A practical precoding scheme for multicell MIMO channels with partial user’s data sharing
Paul de Kerret, Rajeev Gangula, David Gesbert

To cite this version:
Paul de Kerret, Rajeev Gangula, David Gesbert. A practical precoding scheme for multicell MIMO channels with partial user’s data sharing. 2012 IEEE Wireless Communications and Networking Conference Workshops (WCNCW), Apr 2012, France. pp.958 - 962, 10.1109/WCNC.2012.6214513 : hal-00807312

HAL Id: hal-00807312
https://hal.archives-ouvertes.fr/hal-00807312
Submitted on 5 Apr 2013

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L’archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d’enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.
A Practical Precoding Scheme for Multicell MIMO Channels with Partial User’s Data Sharing

Paul de Kerret, Rajeev Gangula and David Gesbert
Mobile Communications Department, Eurecom
2229 route des Crêtes, 06560 Sophia Antipolis, France
{derret,gangula,gesbert}@eurecom.fr

Abstract—In this work\(^1\), we consider a setting where \(K\) Transmitters (TXs) equipped with multiple antennas aim at transmitting to their \(K\) respective Receivers (RXs) also equipped with multiple antennas. Without exchange of the user’s data symbols, this represents a conventional Interference Channel (IC), while it is a so-called MIMO Network Channel if the user’s data symbols are fully shared between all the TXs. The focus of this work is on the intermediate case where the user’s data symbols can be arbitrary shared to the TXs such that only a subset of the TXs has access to the data symbols to transmit to a given RX. We show that we can build a virtual IC so as so have the transmission in that IC equivalent to the transmission in the original setting. In this virtual IC, it is then possible to apply any of the numerous algorithms (Interference Alignment algorithms) initially tailored for the IC. Finally, we let the routing matrix be optimized subject to a constraint on the total number of symbols shared and use a greedy algorithm to find the user’s data allocation. We show by simulations that sharing only few user’s data symbols is sufficient to achieve most of the performance.

I. INTRODUCTION

In future wireless networks, delivering always larger data rates to more users will be required and this has been recognized to be possible only at the condition of an efficient interference management scheme. Particularly, cooperation between transmitters (TXs) appears as a possible solution to reduce, avoid, or cancel interference. When the user’s data are shared to multiple interfering TXs, it is possible to do a joint precoding of the symbols to be transmitted to the receivers (RXs). This scheme is currently considered for next generation wireless networks and is called MIMO Network or Multicell MIMO channel (also CoMP in the 3GPP terminology). With perfect message and Channel State Information (CSI) sharing, the different TXs can then be seen as a unique virtual multiple-antenna array serving all receivers (RXs), in a multiple-antenna Broadcast Channel (BC) fashion [1].

Yet, the sharing of the data symbols and the CSI to the cooperating TXs impose huge requirements on the architecture, particularly as the number of cooperating TXs increases and Coordinated Beamforming has been recognized as a very appealing alternative due to fact that it does not require the user’s data symbols to be shared through the backhaul network which reduces therefore strongly the requirements on the backhaul links and the impact of the cooperation on the existing network infrastructure. Still, they allow for big improvement in performance thanks to the coordination of the precoders used at the different TXs [2], [3].

Especially, the idea of Interference Alignment (IA) in a MIMO interference channel (IC) has been recently developed [4], [5] and has been shown to achieve a larger Multiplexing Gain (MG), than previously thought possible in an IC, i.e., without the sharing of the user’s data symbols. The significant improvement in MGs as well as the practical importance of the IC has drawn the attention of the community and many works have been focused on IA. Consequently, numerous algorithms converging at high SNR to IA have been provided [6]–[8] and reference therein. On the analytical side, a critical question lies in the feasibility of IA for given antenna configurations. Yet, this problem is very intricate, and has not yet been solved in the general case, even though some heuristics exist [9] and exact results have recently been derived for some antenna configurations [10]–[12].

The Multicell MIMO and the Interference Channel (IC) represent two extremal cases of sharing the user’s data symbols to the TXs and correspond respectively to full sharing and no sharing. In this work we aim at bridging the gap between these two settings and we consider the user’s data symbols to be arbitrarily shared to a subset of the TX set. This corresponds to very practical scenarios as for example a cellular setting where a base station shares the data symbols of its users to only some neighbors. This setting has been studied in the case of single antenna RXs in [13], [14]. Yet, this work is fundamentally different as having multiple antennas at the RXs leads to the possibility of aligning the interference at the RXs and therefore to completely different precoding schemes. Indeed, with single antenna RXs, only channel inversion is considered at high SNR while IA will play a key role in a scenario where each RX is equipped with multiple antennas.

Although the MG achieved in a setting where cooperation can occur either on the TX side or on the RX side has been studied in [15], [16], no precoding algorithm has been provided, which is precisely what we will do in our work.

Our main contribution consists in introducing the problem of precoding for the case of the user’s data symbol being only shared to a subset of the TXs and in deriving a novel precoding scheme. Our approach is based on a transformation of the original setting into a virtual MIMO interference channel which allows for the use of conventional IA algorithms from

\(^1\)This work has been performed in the framework of the European research project ARTIST4G, which is partly funded by the European Union under its FP7 ICT Objective 1.1 - The Network of the Future.
transmitting at high SNR, such that IA plays a key role in the achievement of high performance.

B. Interference Alignment

In this work, we consider Interference Alignment (IA) in MIMO IC with a static channel. We characterize an IC by the elements $\prod_{i=1}^{K} (M_i, N_i, d_i)$ as defined above. We recall briefly the main principle of IA and we refer to the large literature on the topic for more details [4]–[8], and reference therein. IA consists in designing the precoders so as to align at each RX the interference in a subspace of dimension small enough so that the intended signal can be received free of interference in the remaining dimensions. Thus, an IA solution is characterized by the fulfillment of the ZF equations:

$$\forall i, j, i \neq j, \quad G_i^H H_j T_j = 0_{d_i \times d_j}. \quad (4)$$

These ZF conditions characterize the IA and express the fact that the TX beamformer and the RX filters should be designed jointly so as to let enough free dimensions at the RX to be able to receive the user’s data free of interference. Finding the TX beamformers satisfying the ZF constraints (4) is not easy and closed form solutions exist only for particular antenna configurations. However, many iterative algorithms have been developed in the literature and provide solutions converging at high SNR to IA solutions when IA is feasible [7], [8], [17]. The IA algorithms are based on the iterative alternative update of the TX and the RX filters to optimize some figure of merit, chosen so as to lead to an IA solution at high SNR.

Maximum SINR Algorithm: We will use for the simulations a simple generalization of the original minimum leakage algorithm [17] where the noise and the strength of the direct signal are taken into account in the optimization to improve the performance at finite SNR. Yet, this corresponds only to one possible choice and any other IA algorithm from the literature could be used instead without changing the main idea of our approach. This precoding scheme is based on the maximization of the per-stream SINR, iteratively between the TX side and the RX side. We recall briefly the main steps for the sake of completeness and we refer to [17] for more details. The algorithm is based on the introduction of a reciprocal network where the roles of the TXs and the RXs are exchanged. Thus, the RX filter becomes the TX beamformer and the TX beamformer is used as RX filter while the power constraint of the TX is transferred to the RX. The algorithm optimizes each stream separately and we present it for the case of single stream transmission as it corresponds to the setting used in the simulations and leads to a simple exposition while the generalization to multiple streams follows directly.

Thus, in a first step, the TX beamformers are considered as being fixed and the RX filters are updated to their optimal value maximizing the per-stream SINR:

$$\forall i, g_i = \frac{\left( I_{N_i} + \sum_{j=1, j \neq i}^{K} H_{ij} t_j H_j^H \right)^{-1} H_{ii} t_i}{\left\| \left( I_{N_i} + \sum_{j=1, j \neq i}^{K} H_{ij} t_j H_j^H \right)^{-1} H_{ii} t_i \right\|} \sqrt{P}. \quad (5)$$
In a second step, the RX filters are fixed and the transmission is considered in the reciprocal network so that it corresponds to fixed TX beamformers and we can apply the same approach as for the first step and obtain:

\[
\forall i, t_i = \frac{\left( I_{M_i} + \sum_{j=1, j \neq i}^{K} H_{ij}^{H} g_j g_j^{H} H_{ji} \right)^{-1} H_{ii} g_i}{\left\| \left( I_{M_i} + \sum_{j=1, j \neq i}^{K} H_{ij}^{H} g_j g_j^{H} H_{ji} \right)^{-1} H_{ii} g_i \right\|} \sqrt{T}.
\]

Feasibility Conditions: The problem of determining feasibility of IA for given antenna configurations has also been studied in the literature, and even though the problem is still open in the general case, partial results have been obtained. The first necessary conditions were obtained in [9] and recently the feasibility of particular configurations has been solved using arguments based on algebraic geometry [10]–[12]. Particularly, we recall now the results for the single stream case.

**Theorem 1.** [18] IA is feasible in the IC \( \prod_{k=1}^{K} (M_k, N_k, 1) \) if and only if

\[
\sum_{k(k,j) \in \mathcal{I}} (M_k - 1) + \sum_{j(k,j) \in \mathcal{I}} (N_j - 1) \geq |\mathcal{I}|, \forall \mathcal{I} \subseteq K.
\]

In the homogeneous IC \( (M, N, 1)^K \), this condition reads as

\[
M + N \geq K + 1.
\]

This formula holds for the feasibility of a conventional IC and we have recalled it because we will show later that the setting considered with partial user’s data sharing has some similarities with the conventional IC channel, and these results will be interesting to offer a reference.

**C. Routing Matrix**

In realistic settings, e.g., cellular networks, it is not relevant to share the user’s data symbols to each antenna individually as the different antennas at a given TX are colocated and can cooperate perfectly. To model the data sharing between TXs, we define the routing matrix which has been first introduced in [13] for single antenna receiver and is extended here to RXs with multiple antennas. It is used to specify to which set of TXs the symbol of any given user is being shared with. Thus, we define the routing matrix as the matrix \( \mathbf{D} \in \{0, 1\}^{K \times d_u} \) whose \((i, \ell)\)-th element is 1 if \( \ell = m_j + p \) with \( m_j \triangleq \sum_{k=1}^{j-1} d_k \) and the \( p\)-th element of the symbol \( s_j \) is allocated to TX \( i \), and 0 otherwise. This slightly involved notation follows simply from the fact that one user can receive several independent data streams and this has to be taken into account in the design of the routing matrix [Cf. Equation (2)]. Furthermore, the total number of user’s data symbols shared through the backhaul network can then be seen to be equal to \( \| \mathbf{D} \|^2 \).

To study the impact of the routing matrix on the transmission, we need to introduce further notations taking into account the antenna configurations at the TXs.

**Lemma 1.** We define the expansion matrix \( \mathbf{E} \in \mathbb{C}^{N_u \times K} \) as

\[
\mathbf{E} \triangleq \left[ \mathbf{A}_1^T \ldots \mathbf{A}_K^T \right]^T
\]

where the matrix \( \mathbf{A}_i \in \mathbb{C}^{N_i \times K} \) is defined as \( \mathbf{A}_i \triangleq \mathbf{1}_{N_i} \cdot e_i^T \). The vector \( \mathbf{1}_{N_i} \in \mathbb{C}^{N_i \times 1} \) has all its elements equal to one and the vector \( e_i \in \mathbb{C}^{K \times 1} \) is the \( i \)-th vector from the canonical basis. Then, we have that

\[
\mathbf{T} = (\mathbf{E} \mathbf{D}) \odot \mathbf{T}.
\]

**Proof:** The proof follows easily by developing the matrix multiplication in (10).

Because TX \( i \) has \( M_i \) antennas, the routing of one symbol to that TX leads to the possibility of transmitting that symbol from the \( M_i \) antennas located at that TX. This is represented by the fact that the lines in the routing matrix \( \mathbf{D} \) are duplicated as many times as there are antennas at the TX. Since the elements of the precoder \( \mathbf{P} \) corresponding to symbols which are not routed to that TX are zero, the Hadamard product with the matrix \( \mathbf{E} \mathbf{D} \) leaves the precoder unchanged.

Equation (10) gives the restrictions coming from the partial sharing of the user’s data symbol. With that constraint on the precoder, conventional IA algorithms cannot be directly applied and we show in the following how efficient precoders can be obtained in a simple way.

**III. PRECODING WITH PARTIAL DATA SHARING**

When the receivers have only one antenna, the adaptation of the ZF scheme to the partial user’s data sharing has been done in [13]. However, with multiple antennas at the RXs, the precoding schemes developed there have to be completely rethought due to the need for Interference Alignment.

Note that in this work, we consider that a symbol vector \( s_i \) is either completely shared to a given TX, or not at all. This is practically meaningful and allows for the simplified approach developed.

**A. Equivalence with a Virtual Interference Channel**

Let consider for the rest of that Subsection an arbitrary routing matrix \( \mathbf{D} \) to be given, and we focus on the optimization of the precoder in that case where the precoder has to take into account the particular structure induced by the routing as described in (10). The partial sharing leads to a setting which differs from an IC because the TXs do not only have access to their own symbols. Yet, we show in the following that it is equivalent to some virtual IC for a certain channel matrix.

**Proposition 1.** Let define an IC with \( K \) TX/RX pairs in which RX \( i \) has \( N_i \) antennas (as in the original setup) while TX \( i \) is equipped with \( M_i' \) antennas and \( M_i' \) is given as the sum of the number of antennas at the TXs where the symbol vector \( s_i \) is shared, i.e.:

\[
M_i' \triangleq \sum_{k=1}^{K} M_k D_{k,(m_j+1)}
\]
with \( m_{ij} \triangleq \sum_{k=1}^{j-1} d_k \). The channel from \( TX \ j \) to \( RX \ i \) is then denoted by \( H_{ij}^m \in \mathbb{C}^{N_t \times M_t} \) and defined as

\[
H_{ij}^m = \begin{bmatrix} H_{ij1} & H_{ij2} & \ldots & H_{ijm} \end{bmatrix}.
\]

(12)

where the \( i_j \) indices correspond to the nonzero elements in the sum in (11), i.e., the indices to the TXs to which symbol \( s_j \) is shared.

The transmission in the original IC \( \prod_{i=1}^{K} (M_t, N_t, d_i) \) with the sharing matrix \( D \) is then equivalent to the transmission in the virtual IC \( \prod_{i=1}^{K} (M_t', N_t, d_i) \) with the channel matrices given in (12) with no data sharing between the TXs.

Proof: Let consider without loss of generality the precoder \( T_j \) transmitting the symbol vector \( s_j \) to RX \( j \). It corresponds to the elements \(((ED) \circ T)_{m_j+1:m_j+d_j} \) of the total multiuser precoder. Removing the zeros elements in the precoder \( T \), which are induced by the zeros in \( D \), we obtain a precoder \( T_j' \) which can be seen to be of size \( M_t' \times d_j \). The signal received at RX \( i \) corresponding to the transmission of symbol \( s_j \) can then be seen to be exactly \( H_{ij}^m T_j' \), which is exactly the transmission in the virtual IC defined above.

The interest of the virtual IC comes from the fact that we can then use IA algorithms from the literature to derive the precoder for the initial setting.

Let denote by \( f \) the precoding function which takes \( D \) and \( H \) as input and gives the multiuser precoder \( T \) as output, such that \( T = f(D, H) \). Applying the max-SINR algorithm described in Subsection II-B to this Virtual IC model leads to a generalization of the max-SINR algorithm [17] which was designed for the conventional IC, to any configuration for the sharing of the user’s data symbols. Furthermore, the proposed algorithm can be seen to coincide with the heuristic max-SINR precoding algorithm for the MIMO BC in [19] for full sharing of the symbols. Therefore, our algorithm bridges the gap between these two algorithms which can be understood as being particular cases of our algorithm. We also conclude this form property that the proposed algorithm should perform well at least in these two limiting settings.

B. Feasibility Conditions with Partial Data Sharing

The feasibility of IA under partial data sharing has been discussed for some configurations in [15], [16]. The analysis is based on results from the field of Algebraic Geometry and has been successful in deriving the following result.

Theorem 2. [15] Let consider the \( K \)-users regular IC \((1,1,1)^K \) with symbol \( s_i \) shared to the \( M_t - 1 \) TXs following TX \( i \) and cooperation between the RXs so that symbol \( s_i \) can be decoded using the received signal at the \( M_t - 1 \) RXs following RX \( i \). The MG is equal to \( K \) (i.e., maximal) if and only if

\[
M_t + M_c \geq K + 1.
\]

(13)

Theorem 2 holds for a given sharing pattern and the transmission of a single stream per user. Yet, it can be believed that any routing pattern fulfilling the condition of the Theorem for each data symbol leads to the maximum MG. Still, no result has been derived for a general case with arbitrary sharing of the user’s data symbols.

On the other side, we have recalled in Subsection II-B feasibility conditions which could potentially be applied to the Virtual IC, thus giving feasibility conditions for IA in the original setting. Nevertheless, the channels from the different TXs are no longer independent in the Virtual IC, due to the duplication of some channel elements to form the virtual channel. To which extent the feasibility results for the conventional MIMO IC extend to our virtual IC is not straightforward and represents an interesting question for future works.

C. Greedy User’s Data Sharing Optimization

We have derived a precoding function \( T = f(D, H) \) for given routing matrix \( D \). We let now the routing matrix \( D \) be optimized under the condition of having the total number of symbols shared through the backhaul network, which is known to be \( \|D\|_F^2 \), lower than a given threshold \( \tau^* \). Thus, the optimization problem is

\[
\maximize_{D \in \{0,1\}^{K \times n_{d,\text{tot}}}} \sum_{i=1}^{K} R_i(D, T = f(D, H)), \text{ s.t. } \|D\|_F^2 \leq \tau^*.
\]

(14)

This problem is of combinatorial nature and a greedy approach developed in [13] for the case of single antenna RXs can be adapted to the current setting with multiple antennas at the RXs. The structure of the algorithm is kept unchanged and it is only necessary to replace the ZF precoding scheme by the precoding function \( T = f(D, H) \) described at the end of Subsection III-A. The idea of the algorithm is simply to add in \( D \) the element leading to the largest increase in sum rate until the constraint on the total sharing is reached.

IV. SIMULATIONS

We simulate a multicell network with \( K \) TX/RX pairs in a Rayleigh fading channel without pathloss so that the elements of the channel matrix \( H \) are generated as \( CN(0,1) \). In Fig. 1, we consider the regular 4-users MIMO IC \((2,2,1)^4 \) and we also use the following values for the routing matrix:

\[
D_1 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}, \quad D_2 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}, \quad D_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}.
\]

In addition to the transmission using these three routing matrices, we also look at the two extreme cases corresponding to full sharing of the users data symbol with \( D = 1_{K \times K} \) (i.e., the BC) and no sharing with \( D = 1_K \) (i.e., the IC). To obtain the precoders in the virtual IC, we use the max-SINR algorithm from [17] described in Subsection II-B. We observe that the MG achieved without cooperation and with the routing matrix \( D_3 \) is zero while the other values for the routing matrix lead to a MG of one for each RX, i.e., IA is feasible, which coincides with the formula from Theorem 2.

In Fig. 2, we keep the same regular 4-users IC and we show the sum rate divided by the sum rate with full data
sharing as a function of the total number of user's data symbols which can be shared through the backhaul link. For each value of the number of user’s data symbols shared \( r^* \) we use the greedy approach from [13] which we have recalled briefly in Subsection III-C. We also consider three different values for the normalized SNR, and we observe that our precoding scheme achieves most of the performance of full cooperation with only few data symbols shared. Particularly, it converges to an IA solution when IA is feasible, as the observation of the curve at very high SNR shows.

V. CONCLUSION

We have discussed the precoding in a multiple antenna static IC where the TXs can only share a limited number of user's data symbols between them. Only a subset of the TXs contributes to the transmission of a given symbol, thus leading to an intermediate setting between an IC and a network MIMO Channel. We have shown that this setting can be transformed into a virtual IC without cooperation so that it is possible to apply the IA algorithms from the literature. The ZF precoding in a BC or the IA algorithms in a conventional IC can then be seen as particular cases of our algorithm. Moreover, our precoding approach is seen to achieve good performance with few data symbols shared when the routing matrix can be optimized. Studying the feasibility of IA under partial data sharing represents an interesting direction of research, and we believe furthermore that the precoding with users data sharing shared only to a subset of TXs is practically relevant for future networks and should be the focus of further analysis.

REFERENCES