c_2 e^{-i\omega L/c} \\ -c_1 i \omega + c_2 e^{i\omega L/c} \\ 2\rho_0 (\dot{U}_j + i\omega \dot{u}_0) \end{pmatrix} \begin{pmatrix} -Z_c + \kappa(\omega) \\ 0 \end{pmatrix} = \begin{pmatrix} -Z_c + c_1 \omega^2 e^{-i\omega L/c} \\ -Z_c + c_1 \omega^2 e^{i\omega L/c} \\ -2\rho_0 (\dot{U}_j + i\omega \dot{u}_0) \end{pmatrix} \begin{pmatrix} c_1 \omega^2 e^{-i\omega L/c} \\ c_1 \omega^2 e^{i\omega L/c} \\ 0 \end{pmatrix} \quad (28)$$

where the constants $c_1$, $c_2$ and $c_3$ are given by (11) and (9), the characteristic impedance $Z_c$ is given by:

$$Z_c = \frac{\rho_0 c}{S_p} \quad (29)$$

and the constant $K$ is obtained by linearizing $\Delta p_m$, the pressure jump due to the jet-drive (see equations (13), (14) and (17)). In fact, after some computations, we obtain the linearized expression $\Delta p_m$:

$$\Delta p_m = \dot{K} u_0(t - \tau) \quad (30)$$

Therefore, to obtain a non trivial solution, the complex modes $\omega = \lambda + i\epsilon$ must satisfy:

$$\det(A(\omega)) = 0, \quad (31)$$

namely, the real numbers $\lambda$ and $\epsilon$ must be solutions of the two real equations:

$$\begin{align*}
\text{Re}(\det(A(\lambda, \epsilon))) &= 0 \\
\text{Im}(\det(A(\lambda, \epsilon))) &= 0
\end{align*} \quad (32)$$

For $L = 0.265 m$ and $p_f = 300 Pa$, the three first frequencies $f_i$, $i = 1, 2, 3$, corresponding to the three first modes $\lambda_i = 2\pi f_i, i = 1, 2, 3$ are approximately, in Hz:

$$f_i \approx 290.8, \quad f_i \approx 903.7, \quad f_i \approx 1474. \quad (33)$$

It can be noticed that:

$$\frac{f_2}{f_1} \approx 3.11, \quad \frac{f_3}{f_2} \approx 5.07 \quad (34)$$

i.e. the frequencies of the modes are close to odd multiples of the first mode frequency, as expected for a closed-open pipe (see [1, 2] for details).
6 Control algorithm and first experimental results

To realize an automatic control law, we have to compute $p_f$ and $L$ or equivalently, the pair $(\bar{U}_j, L)$ such that a desired reference mode $\omega_r = \lambda_r + i\epsilon_r$ (or a desired reference frequency $f_r = \frac{\lambda_r}{2\pi}$) is obtained. Therefore, the control algorithm can be summarized as follows:

- $\lambda_r$ and $\epsilon_r$ being chosen, solve the two equations (32) with respect to the unknown variables $\bar{U}_j$ and $L_r$, using for example a Newton algorithm.

- The resulting asymptotic jet velocity $\bar{U}_j$ will be reached through the servo-valve, asking for a desired steady-state pressure $p_f = \frac{1}{2}\rho_0 \bar{U}_j^2$.

- The resulting length of the pipe $L_r$ will be reached, applying the following simple proportional control law on the piston:

$$\dot{L} = -k(L - L_r), \quad k > 0. \quad (35)$$

For example, to have $\omega_r = 2039$, corresponding to $f_r = f_{j,ideal} = c/4L = 324 \text{ Hz}$, solving equations (32) with initial conditions close to the first mode frequency leads to: $(\bar{U}_j = 8.61 \text{ m/s}; L_r = 0.242)$ (or equivalently $(p_f = 44.5 \text{ Pa}; L_r = 0.242)$).

First experimental results are given in Figure 5. The velocity reference of the piston (black curve) has been chosen with a trapezoidal form to have a smoother behavior than with a simple set point reference position. The actual velocity is the blue curve, the reference position of the piston is the green curve and the actual position is the red one. This example shows that the closed-loop behavior of the system is quite satisfying (for more details on the experiments, one can consult the website https://sites.google.com/site/fluctronic). Some improvements are under study, using more elaborated closed-loop controllers such as $i$-PID (intelligent PID controllers, see e.g. [12, 5] for this notion) to take into account unknown dynamics such as for example friction along the guide.

References


