EM-Based joint symbol and blur estimation for 2D barcode

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***Abstract*— Decoding a severely blurred $2D$ barcode can be considered as a special case of blind image restoration issue. In this paper, we propose an appropriate system model which includes the original image with the particularities related to barcode, the blur and the observed image. We develop an unsupervised algorithm that jointly estimates the blur and detects the symbols using the maximum likelihood (ML) criterion. Besides, we show that when taking into account the spatial properties of the barcode, the prohibitive complexity of the ML algorithm can be reduced without degrading its performance. Simulation results show that the algorithm performs accurate estimation of the blur and achieves good performance for symbol detection which is close to that obtained with supervised algorithm.**

I. Introduction

Introduced in 1952 [1], barcode is an effective means for automatic data identification and is used in many fields such as transport, mail, trade,... [**?**]. The success of this technology is due to the simplicity of data representation and its low cost. Originally, barcodes represent data through the width and the spacing of parallel lines, and are referred to as $1D$ barcodes. Binary data can also be depicted as geometric patterns such as dots or squares within images known as $2D$ barcodes. The latters are progressively flooding the markets of electronic tickets and the traceability of transactions. In comparison with their predecessors, $2D$ barcodes have a bigger capacity of information storage and a more robust code error correction. However, when the barcode acquisition is imperfect, both the bad focalisation and the camera movement give rise to blur which severely degrades the performance of the reader cf. fig 1. Therefore an algorithm for blur identification and image restoration is necessary to recover the initial transmitted information.

Many algorithms [2]–[5] deal with blind restoration of image and depend both on estimation methods and on hypotheses aggregated in a stochastic model of the pair (hidden image, observed image). Some of them deal with interpixel interference coming from blur and are referred as blind deconvolution algorithms [4], [5]. Unlike barcode case, most are concerned by a continuous hidden process modeling degraded photos for instance. On the other hand, amongst works dedicated to barcodes, let's mention the work proposed by Pavlidis et al. [**?**] who have published a study related to information theory fundamentals outlining the process for barcode design using error detection and correction techniques. Tsi et al. [**?**] have developed a method that allows the calculation of the working range in case of a CCD-based reader. Houni et al [**?**] have studied the performance in the framework of information theory. Furthermore, for decoding $2D$ barcode works mainly focus on preprocessing techniques and consist of simple threshold detectors which can't overcome significant blurs [6]. However, in [7] an iterative structure based on a factor graph representation has been developed, this algorithm is suitable for strongly blurred acquisition but has the drawback of assuming the filter impulse response of the blur known. In the framework of $1D$ barcode technologies, we have proposed in [8] a joint blur and symbol estimator that takes advantage of the cyclostationnarity of the hidden process to estimate.

The extension of this algorithm to $2D$ barcode is the main goal of this paper. Its significant bottlenecks are the modeling of the $2D$ data and the induced complexity of the estimation algorithms. Then, the main contributions of the proposed work consist in: i) Modeling the observed image as a Hidden Markov Model and covering the image with a one dimensional path by preserving the Markovien property . ii) Taking into account the spatial properties of the 2D barcode, which induces a cyclo-stationary hidden Markov process iii) This cyclo-stationarity reduces significantly the cardinality of the states set, with trivial state transition probabilities into one period. iv) In consequence, a near optimal reception with low complexity based on the EM algorithm is proposed. This algorithm performs the maximum likelihood estimates of the blur and the noise power and the maximum marginal a posteriori when decoding the data symbols.

The rest of the paper is organized as follows: the model of the blurred $2D$ barcode is presented in section 2. Section 3 describes the Expectation-Maximization algorithm used for the blur identification. The symbol detection criterion is detailed in section 4. In section 5, performances of the proposed algorithm are illustrated via simulations and compared to supervised algorithms (knowing the blur). Finally a conclusion is drawn in section 6.

Notation: : $\lfloor x \rfloor$ returns the nearest integer less than or equal to x. $\lceil x \rceil$ returns the nearest integer greater than or equal to x. $x_{i:j} = (x_i, x_{i+1}, ..., x_j)^T$. x^T is the classical matrix transpose. $x_{i,:}$ designs the vector $(x_{i,0}, ..., x_{i,L-1})^T$. The cardinal of a set Ω is denoted $|\Omega|$. $0_{L,L'}$ is the zero matrix of L rows and L'

Fig. 1. (a):Original image (b): Blurred image

columns. I_L is the L- identity matrix. i mod j is the reminder of the division of i by j.

II. System Model

Let $x'_{0,0},...,x'_{0,N'-1},x'_{1,0},...,x'_{M'-1,N'-1}$ be a set of $M' \times N'$ bits to transmit. Each bit is represented by a dot of size $r_x \times r_y$ in a $2D$ barcode. So each pixel of the barcode is given by $x_{m,n} = x'_{\lfloor \frac{m}{r_x},\frac{n}{r_y} \rfloor}$, $(m,n) \in \{0,...,M-1\} \times \{0,...,N-1\}$, with $M = r_x M'$ and $N = r_y N'$. During acquisition, the barcode is degraded by blur coming from both the optical block and the camera movement which is modeled by a 2D finite impulse response filter of size $L \times L$. Besides, an additive white Gaussian noise takes into account the residual stochastic imperfections of the model.

In order to simplify the presentation, we assume here that the image as well as the dot are square matrices, that is $r_x = r_y = r$ and $M = N$. Using matrix notation, the observed image $Y = [y_{mn}]$ obeys to the following equation:

$$y_{m,n} = H^T X_{m,n} + w_{m,n} \qquad \forall (m,n) \in \{0,...,M-1\}^2 \quad (1)$$

where $H = \left(h^T_{0,:}, h^T_{1,:}, ..., h^T_{L-1,:}\right)^T$ is the impulse response vector made of rows of the $2D$ impulse response filter h. Because the blur is the result of the bad focalisation and the camera motion, it belongs to the class of symmetric and concave blur [3], [9], [10], so the problem of phase ambiguity is avoided in statistical estimation.

$w_{m,n}$ is a white Gaussian noise with variance σ^2.

$$X_{m,n} = \left(x^T_{m,n:n-L+1}, x^T_{m-1,n:n-L+1}, ..., x^T_{m-L+1,n:n-L+1}\right)^T$$

is the interfering pixel vector at the site (m,n).

$X_{m,n}$ verifies the two following equations:

$$X_{m,n} = BX_{m-1,n} + V_{m,n} \quad (2)$$

$$X_{m,n} = B'X_{m,n-1} + V'_{m,n} \quad (3)$$

with, $B = \begin{pmatrix} & 0_{L,L^2} \\ I_{L^2-L} & 0_{L^2-L,L} \end{pmatrix}$,

$V_{m,n} = \left(x^T_{m,n:n-L+1}, 0_{1,L(L-1)}\right)^T$

B' is a block diagonal matrix formed by L elementary matrices $\begin{pmatrix} & 0_{1,L} \\ I_{L-1} & 0_{L-1,1} \end{pmatrix}$,

$$V'_{m,n} = (x_{m,n}, 0_{1,L-1}, ..., x_{m-L+1,n}, 0_{1,L-1})^T$$

Equations (2) and (3) govern respectively vertical and horizontal path in the displayed $2D$ barcode. It is clear from equations (2) and (3) that $(X_s)_{s \in S}$, with $s = (m,n)$ and S the designed path, is a Markov chain whose transition matrix depends on the direction of the move from a pixel to another (cf.fig 2). Any decoder based on these equations can be constructed and will keep the Markovian property. A path in the image could simply be a column by column path, row by row path and also more general path such as the Hilbert Peano one. On the other hand, when considering the barcode spatial properties (the dot resolution $r > 1$), $(X_s)_{s \in S}$ is cyclo-stationary Markov process for which the states and the transitions probabilities depend on the direction of the path and the location relative to a dot (cf.fig 2).

$$P_\theta(Y,X) = P(X_0) \prod_{s=1}^{|S|-1} P(X_s|X_{s-1}) \prod_{s=0}^{|S|-1} f_{Y_s|X_s,\theta}(y_s) \quad (4)$$

where $Y_s|X_s \sim N(H^T X_s, \sigma)$, $\theta = \left(\sigma^2, H\right)^T$ and $P(X_0)$ is the initial probability.

States can be represented in a trellis whose intricacy conditions computation complexity. We evaluate complexity as the number of multiplicative operations needed to compute forward or backward probabilities for all states in a given position (m,n) in the the trellis. In our case this number is simply equal to number of states in the trellis. For standard application, i.e without considering the dot resolution $(r = 1)$, detection is very hard to perform, almost impossible, due to the prohibitive complexity of the trellis which contains 2^{L^2} states. However, when taking into account the resolution $r > 1$, the number of states is not the same for each position (m,n) and depends on the position in the dots. The complexity is then evaluated as the mean number of states among one dot. In general, if $L \geqslant r_x$ and $L \geqslant r_y$, the complexity is given by

$$C = \frac{1}{r_x r_y} \sum_{j=1}^{r_y} \sum_{i=1}^{r_x} 2^{\left(1+\lceil \frac{L-i}{r_x} \rceil\right)\left(1+\lceil \frac{L-j}{r_y} \rceil\right)} \quad (5)$$

It is clear that the complexity is reduced, for images which have the same size it is a decreasing function of $\frac{r}{L}$ (cf. table I). Therefore the number of paths in the trellis decreases and the detection of the optimal sequence is more robust to noise [8].

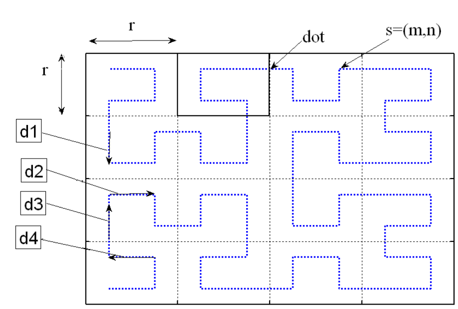

Fig. 2. A path in a displayed 2D barcode

	$L=3$ $r=3$	$L=4$ $r=3$	$L=5$ $r=3$
$r=1$	2^9	2^{16}	2^{25}
$r>1$	$\frac{82}{9}\simeq 9$	16	$\frac{832}{9}\simeq 92$

TABLE I
COMPLEXITY OF THE TRELLIS

III. CHANNEL ESTIMATION

Channel estimation is performed using maximum likelihood criterion which is given by :

$$\hat{\theta} = \arg\max_{\theta} f_{Y,\theta}(y) \quad (6)$$

where $f_{Y,\theta}$ is the likelihood of the observation. The evaluation of the likelihood is not tractable, therefore the maximum is approximated using the Expectation-Maximisation algorithm (EM) [**?**]. At the i^{th} iteration, the two following steps are performed:
E step

$$Q(\theta,\theta^{(i-1)}) = E_{X|Y,\theta^{(i-1)}}[log(P_\theta(X,Y))]$$

M step

$$\theta^{(i)} = \arg\max_\theta Q(\theta,\theta^{(i-1)})$$

For $1D$ detection and a resolution $r=1$, X_s is a stationary Markov process and the estimation algorithm of (H,σ^2) has been proposed by Kaleh et al [11]. The algorithm has been extended to a larger resolution in [8]. For $2D$ detection and when considering the dot resolution $r>1$, the hidden Markov process is a cyclo-stationary process whose states belong to the set $\xi = \{\xi^0,...,\xi^{P-1}\}$. At the i^{th} iteration the channel coefficients and the noise variance estimates are respectively given by these two equations:

$$\left\{\sum_{s=0}^{|S|-1}\sum_{p=0}^{P-1} P_{\theta^{(i-1)}}\left(X_s=\xi^p|Y\right)\xi^{p^T}\xi^p\right\}H^{(i)} = \sum_{s=0}^{|S|-1}\sum_{p=0}^{P-1} P_{\theta^{(i-1)}}\left(X_s=\xi^p|Y\right)y_s\xi^{p^T} \quad (7)$$

$$\sigma^{2^{(i)}} = \frac{1}{|S|-1}\sum_{s=0}^{|S|-1}\sum_{p=0}^{P-1} P_{\theta^{(i-1)}}\left(X_s=\xi^p|Y\right)\times |y_s - H^{(i)}\xi^p|^2 \quad (8)$$

The marginal a posteriori probability $P_{\theta^{(i-1)}}\left(X_s=\xi^p|Y\right)$ is calculated using the Forward Backward algorithm [12].

IV. SYMBOL DETECTION

Symbol detection is based on the maximisation of the marginal a posteriori probability (MAP). For each symbol x'_s, it consists in choosing the value $\{0,1\}$ which maximises $P_\theta(x'_s|Y)$.
We denote $\{X^s\}$ the sequence which includes all vectors of interpixel interference related to the symbol x'_s.
Let $\{\vartheta\}$ be a realisation of this sequence, and Ω is the set of all the realisations of $\{X^s\}$.

$$P_\theta(x'_s=0|Y) = \sum_{\{\vartheta\}\in\Omega} P_\theta(x'_s=0,\{X^s\}=\{\vartheta\}|Y)$$
$$= \sum_{\{\vartheta\}\in\Omega} P_\theta(\{X^s\}=\{\vartheta\}|Y)P_\theta(x'_s=0|Y,\{X^s\}=\{\vartheta\})$$
$$= \sum_{\{\vartheta\}\in\Omega/x'_s=0} P_\theta(\{X^s\}=\{\vartheta\}|Y)$$

The sequence $\{X^s\} = \{X_0^s,...,X^s_{|X^s|-1}\}$, and its realisation $\{\vartheta\} = \{\vartheta_0,...,\vartheta_{|X^s|-1}\}$. Since X is Markovian, then $P_\theta(\{X^s\}=\{\vartheta\}|Y)$ is given by:

$$P_\theta(\{X^s\}=\{\vartheta\}|Y) = P_\theta\left(X_0^s=\vartheta_0|Y\right)\times \prod_{g=1}^{|X^s|-1} P(X^s_{g+1}=\vartheta_{g+1}|X^s_g=\vartheta_g,Y) \quad (9)$$

where, $P(X^s_{g+1}=\vartheta_{g+1}|X^s_g=\vartheta_g,Y)$ is obtained in the channel estimation step.

CHANNEL AND SYMBOL DETECTION ALGORITHM

From previous sections, we deduce the algorithm of joint channel estimation and symbol detection (CESD):

Algorithm 1 Channel Estimation and Symbol Detection algorithm

1: Calculate the number of states and the transition probability matrices.
2: Initialisation, $H^{(0)},(\sigma^2)^{(0)}$.
We denote by ε a predetermined threshold
3: **while** $|\dfrac{Q(\theta,\theta^{(i)})-Q(\theta,\theta^{(i-1)})}{Q(\theta,\theta^{(i-1)})}| > \varepsilon$ **do**
4: Calculate the marginal posterior probability using the Forward Backward algorithm [12].
5: Estimation of H and σ^2 from (7) and (8).
6: **end while**
7: Detection of symbol as described in section IV.

V. SIMULATION RESULTS

The performances of the proposed algorithm are illustrated through numerical simulations. In the following, a set of binary symbols $\{0,1\}$ have been considered and the results have been obtained with $N_s=1000$ Monte Carlo runs. For each run, we generate a new image of size $M\times M$, $M=35$ with a barcode resolution $r=2$ and a new channel of size $L\times L$, $L=2$. The threshold of the EM algorithm is fixed to $\varepsilon=10^{-4}$.
Here we propose to study the impact on symbol detection of the model described in section II, the resolution of the barcode, the estimator bias and the choice of the path in the image. Therefore we consider:

- A supervised algorithm for which θ is perfectly known, and symbol detection is made using the MAP criterion. It is called Supervised MAP ans it provides benchmark performance for unsupervised algorithms.

- Two algorithms constructed without considering the cyclo-stationary of the hidden Markov process, r is supposed to be equal to 1. Both of them are based on the MAP for the symbol detection. The first algorithm a priori knows the blur and the noise power, it is called Supervised MAP-1. Named EM-MAP-1 the second algorithm includes a channel estimator based on the EM.
- An algorithm taking into account the dot resolution, however a row by row path is considered. Each row is treated separately: symbol detection is performed as described in section IV with the difference that the marginal a posteriori probability is calculated given observation $y_{m,:} = (y_{m,0}, ..., y_{m,M-1})$ that is $P_\theta(X_g = \xi_g|Y)$ is replaced with $P_\theta(X_g = \xi_g|y_{m,:})$. Besides, the parameters are supposed to be known, then the algorithm is called Supervised MAP-RR.
- The CESD algorithm described in previous sections, it jointly estimates the channel and the symbols, and takes into account the spatial properties of the barcode. Besides, it used a path that recovers the whole image.

In simulations, E_b/N_0 is the energy per bit to noise power spectral density ratio, with the energy per bit defined as:

$$E_b = r^2 H^T E\left[X_k X_k^T\right] H \tag{10}$$

Channel estimation

The Normalised Root Mean Squared Error (NRMSE), which assesses the quality of the estimation of the channel, is defined as:

$$NRMSE = \sqrt{\frac{(H-\hat{H})^T(H-\hat{H})}{H^T H}}$$

where H and $\hat{H}$ are respectively the true channel coefficients vector and the estimated one. Fig.3 shows the performance of

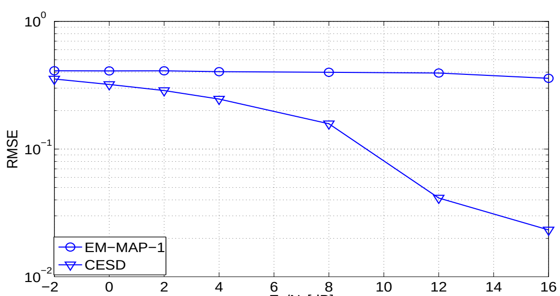

Fig. 3. NRMSE vs E_b/N_0

the estimation of the channel impulse response in term of the NRMSE, with and without considering the dot resolution. The difference between curves is explained by the fact that when taking into account the original image structure, the quality of the estimation of the marginal a posteriori probability $P_{\theta^{(i-1)}}(X_s = \xi^p|Y)$ is improved, which directly improves the estimation of the channel H (cf. equation 7).

Symbol detection

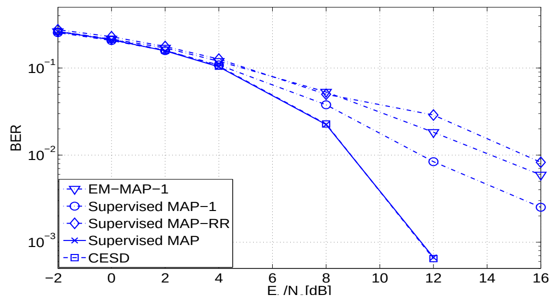

Fig. 4. BER performance

Fig.4 shows the Bit Error Rate (BER) performance versus E_b/N_0. Supervised MAP and Supervised MAP-1 refer respectively to the algorithms with and without considering the dot resolution. As illustrated in Fig.4, when taking into account that the dot resolution r is larger than 1 the BER decreases. Indeed, in this case the number of paths in the trellis is reduced so that the detection of the optimal sequence becomes more robust to noise.

Moreover, the gain of performance obtained with the CESD when comparing to the supervised MAP-RR is explained by the fact that the latter is constructed without considering the dependence between rows i.e each row is treated separately. Indeed, decision on symbol $x'_{m,i}$ is based on the posterior probability using information of the row $y_{m,:}$, that is $P(x'_{m,i}|y_{m,:})$. In contrast with the CESD algorithm, symbol detection is based on a posteriori probability exploiting information given by the whole image $P(x'_{m,i}|Y)$. This clearly illustrates the relevancy of considering a path that covers the whole image and takes into account the dependence between rows and columns.

Finally, it is clear that the BER performance for the proposed algorithm is close to that obtained with a non blind decoder, (CESD and Supervised MAP).

VI. Conclusion

In this paper, we have proposed a joint channel and symbol blind estimator for severely degraded $2D$ barcode. The algorithm takes advantage of both the spatial properties of the $2D$ barcode and the Markovian property of the interfering pixel vectors. We have shown that these properties enable the calculation of the maximum likelihood of the blur, the noise power and the symbols. They also reduce the complexity of the algorithm.

Simulations assess the efficiency of the algorithm and, in particular, they point out the gain obtained by taking into account the spatial properties of the $2D$ barcode. Besides, results attest the impact of taking into account the dependency between rows and columns in the image. Finally, performances of the proposed unsupervised algorithm are close to those obtained by a supervised one, which illustrates the precision of parameter estimation.

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