From welfare to preferences, do decision flaws matter?
The case of tuition fees
Guilhem Lecouteux, Léonard Moulin

To cite this version:
Guilhem Lecouteux, Léonard Moulin. From welfare to preferences, do decision flaws matter? The case of tuition fees. cahier de recherche 2013-06. 2013. <hal-00807687>

HAL Id: hal-00807687
https://hal.archives-ouvertes.fr/hal-00807687
Submitted on 4 Apr 2013

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L’archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d’enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.
FROM WELFARE TO PREFERENCES, DO DECISION FLAWS MATTER? THE CASE OF TUITION FEES

Guilhem LECOUTEUX
Léonard MOULIN

April 2013

Cahier n° 2013-06
From welfare to preferences, do decision flaws matter? The case of tuition fees

Guilhem Lecouteux* and Léonard Moulin†

April 2, 2013

Abstract

We investigate the relation between welfare and preference satisfaction in economics, and show that the extension of the scope of economic analysis through the 20th century forces economists to question the validity of the preference satisfaction criterion as a normative criterion for evaluating public policies. We then argue that welfare economists should clarify the normative content of this criterion in order to properly design public policies. We illustrate our point with the case of the selection at the entry of university, and show that, according to the normative criterion used by the social planner, the optimal policy radically changes. In particular, we argue that — if we adopt a paternalistic conception of welfare — it becomes necessary to establish a clear distinction between preference and welfare, and to integrate the different social and psychological biases that can influence the decision of the individuals in the design of public policies.

JEL classification: B41, D60, I24.

Keywords: preferences, welfare, prospect theory, tuition fees.

*École Polytechnique, Laboratoire d’économétrie PREG-CECO (CNRS, UMR 7176), Département d’Économie, Route de Saclay, 91128 Palaiseau, France. E-mail: guilhem.lecouteux@polytechnique.edu

†Université Paris 13, Sorbonne Paris Cité, CEPN (CNRS, UMR 7234), UFR de Sciences Économiques, 99 Avenue Jean Baptiste Clément, 93430 Villetaneuse, France. E-mail: leonard.moulin@ens-cachan.org
1 Introduction

As argued by McQuillin & Sugden (2012), the recent development of behavioural economics raises some issues in welfare economics concerning the interpretation of the preference-satisfaction criterion as a normative criterion. They argue that this criterion can be interpreted in three conceptually different ways — as an evaluation in terms of happiness, self-assessed well-being or freedom — which lead to the same prescriptions as long as the preferences of the individuals are coherent. However, there exists a large literature showing the existence of numerous and systematic incoherences in the preferences of the individual (see Kahneman & Tversky (2000) and Camerer (2003)), such as framing effects: the different interpretations of the preference-satisfaction criterion does not lead to the same prescriptions any more, and it seems that economists should choose one of these interpretations in order to clarify the normative content of their prescriptions. In particular, we can highlight a major difference between on the one hand the interpretation of the satisfaction of preferences in terms of happiness or self-assessed well-being and on the other hand in terms of freedom: in the first case, the economist considers that the individual should be “helped” in order to achieve a given objective — whether it be the maximization of her own happiness (objectively defined in a “mental state” perspective1) or of her well-being (subjectively defined as what the individual values as preferable to herself2) — and therefore that her eventual decision flaws should be corrected, whereas the interpretation of the preference-satisfaction criterion in terms of freedom of choice is in a more deontological perspective, which implies that the economist should not pay attention to the effective choice of the individuals, but only to the opportunity they had making choices among a wide range of alternatives3. According to the normative criterion economists are referring to, the issues to be tackled are quite different: in the case of a paternalist criterion, the economists should study the connections between the preferences of the individual, defined as the relation over the space of actions that determines her effective choices, and her “welfare”4; and in the case of a non-paternalistic criterion, the economists should focus on the conditions under which the individuals can have access to the wider range of choice, independently of any consideration about the coherence of their preferences5.

1According to this approach, some mental states are intrinsically good, such as pleasure or happiness in the spirit of Bentham’s hedonism (see Hausman (2012, page 79)).

2This position is defended among others by Sunstein & Thaler (2003): their argument is that the individual should be protected against her mistakes while respecting her subjectivity.

3This argument has been developed by Sugden (2004, 2007), who suggests using a normative criterion of opportunity rather than preference-satisfaction: the relevant point a social planner should focus on is the ability for the individual of having at her disposal a wide range of available actions, and not the consequences of the choice the individual effectively makes.

4The welfare can be defined as the relation over the space of actions a paternalistic planner is caring about — either a relation objectively defined by the planner if he seeks to maximize the happiness of the individual, or a relation subjectively defined by the individual which represents her “true” preferences, i.e. the relation that would have determined her choice if she had “complete information, unlimited cognitive abilities, and no lack of willpower” (Sunstein & Thaler 2003, 1162).

5Sugden (2004) suggests for instance using an “opportunity criterion” in order to evaluate the opportunities an individual has at her disposal, and considers that it “should be given an analogous status [than the Pareto criterion in conventional welfare economics] in an opportunity-based form of normative economics” (page 1021).
In this paper, we suggest highlighting the impact of the choice of one of these three interpretations on the design of public policies, as well as the major consequences that can produce the introduction of behavioural findings in the design of public policies: we will then argue that economists should clarify the normative content of the preference satisfaction criterion, and also that — in the case of a paternalistic normative criterion — welfare economists should make a clear distinction between the preferences of the individuals (which will determine their choices) and their welfare (the “component” of the preferences the planner cares about): they must therefore produce theories in order to connect both relations, i.e. theories that explain how to deduce the welfare of an individual from her preferences, and vice versa. We will focus here on the case of educational choices: there indeed exists many issues of information asymmetries (concerning for instance the abilities of the students, implying an adverse selection issue), and — in order to maximize social welfare — it can be optimal to implement a selection at the entry of university\(^6\). A possible policy is the implementation of tuition fees: Gary-Bobo & Trannoy (2005, 2008) showed for instance that such a policy is efficient because it enables selecting students by sorting them on the basis of their abilities. However, as stressed by Flacher & Harari-Kermadec (2013) and Flacher, Harari-Kermadec & Moulin (2013), their reasoning is based on a a-psychological and a-sociological conception of students’ behaviours, and the introduction of cognitive biases can question the efficiency of a policy of tuition fees.

There currently exists a strong debate about the soundness of a policy of tuition fees in higher education, as illustrated by the recent social movements of great magnitude in the United Kingdom, Chile and Quebec, that have followed the announcement of an increase in the level of tuition fees. We suggest therefore in this paper evaluating a policy of tuition fees implementation according to each of those criteria. We show that, if we consider a paternalistic normative criterion, the implementation of a unique level of tuition fees is an optimal policy for unbiased individuals, i.e. for whom there is no difference between their welfare and their preferences, whereas it can be suboptimal when the individuals can make decision flaws, i.e. for whom the preference relation that determines their choices is not the relation that determines the welfare they get from their choices. In the case of a non-paternalistic criterion, there is no specific reason to implement a selection at the entry of university, since the objective of the planner is not in terms of social welfare but in terms of opportunity.

The paper is organised as follows. In section 2, we argue that — due to the evolution of the scope of economic analysis — preference satisfaction cannot be considered as welfare, and we present the consequences in welfare economics of distinguishing between welfare and preferences. Section 3 illustrates our main argument by considering the economic analysis of the selection at the entry of university. In section 4 we evaluate the efficiency of the implementation of tuition fees according to the different normative criteria mentioned above. Section 5 concludes.

2 Preference satisfaction and welfare

In this section, we argue that the evolution of the scope of economic analysis does not allow economists to identify preference satisfaction with welfare any more. We firstly precisely define the different notions of preferences and welfare, and then highlight that the definition of economics as the science of logical actions by Pareto (1909, 1916) enabled the three interpretations of the preference-satisfaction criterion — in terms of objective, subjective well-being or freedom — to converge to the same prescriptions. We then show that the extension of the scope of economic analysis during the second half of the 20th century questioned the convergence of those three interpretations.

2.1 Preference and welfare

The concept of preference is central in economics, but it appears that there exists some misconceptions about what preferences actually are (Hausman 2011). One of the main questions about the definition of preferences is to know if it is a substantive or a formal notion: Sen suggested for instance that we could define several concepts of preferences, one “as the underlying relation in terms of which individual choices can be explained” and another “in the usual sense with the property that if a person prefers $x$ to $y$ then he must regard himself to be better off with $x$ than with $y$” (Sen 1973, page 67). It resulted from this a certain fuzziness about the mere definition of some of the notions that constitute the identity of economics, such as the one of utility or welfare. We can for instance find such a confusion in Becker’s work, who considers on the one hand that “individuals behave so as to maximize utility while extending the definition of preferences to include personal habits and addictions, peer pressure, parental influences on the tastes of children, advertising, love and sympathy, and other neglected behavior” (Becker 1996, page 4) — suggesting here a purely formal notion of preferences — but also on the other hand that “individuals maximize welfare as they conceive it” (Becker 1993, page 386), and that preferences are equivalent to “tastes” (Becker 1996, page 3) — suggesting in this case a much more substantive notion of preferences.

As underlined by Hausman (2012, page 1-page 2), we can list four main concepts of preference when English-speakers talk about preferences:

1. enjoyment comparison: saying that Anna prefers $x$ to $y$ means that Anna enjoys more $x$ than $y$.

2. comparative evaluations: saying that Bob prefers $x$ to $y$ means that Bob judges $x$ as better than $y$ in some regard (either according to a specific criterion, or to any relevant criterion);

3. favouring: Hausman takes here the example of affirmative action and “racial preferences”, i.e. that affirmative action favours racial minorities (this use of the word preference is quite different from the previous ones, since it does not imply any comparison among a set of available actions);

4. choice ranking: saying that Carla prefers $x$ to $y$ means that if she is faced with a choice between $x$ and $y$, she will choose $x$. 
Hausman argues that economists, in order to model human behaviour, have chosen a modelling strategy that establishes a tight connection between preference and choice: given a set of constraints and the beliefs of an individual, her preferences should determine her choices (Hausman 2012, page 131). The corollary of this approach is that the preferences of an individual should integrate everything that is relevant to the choice from her point of view. Preferences in economics should therefore be defined as total subjective comparative evaluations:

“To say that Jill prefers $x$ to $y$ is to say that when Jill has thought about everything she takes to bear on how much she values $x$ and $y$, Jill ranks $x$ above $y$. [...] Because Jill’s total subjective ranking does not leave out anything that she regards as relevant to the evaluation of alternatives, it combines with beliefs to determine her choices” Hausman (2012, page 34).

We can here notice that this definition is quite ambiguous: Hausman indeed considers that the preferences determine the effective choice of the individual, but also that the individual is supposed having “thought about everything she takes to bear on how much she values $x$ and $y$”, which presupposes that the individual has unlimited cognitive abilities. In addition, Hausman argued that the internal consistency of preferences, and the respect of the axioms of rationality such as the one of transitivity are logical consequences of comparative evaluations, i.e. that the preferences of the individual should be rational: this seems to contradict the assumption that the preferences of the individual, combined with her beliefs, determine her choices, since the individuals can make decision flaws.

Since there can exist some ambiguities about the notions of welfare, utility and preferences, we suggest precisely defining them in order to avoid any misinterpretation. We assume that each individual has some beliefs about the set of available actions at her disposal, and about the set of consequences that results from any action (for instance a monetary outcome). We assume that each action is evaluated through its consequences and that:

- the planner is able to rank the different actions in terms of an objective notion of welfare — such as happiness — the individual will get from her choices;
- the individual is able to make total subjective comparative evaluations and therefore to rank the different actions in terms of a subjective notion of welfare.

If the relation in terms of an objective (respectively subjective) notion of welfare over the set of actions is complete and transitive, then it can be represented by a function of objective (subjective) welfare. We define then the preferences of the individual as a relation over the set of actions that determines her effective choices\(^7\). If this preference relation is complete and transitive, then it can be represented by a utility function, whose maximization over the set of actions

\(^7\)The individual is able to rank the available actions, and we assume that she tries to satisfy those preferences, i.e. to choose the action with the highest rank.
determines the choice of the individual.

A question that results from those definitions is then to know if preference satisfaction is welfare (in the objective or subjective sense). Indeed, if this is the case, since the observation of the choices of an individual can give us some information about her preferences (knowing the beliefs of this individual), then a paternalist social planner will be able to get information about the welfare of the individual from her preferences. However, if preference satisfaction is not welfare, then it implies that a paternalist social planner should distinguish between the preferences and the welfare of the individual. This raises some serious issues, since, as underlined by Hausman & McPherson (2009), “[t]he tenuous claims of cost-benefit analysis to guide policy so as to promote welfare turn on measuring welfare by preference satisfaction and taking willingness-to-pay to indicate preferences” (page 2). This means that the willingness to pay determined in cost-benefit analysis gives information only about the preferences of the individuals, and not about their welfare: from the point of view of a paternalist social planner, cost-benefit analyses should therefore be used with caution.

We suggest now showing that the notion of preferences as the ultimate determinant of choice has always been a central element in economics, but that the connection between preference and welfare progressively disappeared through the extension of the scope of economic analysis. The main consequence of this evolution is that we cannot any more easily infer welfare from preference satisfaction, and that the traditional three interpretations of the preference-satisfaction criterion progressively diverged.

2.2 The neoclassical representation of human behaviour

In the 19th century, economics was tightly intertwined with psychology; and some of the first neoclassical economists — Edgeworth, Jevons for instance — claimed the scientific validity of their works because those were based on empirically robust psychological laws. The objective of those theoreticians was then to explain the economic behaviours of individuals as they manifested themselves in reality; and the validity of the psychological hypothesis, on which were based these economic theories, were assessed through experimentation. In the 20th century the methodology of economics underwent a major change known as the “paretian turn”, which shifted the foundations of microeconomics from theories of psychology about human behaviour to principles of rational choice.

2.2.1 Pareto’s science of logical action

Pareto wished to eliminate all psychological interpretation within economics, in order to establish a theory based on principles of rational choice, and therefore create a new science, different form psychology or sociology — a science of logical action. Actions are logical as soon as they “logically conjoin means to ends not only from the standpoint of the subject performing them, but from the standpoint of other persons who have a more extensive knowledge” (Pareto 1916, par.150). Logical actions therefore belong to a restricted range of human actions, characterized by (1) their repetition, and (2) the pursuit of the satisfaction

---

8For an analysis of the approach of Pareto, see Demeulenaere (1996) as well as Bruni & Sugden (2007).
of one’s tastes\footnote{Pareto extended later the definition of logical actions to the pursuit of one’s interests (Pareto 1916, par.2146), defined as the tendency to appropriate useful material goods, or only pleasant to life, as well as to look for consideration and honours (Pareto 1916, par.2009).} (Pareto 1909, chap.3, par.1). The repetition of the action should ensure that people’s beliefs coincide with objective facts, and the objective of satisfaction of one’s tastes imply that the individuals are actuated by hedonistic motives, such as the maximization of happiness.

In the continuation of Mill’s epistemological principles (1843), Pareto suggested then the concept of the \textit{homo economicus} as the “dimension” of human being which deals with logical actions, by analogy with the study of a concrete body, which can be seen under different perspectives (chemical, geometrical...). Therefore in reality man is an aggregate of all these different dimensions (the \textit{homo economicus}, the \textit{homo religiosus}, the \textit{homo ethicus}...) and apart from a few very specific contexts, the behaviour of an actual individual is most of the time different from the behaviour of the \textit{homo economicus} (Pareto 1909, chap.1). Pareto then stressed the imperative “reconstruction” of the individual, when one moves away from pure economics to applied economics (Pareto 1909, chap.1, par.26). In this perspective, and regarding the prescription of concrete economic measures, it is therefore imperative to complete the analysis of the \textit{homo economicus} by taking into account its other dimensions in order to verify the relevance of a measure.

This definition of \textit{homo economicus} confers a relatively restricted scope upon economic analysis, in the sense that the behaviour of a real individual can be approximated by the behaviour of a fictive \textit{homo economicus} if and only if she is doing a logical action. The study of non-logical actions falls for instance within the framework of sociology.

This means that economic analysis was originally connected to the study of a restricted range of human behaviour, characterized not only by the pursuit of self-interest, but also by objective beliefs about the world. In this situation, the preferences of the individual — defined as the relation that determines her choices — correspond by construction to her subjective welfare relation\footnote{Pareto indeed recognized that the \textit{homo economicus} could be actuated by non selfish motives (Pareto 1909, chap.3, par.11): her preferences can therefore — in this case — be defined as total subjective comparative evaluations, i.e. in terms of subjective welfare, and not in a purely hedonistic conception.}, since it is the only factor that motives the choice of the individual, and that the repetition of these kinds of actions enabled her to correct her eventual past decision flaws. We can also find a restricted scope of analysis for economics in the work of Jevons and Edgeworth, since Jevons explicitly considered that economics only deals with human actions motivated by “the lowest rank of feeling” (Jevons 1871, page 27), and that Edgeworth stated that “the first principle of Economics is that every agent is actuated only by self-interest” (Edgeworth 1881, page 16).

Since preferences were defined in terms of welfare, and that the individuals were supposed to have objective beliefs about the world, it was therefore possible to assimilate preference satisfaction to welfare\footnote{This is true by definition for logical actions, and conditioned to the absence of any false beliefs or issues of information in the case of Jevons (1871) and Edgeworth (1881).}: the preference-satisfaction criterion was therefore equivalent to a criterion in terms of welfare (either objective if we consider the hedonistic approach of Jevons and Edgeworth, or subjective if we refer to Pareto’s conception of logical action).
2.2.2 Economic analysis of human action

Despite Pareto’s precautions, economists progressively extended the scope of economic analysis\textsuperscript{12}, and the methodological principles suggested by Friedman (1953)\textsuperscript{13} enabled economists not to question the implications of this evolution on the definition and meaning of central notions in economics, such as utility, preferences, and the homo œconomicus. In his \textit{Essays in Positive Economics}, Friedman indeed placed at the core of his economic methodology the necessity to test economic theories according to their capacity to “explain much by little” (page 11) — and not according to the empirical validity of their assumptions. This approach is today implicitly accepted in most of the economic works, and corresponds to the idea that theory must not be judged according to the realism of their assumptions (which are necessarily wrong in any modelisation), but according to their predictive power: it is assumed that individuals act \textit{as if} these assumptions were true. In the case of economics, it is generally assumed that the individuals act \textit{as if} they were rational, perfectly aware of their objectives and of the consequences that could result from their actions, without any cognitive limitation.

Based on this suggestion, the concept of the homo œconomicus deeply evolved, from a mere dimension of the human being (tightly linked to the study of other dimensions — psychological, religious, ethical), to a full representation of the individual which presupposes that she acts rationally in all circumstances, maximizing on her set of available actions her utility function. This evolution sheds light on one of the major issues of economic analysis, which is that of the very definition of economic actions. Indeed, economists are continuously torn between two opposite goals — on one hand to suggest a purely economic theory of behaviour, separate from psychology and sociology, with a restricted scope of analysis, and on the other to strive towards a universal nature of economic action by considering all human actions as economic actions (this school of thought has greatly developed itself since the works of Becker (1996) and Coleman (1976), but we should also name the work of Von Mises and the Austrian school praxeology, according to which it is illegitimate to make a distinction between different “kinds” of human action\textsuperscript{14}). The issue at stake here is the meaning of preferences, i.e. whether we should define preferences as a substantive or formal notion: in the work of the first neoclassical economists, preferences were defined as a matter of tastes (with a substantive content, and providing information about the welfare of the individual), and — by definition of the scope of economic analysis — those preferences also determined the choices of the individual in the case of economic actions; with the fictionalist methodology initiated by Friedman, the preferences of an individual became merely the formal relation over the space of strategies that determines the choices of an individual: therefore a preference relation is only a mathematical object, and does not present any direct correspondence in the real world in terms of welfare.

We can here notice that, even though the field of study of economics ex-

\textsuperscript{12}We can think of the development of the neoclassical rational choice theory in sociology, with for instance the development of the “economics of marriage” or of crime.

\textsuperscript{13}See Maki (2003) about the implications of Friedman’s methodology, and in particular the ambiguity of the “as if” formulation of theories.

\textsuperscript{14}The title of the main work of Von Mises — \textit{Human Action: A Treatise on Economics} — is quite meaningful.
panded, the economists kept the same model of human behaviour, and kept representing individuals as self-seeking and perfectly rational *homo economicus*, whereas this model was explicitly designed for the study of logical actions. This implies among other things that the satisfaction of the preferences does not necessarily provide clear information about the welfare of the individuals. Behavioural economics is precisely a criticism of this approach: unlike Pareto and his will of creating a science distinct from psychology, behavioural economists suggest grounding microeconomics and the model of individual behaviour on psychology theories, rather than on principles of rational choice.

We have shown that the evolution of economics and the growing place of economic models in sociology (thanks to the neoclassical rational choice theory) imply that the tight connection that existed between self-interested, perfectly informed tastes and choice — i.e. the identity between welfare and preference satisfaction — in the case of logical actions is not valid any more, since economists are studying a wider range of human behaviour, for which the individuals can present more complex preferences than the simple pursuit of self-interest, as well as decisions flaws. Hausman & McPherson (2009) developed a similar argument, by identifying two major objections to a preference satisfaction view of welfare: non self-interested motives (Parfit 1986) and mistaken beliefs (Hausman & McPherson 1994). The existence of mistaken beliefs create a distinction between the interpretation of the preference satisfaction criterion in terms of freedom and in terms of welfare, and the existence of non self-interested motives between the interpretations in terms of objective and subjective welfare.

### 2.3 Preferences and welfare economics

One of the main consequences of distinguishing between welfare and preferences is that we cannot any more identify preference satisfaction with welfare. This raises some serious issues in welfare economics, since it becomes harder to properly define and therefore measure individual welfare. Preferences are for instance obviously influenced to some extent by the pursuit of happiness (or any other objective measure of welfare)\(^\text{16}\), but also by different other factors, such as ideals, personal commitments (constitutive of a subjective welfare), psychological and sociological biases... (Hausman 2012, page 134). There exists therefore an issue of identification: if we can partially observe the preferences of the individual through her choices, it is then not possible to distinguish the influence of individual welfare from these others factors in the preferences. It becomes therefore necessary to introduce a substantive theory of welfare in economics so that we could define *a priori* the welfare of the individuals. Unfortunately, there does not currently exist any accepted theory of well-being in philosophy, and it seems that economists should accept “common platitudes concerning what makes people better off” (Hausman 2012, page 92) to evaluate individual welfare, for lack of anything better.

---

15They also list four subsidiary difficulties: preferences change (Gibbard 1986), conflicts among preferences (Sen 1977), immoral preferences (Arneson 1990, Harsanyi 1982) and preferences which result from problematic psychological mechanisms — we can for instance want something only because we cannot precisely get it.

16For instance, we can reasonably assume that if an individual has the choice between two strictly identical options in terms of any relevant considerations but happiness, and that one of the available options gives a greater level of happiness, then she will prefer this last option.
As soon as we consider a paternalistic normative criterion, isolating welfare from other factors in the preferences of the individual becomes a crucial point: since the satisfaction of the preferences does not necessarily maximize the welfare of the individual, their simple aggregation cannot maximize social welfare. There are here two issues that the planner should tackle: (i) identify what individual values as good for themselves, and (ii) anticipate the actual behaviour of the individual in order to design public policies. This double objective precisely corresponds to the identification of both welfare and preference relations. Since the welfare relation is probably a central component of the preferences of the individual, it seems that if the planner can obtain some information about either the welfare or the preference relation, she would be able to identify the other relation. We can for instance notice that the preferences of the individual are partially observable through their past choices, and can therefore provide some relevant information on what makes people better off, as long as we “clean up” those preferences in order to reveal the underlying welfare relation\(^{17}\), i.e. the component of the preferences the planner cares about. Conversely, we can build a priori the welfare relation of the individual, relying on “common platitudes” about what makes people better off, and try to deduce the preferences of the individuals that could result from this welfare relation. Welfare economists should therefore investigate the relations that can exist between the welfare and the preference relations of the individual, in order to properly design public policies. We can also notice that this “construction” of preferences from a welfare relation (or conversely the isolation of a welfare component in the preferences) is quite similar with the methodological approach of Pareto, who considered that, in order to study human behaviour, we should isolate an “economic dimension” — i.e. an *homo oeconomicus* whose only motive is the satisfaction of self-interest — and that we need to aggregate the different dimensions (the *homo oeconomicus*, the *homo religiosus*, the *homo ethicus*...) in order to design public policies. This approach — defended for instance by Sunstein & Thaler (2003) — is therefore quite similar to the one suggested by Pareto. We indeed showed that the behaviour of an actual individual is determined by her preferences, and that in the particular case of logical actions, her preferences are equivalent to her welfare relation: the pursuit of welfare determines the behaviour of a fictive *homo oeconomicus* — who corresponds to the behaviour of an actual individual in the case of logical actions — and preference satisfaction determines the behaviour of an actual individual. When we consider the consequences of an economic policy, we need to model the actual behaviour of the individuals, and not only the behaviour of the *homo oeconomicus*.

In this section we argued that, from a paternalistic perspective, the design of public policies should integrate the different factors that differentiate preferences from welfare, such as psychological biases. In the next sections, we will focus on the specific case of the implementation of tuition fees at university, so that we will be able to show that the introduction of even slight differences between preferences and welfare can considerably change the design of optimal policies: this will support our argument that welfare economists should systematically

\(^{17}\)This argument is not contradictory with the idea that preference satisfaction is not welfare, since “the preferences [of the individuals] will correspond to what is good for them, because their preferences depend on their judgments of what is good for them, rather than because their preferences make things good for them.” Hausman (2012, page 88).
investigate the possible differences between the preference and the welfare relations, and also that they should leave the preference-satisfaction criterion in order to clarify the normative content of their prescriptions.

3 The special case of tuition fees

According to Friedman (1962), there exists numerous issues of information asymmetry in higher education which affect both students and universities: no-one is able to perfectly observe the “quality” of the other agent, and students often have a poor estimate of their own abilities (Gary-Bobo & Trannoy 2008, Jongbloed 2003, Teixeira 2006). Higher education is characterized by adverse selection — when the student chooses her university, and reciprocally when the university tries to select its future students — and moral hazard phenomena — since the level of effort of both students and universities is not perfectly observable \textit{ex post}. Two kinds of selection mechanisms have then been suggested by economists in order to solve these information issues: exams and tuition fees (Del Rey & Romero 2004, Fernandez 1998, Fernandez & Gali 1999, Gary-Bobo & Trannoy 2008). Tuition fees have the property of selecting the students on their willingness to pay their studies, and therefore on their “real” motivation in joining the university: tuition fees therefore reveal the preferences of the individual, whereas exams — if they are efficient — reveal the quality of prospective students. The optimality of the implementation of tuition fees had for instance been defended by Gary-Bobo & Trannoy (2008) who show their efficiency when students are not able to perfectly observe their own abilities.

However, the educational choice cannot really be assimilated to a logical action — at least from the point of view of prospective students. Even though it does not seem too unrealistic to assume that prospective students choose their university in order to maximize their expected outcome, the different issues of information imply an indeterminacy of several key elements in the educational choice. We can for instance consider — unlike Gary-Bobo & Trannoy (2008) — that the students have a biased perception of their own academic abilities or of the outcomes of their educational choice, resulting from both psychological and sociological factors. This implies in particular that, in order to correctly predict the behaviour of the students, we should integrate in the preferences of the prospective students different biases.

Among others, Bourdieu (1974) and Haveman & Wolfe (1995) showed that the social origin generates heterogeneity between students’ perceptions\footnote{According to Bourdieu (translated by us), “the adolescent will behave such that he will achieve what he perceives as an established fact: when you belong to a disadvantaged background, you cannot join University. [...] The competence required by the ‘choice’ of the best objective strategies (the choice of an investment, a school of education or a professional career) is quite unequally distributed” (Bourdieu 1974, page 6-page 8). Similarly, “children who grow up in a poor or low-income family tend to have lower educational and labor market attainments than children from more affluent families, suggesting that parental choices or attributes that result in reduced access by children to economic resources or opportunities increase the chances of low attainment.” (Haveman & Wolfe 1995, page 1870).}. Some economists have theorized the existence of these biases with the concept of relative risk aversion (Breen & Goldthorpe 1997, Holm & Jaeger 2008): this theory
emphasizes that social classes — in addition to differences in allocations and therefore in the cognitive resources available to them — differ in the gains they associate to education: the desire of the most favoured students to avoid downward social mobility is stronger than the desire of the less favoured students to upward social mobility. Brodaty, Gary-Bobo & Prieto (2009) argue that risk aversion is lower for less favoured students because they are those who have the least to lose. Relative risk aversion theory is consistent with the sociological work of Boudon (1974, 1994) who showed that inequality between students pathways can be explained by the differences between the strategies of different social classes: the existence of distinct social positions leads to the existence of different systems of expectations and decisions whose effects on inequality of opportunity are multiplicative (Boudon 1974, page 211). Some of the sociological biases that affect the students’ decision-making process can also result from differences in level of aspiration: the lower the student social class is, the lower her social aspiration will be (Krauss 1964, Page 2005).

The issue of educational choices is therefore particularly relevant for our investigation about the distinction between the different interpretations of the preference-satisfaction criterion: it indeed empirically appears that educational choice is characterized by multiple decision flaws, and therefore that the preferences of prospective students are different from their welfare relation. We suggest now building a simple model of educational choice, in order to evaluate according to different normative criteria the soundness of the implementation of tuition fees in higher education: if the prescriptions diverges, then it would mean that economists should leave the preference-satisfaction criterion.

4 Optimality of the implementation of tuition fees

In this section, we build a simple framework of educational choice, and evaluate the policy of the implementation of tuition fees in higher education according to different normative criteria. We firstly present our general framework, discuss the objective of the planner and then determine the optimal policy the planner should implement in order to achieve her objective.

4.1 Analytical framework

We present here a simple model of educational choice, and assume that prospective students do not interpret the outcome of their choices in absolute quantities, but in terms of gains or losses\(^1\). We will therefore model the behaviour of the students thanks to the prospect theory developed by Kahneman & Tversky (1979).

We assume that there exists a population of prospective students \(N = \{1; \ldots; n\}\),

\(^1\)A particularly relevant illustration of this phenomenon is the experiment conducted by Tversky & Kahneaman (1981) concerning the choice of a medical program as a response to a disease that affects 600 people: according to the way the programs were presented to the subjects (in terms of the number of dead and saved lives), almost half of them changed their choices, even if the two situations lead to the exact same consequences, and therefore the same level of welfare (in an objective or subjective notion). The preferences of the individuals therefore changed during the experiment, but not the welfare they would have get from their choice.
characterized by (1) a function of subjective welfare, (2) a reference point and (3) academic abilities. Since we want to focus on the impact of the introduction of decision flaws, we will produce the simplest possible model of educational choice, by assuming that there does not exist any borrowing constraint for prospective students, and that each student is perfectly informed about her probability of success at university. Each student is faced to the following choice:

1. stop one’s study, and get a low-qualified work, with a low salary $G_L$;
2. continue one’s study in order to get in fine a high-qualified work, whose discounted salary $G_H$ is strictly higher than $G_L$, under the condition that the individual passes her final exam, knowing that:
   - each student $i$ is paying a tuition fee $F_i \in [0; G_H - G_L]$\(^{20}\) in order to join the university;
   - each student $i$ has a probability $p_i \in [0; 1]$ of passing her final exam, which depends on her academic abilities;
   - if $i$ fails her final exams, then she will not be able to get a high-qualified work, and will be constrained to accept a low qualified one, with a low salary $G_L$: since she only gets her salary after several years of unpaid studies, the student perceives this gain as weighted by a discount parameter $\delta < 1$.

The university is assumed to be a public one (i.e. its tuition fee policy is determined by the social planner) and supports a fixed cost $c > 0$ per student, such that $G_H - c \geq G_L$, i.e. there is a net gain for the society if an individual gets a high-qualified work. This educational choice can therefore be represented as a choice between two prospects $H$ (study) and $L$ (work)\(^{21}\):

$$\begin{align*}
H &= (G_H - F_i, p_i; \delta G_L - F_i, 1 - p_i) \\
L &= (G_L, 1)
\end{align*}$$

We now assume that all the individuals have the same function of subjective welfare $U_i(P) : \mathcal{P} \mapsto \mathbb{R}$:

$$U_i(P) = \sum_{k \in K} p_k u(x_k)$$

with $P \in \mathcal{P}$ a prospect and $u(x)$ a Von Neumann-Morgenstern utility function. We consider that each player has a reference point, i.e. a level of outcome $\bar{x}_i$ such that:

- if $i$ gets an outcome $x > \bar{x}_i$, then she will perceive $x$ as a gain;
- if $i$ gets an outcome $x < \bar{x}_i$, then she will perceive $x$ as a loss.

\(^{20}\)We do not consider the possibility of negative tuition fees — which would correspond to a kind of scholarship — and assume that a prospective student who is certain to succeed in higher education will always study.

\(^{21}\)We reproduce here the notation of Kahneman & Tversky (1979). The prospect $P = (x_1; p_1; \ldots; x_K; p_K)$ means that the player get the outcome $x_k$ with a probability $p_k$, $\forall k \in [1; K]$. Let $\mathcal{P}$ denote the set of prospects.
According to the experimental results of Kahneman & Tversky (1979), the individuals have a tendency to be more risk averse when they are facing prospects with gains, and more risk-seeking when facing losses. This reference point can be interpreted as the social aspiration of the individual in terms of outcome: we can therefore assume that this reference point directly depends on the social origins of the prospective students, and that an individual from a disadvantaged background will have a relatively lower reference point than a more favoured individual.

From the function of subjective welfare, it is now possible to define the preferences of the individual, i.e. her utility function $V_i : P \mapsto \mathbb{R}$:

$$V_i(P) = \sum_{k \in K} p_k v(y_{ik})$$

with $y_{ik} = u(x_k) - u(\bar{x}_i)$

with $P$ a prospect, and $v$ a value function that integrates the perception of the outcome as a gain or a loss. The perception bias is supposed here to concern the subjective welfare the individual gets from her choice, and not directly the monetary outcome. Kahneman & Tversky (1992) suggested that this value function is (i) concave for gains, (ii) convex for losses, and (iii) steeper for losses than for gains. We have therefore:

$$\begin{align*}
  v(0) &= 0 \\
  \frac{\partial v}{\partial x}(x) &> 0 \quad \forall x \neq 0 \\
  \frac{\partial^2 v}{\partial x^2}(x) &\leq 0 \quad \text{if } x > 0 \\
  \frac{\partial^2 v}{\partial x^2}(x) &\geq 0 \quad \text{if } x < 0 \\
  \frac{\partial v}{\partial x}(-x) &\geq \frac{\partial v}{\partial x}(x) \quad \text{if } x > 0
\end{align*}$$

In this framework, the prospective students have the choice between a risky prospect (continue their studies) and a riskless one (get a low qualified work). The welfare of both prospects can be evaluated on an objective scale, such as the monetary outcome the individual gets, and also on a subjective scale, which corresponds to a self-assessed well-being. The individuals present a psychological bias, and evaluate the results of the prospects as gain or loss in terms of subjective welfare. An individual $i$ will choose to continue her studies if and only if she prefers the prospect $H$ to the prospect $L$, i.e. if and only if:

$$V_i(H) \geq V_i(L) \iff p_i \geq \bar{p}_i$$

with $\bar{p}_i = \frac{v[u(G_L) - u(\bar{x}_i)] - v[u(\delta G_L - F_i) - u(\bar{x}_i)]}{v[u(G_H - F_i) - u(\bar{x}_i)] - v[u(\delta G_L - F_i) - u(\bar{x}_i)]}$

$\bar{p}_i$ can be interpreted as a threshold of self-selection: prospective students whose abilities are less than $\bar{p}_i$ will choose the riskless prospect, whereas the sufficiently talented students will prefer continuing their studies.
4.2 Objective of the planner

We suggest now discussing the objective of the planner according to the normative criterion she refers to. Consider firstly the interpretation of the preference-satisfaction criterion as freedom. The planner then cares about the possibility for the individuals of choosing their action among the widest range of options, as long as it does not interfere with the freedom of choice of another individual: this criterion is not grounded on consequentialist considerations, therefore the planner will not evaluate her policy in terms of social welfare. Since we assume in our framework the absence of any borrowing constraints, the individuals are perfectly able to choose both options, and there is therefore no policy to implement\(^\text{22}\): there exists no specific reason for implementing tuition fees\(^\text{23}\).

We now focus on the two paternalist criteria in order to evaluate the implementation of tuition fees. In both cases, the objective of the planner is to help the individuals choosing the option that will maximize social welfare, either defined as the sum of the functions of objective welfare or as the sum of the functions of subjective welfare of the agents. Since the self-selection threshold \(\hat{p}_i\) of the individual \(i\) does not depend on the cost of studies \(c\) supported by the university, the planner can try to implement a threshold \(p^*\) such that only the sufficiently talented students (whose probability of success \(p_i\) is higher than \(\hat{p}_i\)) go to university: the less talented students have indeed only a few chance of passing their final exam, and their expected gain does not cover the cost supported by the university. The planner needs therefore to determine the optimal level of \(p^*\), and will then implement tuition fees \(F_i\) such that \(\bar{p}_i = p^*\), \(\forall i \in N\), i.e. such that the self-selection threshold of each individual \(i\) corresponds to the optimal threshold.

The social planner needs therefore to define the social welfare function: this will depend on the normative criterion she will refer to. If the planner seeks to maximize an objective measure of welfare, such as the monetary outcome the individuals get from their choice, the social welfare function will be the following:

\[
SW(\bar{p}) = \sum_{i \in N, p_i \geq \bar{p}} [p_i G_H + (1-p_i)\delta G_L - F_i] + \sum_{i \in N, p_i < \bar{p}} G_L + \sum_{i \in N, p_i \geq \bar{p}} (F_i - c)
\]  

(6)

On the contrary, if the planner seeks to maximize the self-assessed well-being of the individuals\(^\text{24}\)

\[
SW(\bar{p}) = \sum_{i \in N, p_i \geq \bar{p}} U_i(H) + \sum_{i \in N, p_i < \bar{p}} U_i(L) + \sum_{i \in N, p_i \geq \bar{p}} (F_i - c)
\]  

(7)

\(^{22}\)If we assume that there exists borrowing constraints, then the planner should try to improve the credit market in order to suppress them. We can find a similar spirit in the procedural conceptions of justice, such as the ones of Rawls (1971) or Hayek (1976), which consider that we must evaluate the justice of a situation according to the "basic structure" of the society, and not according to the effective achievements of the individuals.

\(^{23}\)The only motive that could justify such a policy is the funding of higher education. However, this question will remain for all the different criteria: our result only means that referring to the normative criterion of freedom does not provide any additional reason for implementing tuition fees.

\(^{24}\)For convenience, we assume here that we can proceed to interpersonal comparisons of this subjective measure of welfare.
It clearly appears that the optimal threshold $\bar{p}^*$ depends on the specification of the social welfare function, i.e. on the normative criterion the social planner refers to. The solutions will indeed be generically different as long as the Von Neumann - Morgenstern utility function $u$ is not linear (in this situation, the subjective and objective measures of welfare will coincide). We can therefore see that according to the normative criterion we use in order to evaluate public policies, the objective of the planner deeply changes: with a non paternalist approach, the implementation of tuition fees in order to select the prospective students is pointless, and with a paternalist approach, the objective of the planner (i.e. the threshold of selection she wants to implement) depends on the definition of welfare — either an objective measure, such as happiness or more pragmatically a monetary outcome, or a subjective one. We can also underline an other point: in our model, we assumed that the planner know the functions of objective and subjective welfare, as well as the preferences of the individual. More realistically, since there does not exist any accepted theory of well-being in philosophy, it is hardly arguable that the planner knows the function of subjective welfare (the preferences can be determined thanks to cost-benefit analysis or through the past choices of the individuals, and the function of objective welfare is defined by the planner herself). This means that if we consider a paternalist criterion, it becomes essential to produce theories that could explain how we can deduce the function of subjective welfare from the preferences, i.e. how we can deconstruct them in order to identify the underlying welfare relation of the individual\(^{25}\).

Since we showed that it can be optimal to implement a threshold $\bar{p}^*$ at the entry of university in order to maximize social welfare, we now characterize the optimal level of tuition fees the planner should implement.

### 4.3 Optimal level of tuition fees

For convenience, we will assume that the measures in terms of objective and subjective welfare are identical, i.e. that the functions $u_i$ are linear (our results remain the same if we consider non linear Von Neumann - Morgenstern functions). The self-selection threshold $\bar{p}_i$ is therefore:

$$
\bar{p}_i = \frac{v(G_L - \bar{x}_i) - v(\delta G_L - F_i - \bar{x}_i)}{v(G_H - F_i - \bar{x}_i) - v(\delta G_L - F_i - \bar{x}_i)}
$$

(8)

Since the objective of the planner is to send to university the sufficiently good students, it implies that a necessary condition of its policy is that two individuals with the same probability $p_i$ must take in the same decision.

We will now study the effect of a variation in the level of tuition fees and in the level of the reference point on the threshold $\bar{p}_i$: if the level of the reference point impacts the threshold, then it will mean that the planner should put in place

\(^{25}\)However, as argued in the second section, we are faced with an issue of identification: since we know that the preferences of an individual are constituted of her welfare plus certain psychological biases, it is possible to define this subjective welfare relation either (i) by defining it a priori or (ii) by eliminating the different elements that can lead to decision flaws: in the first case, we need a substantive theory of well-being — that we do not have currently in economics — and in the second case, we need to know what are the biases that influence the choice of the individual. Since we cannot determine a priori the biases that play a role in the decision process of the individuals, it seems quite delicate to properly deconstruct the preferences of the individual.
different levels of tuition fees according to the reference point of the individual, such that the difference in tuition fees compensates the effect of the difference in reference points.

We can firstly show that the level of tuition fees has a positive impact on the threshold $\bar{p}_i$, i.e. that higher fees imply a stronger self-selection (details in appendix):

$$\frac{\partial \bar{p}_i}{\partial F_i}(\bar{x}_i; F_i) > 0$$ (9)

In addition, we can determine under which conditions the threshold $\bar{p}_i$ decreases when the level of the reference point increases, i.e. the level of $\bar{x}_i$ such that:

$$\frac{\partial \bar{p}_i}{\partial \bar{x}_i}(\bar{x}_i; F_i) < 0$$ (10)

We can show that (details in appendix), (i) for intermediate values of $\bar{x}_i$, the condition necessarily holds for any function $v$ that respects the conditions (4), (ii) for low values of $\bar{x}_i$, it holds if $v$ is sufficiently concave for gains, and (iii) for high values of $\bar{x}_i$, the condition holds if $v$ is sufficiently convex for losses. It means that, if $v$ is sufficiently concave for gains and convex for losses, then $\bar{p}_i$ always decreases in $\bar{x}_i$: the individuals from a disadvantaged background — presenting therefore a low social aspiration and a relatively low reference point $\bar{x}_i$ — will have a higher threshold than an individual from a more favoured background — presenting a higher reference point. This means that two individuals with identical academic abilities and different reference points can take different decisions, which is not optimal in terms of social welfare. In particular, it means that the planner should correct this deformation by the implementation of different levels of tuition fees, according to the level of the reference point of the individuals.

We can highlight here an interesting mechanism, consistent with the findings of (Breen & Goldthorpe 1997, Holm & Jaeger 2008): individuals with a relatively high social aspiration are considering both prospects as losses, and are therefore risk-seeking, unlike the students with a more modest objective, who evaluate the outcomes of the prospects as gains, and are therefore risk averse. We can therefore expect that those individuals will behave more cautiously and take less risk than the students from a more favoured background.

The introduction of a difference between the preferences and the welfare of an individual considerably changes the design of the optimal selection policy. In particular, if we assume that the value function $v$ is sufficiently concave for gains and convex for losses — i.e. if there exists an important psychological bias —, then prospective students with modest ambitions need to have a stronger incentive than the prospective students with higher aspirations, and therefore should pay less tuition fees. We have here an interesting justification of the introduction of decreasing tuition fees: individuals with a disadvantaged background need to pay less tuition fees, not because they are facing some funding issues, but because their modest social aspirations imply a too cautious behaviour. We can however question the accuracy of the implementation of decreasing tuition fees in reality: we can indeed notice that the actual population in different
universities is quite heterogeneous, i.e. that the proportion of students from a disadvantaged background is much higher in some universities than others. This implies that the implementation of decreasing tuition fees will enable universities with a high proportion of students from a favoured background of raising important funds through the payment of tuition fees, whereas the universities for which the proportion of disadvantaged students is high will not collect many funds from the implementation of tuition fees. Such a policy of decreasing tuition fees, even if it appears to be optimal on an individual level, is therefore difficultly implementable in reality, since it will generate serious inequalities between the universities according to the population of their students.

5 Conclusion

The development of behavioural economics and the evidences of systematic mistakes and decision flaws question the accuracy of the criterion of preference-satisfaction in normative economics. In this paper, we showed that, in the case of educational choices, the three traditional ways of justifying this criterion — happiness, self-assessed well-being and freedom — lead to radically different prescriptions. We indeed showed that the evolution of the scope of economic analysis progressively created divergences between those three criteria, by the introduction within the scope of economics of actions guided by non hedonistic motives (differentiating the analysis in terms of objective and subjective welfare), as well as choice situations characterized by decision flaws and mistaken beliefs (differentiating the paternalist criteria from the non paternalist one). We suggest therefore that economists should leave the preference satisfaction criterion, as long as they do not clarify the normative content of this principle. In particular, it implies that economists should cautiously use cost-benefit analysis in order to evaluate the soundness of a public policy: they are indeed not evaluating the welfare of the individual — which is fundamentally the relevant variable that a paternalist planner wants to maximize — but her preferences. We have then suggested that welfare economists — in a paternalistic perspective — should develop models that could explain the formation of preferences from the welfare of the individuals, and reciprocally how we could deduce the welfare of an individual from her observed preferences.

In the case of the implementation of tuition fees in higher education, we have shown that not only the implementation of a selection mechanism at the entry of university is not necessarily supported by the preference-satisfaction criterion (if we consider its liberal interpretation in terms of “consumer sovereignty”), but also that the implementation of a unique level of tuition fees can be suboptimal in terms of social welfare: since the individuals from a disadvantaged background have lower social aspirations, they have a tendency to act too cautiously, and relatively good students will not be sufficiently encouraged to join university. Nevertheless, although the implementation of decreasing tuition fees is a better solution than the implementation of a unique level of fees on a theoretical level, its actual application can create serious inequalities in terms of funding between the universities.
Appendix

We firstly show the relation (9), i.e. that the threshold of self-selection increases with the level of tuition fees, and then investigate the conditions under which (10) holds, i.e. the threshold of self-selection decreases with the level of the reference point.

The relation (9) holds if and only if:

\[
\frac{\partial \bar{p}_i}{\partial F_i} = \frac{\partial v}{\partial x} (G_H - F_i - \bar{x}_i)(v(G_L - \bar{x}_i) - v(\delta G_L - F_i - \bar{x}_i)) + \frac{\partial v}{\partial x} (\delta G_L - F_i - \bar{x}_i)(v(G_H - F_i - \bar{x}_i) - v(G_L - \bar{x}_i)) \frac{(v(G_H - F_i - \bar{x}_i) - v(\delta G_L - F_i - \bar{x}_i))^2}{(v(G_H - F_i - \bar{x}_i) - v(G_L - \bar{x}_i))^2} > 0
\]

which is true by construction, since \(\frac{\partial v}{\partial x}\) is positive \(\forall x\), and \(G_H - F_i - \bar{x}_i > G_L - \bar{x}_i > \delta G_L - F_i - \bar{x}_i\).

The relation (10) holds if and only if:

\[
\frac{\partial \bar{p}_i}{\partial \bar{x}_i} = \frac{\partial v}{\partial x} (G_H - F_i - \bar{x}_i)(v(G_L - \bar{x}_i) - v(\delta G_L - F_i - \bar{x}_i)) + \frac{\partial v}{\partial x} (\delta G_L - F_i - \bar{x}_i)(v(G_H - F_i - \bar{x}_i) - v(G_L - \bar{x}_i)) \frac{(v(G_H - F_i - \bar{x}_i) - v(\delta G_L - F_i - \bar{x}_i))^2}{(v(G_H - F_i - \bar{x}_i) - v(G_L - \bar{x}_i))^2} < 0
\]

which is equivalent to:

\[
\frac{\partial v}{\partial x} (G_L - \bar{x}_i) \geq \frac{\partial v}{\partial x} (G_H - F_i - \bar{x}_i) \frac{v(G_L - \bar{x}_i) - v(\delta G_L - F_i - \bar{x}_i)}{v(G_H - F_i - \bar{x}_i) - v(\delta G_L - F_i - \bar{x}_i)} + \frac{\partial v}{\partial x} (\delta G_L - F_i - \bar{x}_i) \frac{v(G_H - F_i - \bar{x}_i) - v(G_L - \bar{x}_i)}{v(G_H - F_i - \bar{x}_i) - v(\delta G_L - F_i - \bar{x}_i)}
\]

We suggest now studying the first order derivative \(\frac{\partial v}{\partial x}(x)\) in \(x = \delta G_L - F_i - \bar{x}_i\), \(x = G_L - \bar{x}_i\), and \(x = G_H - F_i - \bar{x}_i\). Knowing that \(v\) is concave for gains, convex for losses and steeper for losses than for gains, we can deduce from the conditions (4):

\[
\frac{\partial v}{\partial x}(y) \geq \frac{\partial v}{\partial x}(x) \quad \forall y \in [-x; x], \quad x \geq 0
\]

We have therefore:

\[
\bar{x}_i \geq \frac{G_L + \delta G_L - F_i}{2} \Rightarrow \frac{\partial v}{\partial x}(G_L - \bar{x}_i) \geq \frac{\partial v}{\partial x}(\delta G_L - F_i - \bar{x}_i)
\]

\[
\bar{x}_i \leq \frac{G_L + G_H - F_i}{2} \Rightarrow \frac{\partial v}{\partial x}(G_L - \bar{x}_i) \geq \frac{\partial v}{\partial x}(G_H - F_i - \bar{x}_i)
\]

These last conditions imply that, \(\forall \bar{x}_i \in \left[\frac{G_l + \delta G_L - F_i}{2} ; \frac{G_L + G_H - F_i}{2}\right]\), the condition (10) necessarily holds, and therefore that \(\bar{p}_i\) is always decreasing for those
In addition, we can notice that the first order derivative of $v$ is always higher in $x = (G_L - \bar{x}_i)$ than either in $x = (G_H - F_i - \bar{x}_i)$ or $x = (\delta G_L - F_i - \bar{x}_i)$. In particular, for relatively low values of $\bar{x}_i$ (for instance if $\bar{x}_i < (\delta G_L - F_i)$, which implies that the individual perceives all the possible outcomes as gains), we have the following inequalities:

$$\begin{aligned}
\frac{\partial v}{\partial x}(G_L - \bar{x}_i) &\geq \frac{\partial v}{\partial x}(G_H - F_i - \bar{x}_i) \\
\frac{\partial v}{\partial x}(G_L - \bar{x}_i) &\leq \frac{\partial v}{\partial x}(\delta G_L - F_i - \bar{x}_i)
\end{aligned}$$

Under those circumstances, we can see that the condition (10) holds if $v(G_L - \bar{x}_i)$ is relatively close to $v(G_H - F_i - \bar{x}_i)$, i.e. if $v$ is sufficiently concave for gains.

On the contrary, if the reference point of the individual is sufficiently high (with for instance $\bar{x}_i > (G_H - F_i)$, which implies that the individual perceives all the possible outcomes as losses), then we have the following inequalities:

$$\begin{aligned}
\frac{\partial v}{\partial x}(G_L - \bar{x}_i) &\leq \frac{\partial v}{\partial x}(G_H - F_i - \bar{x}_i) \\
\frac{\partial v}{\partial x}(G_L - \bar{x}_i) &\geq \frac{\partial v}{\partial x}(\delta G_L - F_i - \bar{x}_i)
\end{aligned}$$

In this situation, the condition (10) holds if $v(G_L - \bar{x}_i)$ is relatively close to $v(\delta G_L - F_i - \bar{x}_i)$, i.e. if $v$ is sufficiently convex for losses.

More generally, let $x^+_i$ and $x^-_i$ denote the levels of the reference point $\bar{x}_i$ such that:

$$\begin{aligned}
\frac{\partial v}{\partial x}(G_L - x^+_i) &= \frac{\partial v}{\partial x}(G_H - F_i - x^+_i) \\
\frac{\partial v}{\partial x}(G_L - x^-_i) &= \frac{\partial v}{\partial x}(\delta G_L - F_i - x^-_i)
\end{aligned}$$

We have then:

- $\forall \bar{x}_i \in [x^-_i ; x^+_i]$, (10) always holds
- $\forall \bar{x}_i \leq x^-_i$, (10) holds if $v$ is sufficiently concave for gains
- $\forall \bar{x}_i \geq x^+_i$, (10) holds if $v$ is sufficiently convex for losses

References


