Robust dynamic output feedback fuzzy Lyapunov stabilization of Takagi-Sugeno systems - A descriptor redundancy approach

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ROBUST DYNAMIC OUTPUT FEEDBACK FUZZY LYAPUNOV STABILIZATION OF TAKAGI-SUGENO SYSTEMS – A DESCRIPTOR REDUNDANCY APPROACH

Kevin Guelton*, Tahar Bouarar, Noureddine Manamanni

**CReSTIC, EA3804, URCA, Université de Reims Champagne-Ardenne*

Moulin de la House BP1039, 51687 Reims Cedex 2, France

Tel : (+33) 3 26 91 83 86 fax : (+33) 3 26 91 31 06

**Corresponding author: kevin.guelton@univ-reims.fr*

Abstract: This paper deals with Takagi-Sugeno (T-S) systems stabilization based on dynamic output feedback compensators (DOFC). In fact, only few results consider DOFC for T-S systems and most of them propose quadratic Lyapunov stability function to provide stability conditions, which may lead to conservatism. In this work, to overcome this drawback and enhance the closed-loop transient response, we provide for T-S uncertain closed-loop systems non quadratic stability conditions. Based on a fuzzy Lyapunov candidate function and the descriptor redundancy property, these stability conditions are written in terms of linear matrix inequalities (LMI). Afterward, the DOFC is designed with H_∞ criterion in order to minimize the influence of the external disturbances. Finally, a few academic examples illustrate the efficiency of the proposed approach.

Keywords: Fuzzy Control, Takagi-Sugeno Systems, Dynamic Output Feedback, Non Quadratic Stabilization, Fuzzy Lyapunov Function, Descriptor Redundancy.

INTRODUCTION

In the past few decades, with the growing complexity of dynamic systems, nonlinear control theory has attracted a great interest. Among nonlinear control theory, the Takagi-Sugeno (T-S) fuzzy model-based approach has nowadays become popular since it constitutes universal approximators of nonlinear systems. Indeed, Takagi and Sugeno have proposed a class of fuzzy models to describe nonlinear systems as a collection of linear time invariant models blended together with nonlinear functions [32]. Based on this modeling approach, stability conditions have been derived from the direct Lyapunov methodology [33]. Then T-S control laws have been proposed to stabilize such nonlinear systems. The most commonly used are based on the so-called Parallel Distributed Compensation (PDC) scheme and remain to associate inferred state feedback to each local subsystem [42].

The stability of T-S models and the design of T-S control laws are, in most of the case, investigated via the direct Lyapunov approach leading to a set of linear matrix inequalities (LMI), in the better case, or bilinear matrix inequalities (BMI) [5]. These matrix inequalities can be solved, when a solution exists, by classical convex optimization algorithms [10]. Most of the proposed approaches consider a quadratic Lyapunov function where common matrices to each subsystem have to be found (see e.g. [30][36] and references therein). The interest of these approaches is that the obtained solutions are not depending on the nonlinearities (membership functions) allowing to extend the involved linear control theory to nonlinear control design. Nevertheless, the obtained conditions lead to conservatism. Thus, numerous works have been proposed to relax (reduce the conservatism) such conditions. Some of them propose the use of matrix transformations on the sum structure of the closed-loop T-S system

[41]. Some others introduce new decision variables in order to provide much more degrees of freedom to the LMI problem [19][21]. One other way is to reconsider stability conditions on the basis of other candidate Lyapunov functions. Thus, stability and stabilization have been considered via piecewise quadratic Lyapunov functions (PWLF) [18], non quadratic or fuzzy Lyapunov functions (FLF) [8][13][29][37]. More recently, it has been shown that using a descriptor redundancy approach leads to less computational cost of LMI solutions [11][38]. Moreover, descriptor redundancy may also be interesting since using a descriptor formulation may lead to less conservatism [9][12].

Complementary to the works related to the relaxation of LMI conditions and with the growing interest on engineering applications of T-S models based stabilization, some studies have been done regarding to robust and/or output stabilization of T-S fuzzy models. Indeed, a lot of works involving various specifications are now available for state feedback: Robustness with bounded uncertainties [7][27][34], time delay models with or without uncertainties [6][43], performance specification using a H_2 or a H_∞ criterion [21][26], using the circle criterion [3] or the Popov criterion [4], adaptive control [39], decentralized control [40], etc.

Output stabilization can be considered through three approaches. The first one is based on the introduction of a state observer [22][23][24][35][44]. This approach is interesting when the state is not entirely available from measurements and a separation principle is only available when the premises variables are measurable. However, stability conditions have been proposed in the case of non measurable premises variables [14][25][45]. The second approach for output stabilization is called “static output feedback”. This one is interesting to reduce real time computational cost when implementing practical applications since it doesn’t need any

ODE solving [31]. Thus, static output feedback controller design for fuzzy T-S models has been recently proposed [16][17] but the results are provided in terms of BMI. Finally, the third way to address the problem of output feedback stabilization is to use a “dynamic output feedback compensator” (DOFC) [31]. To improve the closed-loop dynamics control law’s performances, robust control based on DOFC controllers has been extensively studied in various kinds of linear systems (Linear Time Invariant (LTI), Linear Parameter Varying (LPV), Linear Time Varying (LTV)…), see e.g. [1][48]. Indeed, due to its dynamical behavior, this kind of controller is a good way to improve the closed-loop transient response. These techniques are often based on the Linear Fractional Transformation (LFT) paradigm [28]. Nevertheless, few tractable results have been proposed in the case of T-S fuzzy control. In fact, using the Redheffer product to write the closed-loop dynamic of a DOFC T-S fuzzy control plant leads to high conservatism since the obtained LMI or BMI stability conditions involve numerous crossing terms between system’s and controller’s matrices and lead to a strong membership interconnection structure [2][20][46]. Moreover, one can point out that, in the previous literature, LMI DOFC based design approaches are only suitable for a restrictive class of nonlinear systems. The latter consider models where the output equation is supposed to be linear (with a common output matrix) and without direct transfer between inputs and outputs. Note that the presence of crossing terms ruins tentative to derive non-quadratic Lyapunov LMI stability condition when using the Redheffer product. These lack of results regarding to DOFC design, understood as the deficiency of LMI formulation in the general case, lead to the aim of this paper.

Recently, a preliminary study has introduced new conditions for the stabilization of T-S fuzzy systems via a DOFC using the descriptor redundancy [11]. In the present paper, a generalization of this preliminary work is proposed with new LMI conditions for a robust DOFC design for uncertain and disturbed T-S fuzzy models. It will be shown that, using a descriptor representation of the closed-loop systems allows avoiding numerous crossing terms in the LMI formulation since the Redheffer product is no longer required. Thus, unlike to the previous works on T-S DOFC based stabilization, it is now possible to provide less conservative LMI conditions by the use of a FLF for a large class of T-S systems with parametric uncertainties, subject to external disturbances, including both a non linear behaviour within the output equation and a direct transfer between inputs and outputs.

The paper is organized as follows. The next section provides useful notations and lemmas. Rewriting the closed loop dynamics in the descriptor form, the problem statement of the proposed output feedback controller design is formulated in section 3. Afterward, in section 4, the design of DOFC controllers for uncertain T-S systems without external disturbance is provided through a non quadratic FLF approach. Then, these conditions are extended with a well-known H_∞ criterion in order to design a DOFC controller minimizing the influence of the external disturbances on the state. Finally, in the last section, designed examples are given to illustrate the efficiency of the proposed approaches.

1. USEFUL NOTATIONS AND LEMMAS

Let us consider the scalar functions $h_i(z)$, the matrices Y_i and T_{ij} for $i \in \{1,\dots,r\}$ and $j \in \{1,\dots,l\}$ with appropriate dimensions, we will denote $Y_h = \sum_{i=1}^{r} h_i(z) Y_i$,

$T_{hv} = \sum_{k=1}^{l} \sum_{i=1}^{r} v_k(z) h_i(z) T_{ik}$. Moreover, in some cases, a subscript $\bar{h}$ will be used to indicate submatrices that are depending on the same summation structure, for instance $M_h = X_{\bar{h}} Y_{\bar{h}} = \sum_{i=1}^{r} h_i(z(t)) X_i Y_i$. Note that h and $\bar{h}$ will be identically used as subscript or superscript in order to lighten the notations. Also for more simplicity, we will use the subscript $\underline{h}$ to indicate a matrix depending on inverse summation structures as $Q_{h\underline{h}} = L_h (M_h)^{-1}$. Finally, as usual, in a matrix, $(*)$ indicates a symmetrical transpose quantity.

In the sequel, when there is no ambiguity, the time t in a time varying variable will be omitted for space convenience.

Lemma 1 [47]:

For real matrices X, Y with appropriate dimensions and a positive scalar ε, the following inequalities hold:

$$X^T Y + Y^T X \leq \varepsilon X^T X + \varepsilon^{-1} Y^T Y \quad (1)$$

Lemma 2 [41]:

Consider the proposition "*For all combinations of* $i, j = 1, 2, \ldots, r$ *we have* $\Gamma_{ij} < 0$".

This proposition is equivalent to: "*For all combinations of* $i, j = 1, 2, \ldots, r$*, we have* $\Gamma_{ii} < 0$ *and for* $1 \leq i \neq j \leq r$*, we have* $\frac{1}{r-1}\Gamma_{ii} + \frac{1}{2}(\Gamma_{ij} + \Gamma_{ji}) < 0$".

2. PROBLEM STATEMENT

Let us consider the class of uncertain and disturbed T-S fuzzy systems described by:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} h_i(z(t))\left[(A_i + \Delta A_i(t))x(t) + (B_i + \Delta B_i(t))u(t) + F_i\varphi(t)\right] \\ y(t) = \sum_{i=1}^{r} h_i(z(t))\left[(C_i + \Delta C_i(t))x(t) + (D_i + \Delta D_i(t))u(t) + G_i\varphi(t)\right] \end{cases} \tag{2}$$

where r represents the number of fuzzy rules. $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, $y(t) \in \mathbb{R}^q$ and $\varphi(t) \in \mathbb{R}^{d \leq n}$ represent respectively the state, the input, the output and the external disturbances vectors. $h_i(z(t))$ are positive membership functions satisfying the convex sum proprieties $0 \leq h_i(z(t)) \leq 1$ and $\sum_{i=1}^{r} h_i(z(t)) = 1$. $A_i \in \mathbb{R}^{n\times n}$, $B_i \in \mathbb{R}^{n\times m}$, $C_i \in \mathbb{R}^{q\times n}$, $D_i \in \mathbb{R}^{q\times m}$, $F_i \in \mathbb{R}^{d\times n}$, $G_i \in \mathbb{R}^{d\times q}$ are real matrices. $\Delta A_i(t) \in \mathbb{R}^{n\times n}$, $\Delta B_i(t) \in \mathbb{R}^{n\times m}$, $\Delta C_i(t) \in \mathbb{R}^{q\times n}$ and $\Delta D_i(t) \in \mathbb{R}^{q\times m}$ are Lesbegue measurable uncertainties defined as [47]: $\Delta A_i(t) = H_a^i f_a(t) \mathrm{N}_a^i$, $\Delta B_i(t) = H_b^i f_b(t) \mathrm{N}_b^i$, $\Delta C_i(t) = H_c^i f_c(t) \mathrm{N}_c^i$, $\Delta D_i(t) = H_d^i f_d(t) \mathrm{N}_d^i$. In that case, for the subscript $s = a, b, c$ or d one has H_s^i, N_s^i constant matrices with appropriate dimensions and $f_s(t)$ uncertain matrices bounded such as: $f_s^T(t) f_s(t) \leq I$.

Let us consider the following non PDC DOFC:

$$\begin{cases} \dot{x}^*(t) = \left(\sum_{i=1}^{r} h_i(z(t))A_i^*\right)\left(\sum_{i=1}^{r} h_i(z(t))W_6^i\right)^{-1} x^*(t) + \left(\sum_{i=1}^{r} h_i(z(t))B_i^*\right)\left(\sum_{i=1}^{r} h_i(z(t))W_{11}^i\right)^{-1} y(t) \\ u(t) = \left(\sum_{i=1}^{r} h_i(z(t))C_i^*\right)\left(\sum_{i=1}^{r} h_i(z(t))W_6^i\right)^{-1} x^*(t) + \left(\sum_{i=1}^{r} h_i(z(t))D_i^*\right)\left(\sum_{i=1}^{r} h_i(z(t))W_{11}^i\right)^{-1} y(t) \end{cases}$$

(3)

where $x^*(t) \in \mathbb{R}^n$ is the controller state vector. $A_i^* \in \mathbb{R}^{n\times n}$, $B_i^* \in \mathbb{R}^{n\times q}$, $C_i^* \in \mathbb{R}^{m\times n}$ and $D_i^* \in \mathbb{R}^{m\times q}$ are real matrices to be synthesized as well as $W_6^i \in \mathbb{R}^{n\times n}$ and $W_{11}^i \in \mathbb{R}^{q\times q}$ where $\sum_{i=1}^{r} h_i(z(t))W_6^i$ and $\sum_{i=1}^{r} h_i(z(t))W_{11}^i$ are nonlinear Lyapunov dependent non singular matrices (see remark 3, section 4).

In [38], LMI based design for state feedback controller using the descriptor redundancy has been proposed to reduce computational cost. To take advantage of a descriptor redundancy formulation, (2) and (3) can be easily rewritten with the above defined notations respectively as:

$$\begin{cases} \dot{x}(t) = (A_h + \Delta A_h(t))x(t) + (B_h + \Delta B_h(t))u(t) + F_h\varphi(t) \\ 0\,\dot{y}(t) = -y(t) + (C_h + \Delta C_h(t))x(t) + (D_h + \Delta D_h(t))u(t) + G_h\varphi(t) \end{cases} \quad (4)$$

and

$$\begin{cases} \dot{x}^*(t) = A_h^*\left(W_6^h\right)^{-1} x^*(t) + B_h^*\left(W_{11}^h\right)^{-1} y(t) \\ 0\dot{u}(t) = -u(t) + C_h^*\left(W_6^h\right)^{-1} x^*(t) + D_h^*\left(W_{11}^h\right)^{-1} y(t) \end{cases} \quad (5)$$

Note that, here, the descriptor redundancy consist on introducing virtual dynamics in the outputs equations of both (4) and (5). Then, a descriptor formulation can be obtained considering the extended state vector $\tilde{x}(t)=\begin{bmatrix} x^T(t) & x^{*T}(t) & y^T(t) & u^T(t)\end{bmatrix}^T$ and the closed loop dynamics can be expressed as:

$$\tilde{E}\dot{\tilde{x}}(t)=\left(\tilde{A}_{h\underline{h}}+\Delta\tilde{A}_h\right)\tilde{x}(t)+\tilde{F}_h\varphi(t) \tag{6}$$

with $\tilde{E}=\begin{bmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, $\tilde{A}_{h\underline{h}}=\begin{bmatrix} A_h & 0 & 0 & B_h \\ 0 & A_h^*\left(W_6^h\right)^{-1} & B_h^*\left(W_{11}^h\right)^{-1} & 0 \\ C_h & 0 & -I & D_h \\ 0 & C_h^*\left(W_6^h\right)^{-1} & D_h^*\left(W_{11}^h\right)^{-1} & -I \end{bmatrix}$,

$\Delta\tilde{A}_h=\begin{bmatrix} \Delta A_h(t) & 0 & 0 & \Delta B_h(t) \\ 0 & 0 & 0 & 0 \\ \Delta C_h(t) & 0 & 0 & \Delta D_h(t) \\ 0 & 0 & 0 & 0 \end{bmatrix}$ and $\tilde{F}_h=\begin{bmatrix} F_h \\ 0 \\ G_h \\ 0 \end{bmatrix}$.

Therefore, (2) is stabilized via the control law (3) if (6) is stable. Thus, the goal is now to provide LMI stability conditions allowing to find the matrices A_h^*, B_h^*, C_h^*, D_h^*, W_6^h and W_{11}^h ensuring the stability of (6).

Remark 1: Unlike previous studies using the Redeffher products [2] [20][46], rewriting the closed-loop system (6) by the use of descriptor redundancy allows to avoid appearance of crossing terms between the state space matrices and the controller's ones. Therefore, the benefit of this descriptor formulation will be emphasized in the following section since it makes easier the LMI formulation of non quadratic stability conditions.

3. FUZZY LYAPUNOV LMI BASED DESIGN FOR DOFC WITHOUT EXTERNAL DISTURBANCES

First, let us focus on the non quadratic stabilization of uncertain T-S systems (2) but without external disturbances ($\varphi(t)=0$). The main result is summarized in the following theorem.

Theorem 1: *The T-S fuzzy model* (2) *(with* $\varphi(t)=0$*) is globally asymptotically stable via the non PDC dynamic output feedback compensator* (3) *if there exist, for* $i, j = 1,\ldots,r$, *the matrices* $W_1^i = W_1^{iT} > 0$, $W_6^i = W_6^{iT} > 0$, W_{11}^i, W_{13}^i, W_{14}^i, W_{15}^i, W_{16}^i, A_i^*, B_i^*, C_i^* *and* D_i^*, *the scalars* ε_{1a}^{ij}, ε_{6a}^{ij}, ε_{13b}^{ij}, ε_{14b}^{ij}, ε_{15b}^{ij}, ε_{16b}^{ij}, ε_{1c}^{ij}, ε_{6c}^{ij}, ε_{13d}^{ij}, ε_{14d}^{ij}, ε_{15d}^{ij} *and* ε_{16d}^{ij} *such that the following LMI conditions are satisfied:*

- for $i = 1,2,\ldots,r$, $\Gamma_{ii} < 0$ (7)
- for $i = 1,2,\ldots,r$ and $1 \le i \ne j \le r$, $\frac{1}{r-1}\Gamma_{ii} + \frac{1}{2}\left(\Gamma_{ij} + \Gamma_{ji}\right) < 0$ (8)
- for $i = 1,2,\ldots,r-1$, $W_1^i - W_1^r \ge 0$ and $W_6^i - W_6^r \ge 0$ (9)

where $\Gamma_{ij} = \left[\begin{array}{cccc:c} \Gamma_{ij}^{(1,1)} & (*) & (*) & (*) & (*) \\ \Gamma_{ij}^{(2,1)} & \Gamma_{ij}^{(2,2)} & (*) & (*) & \\ \Gamma_{ij}^{(3,1)} & C_i W_6^{\,j} + D_i W_{14}^{\,j} + B_i^{*T} & \Gamma_{ij}^{(3,3)} & (*) & \\ C_i^* - W_{13}^{\,j} + W_{16}^{\,jT} B_i^T & C_i^* - W_{14}^{\,j} & D_i^* + W_{16}^{\,jT} D_i^T - W_{15}^{\,j} & -W_{16}^{\,j} - W_{16}^{\,jT} & \\ \hdashline & \tilde{Z}_{ij} & & & \tilde{P}_{ij} \end{array}\right]$,

$$\Gamma_{ij}^{(1,1)} = A_i W_1^{\,j} + W_1^{\,j} A_i^T + B_i W_{13}^{\,j} + W_{13}^{\,jT} B_i^T - \sum_{k=1}^{r-1} \phi_k \left(W_1^{\,k} - W_1^{\,r} \right) + \left(\varepsilon_{1a}^{ij} + \varepsilon_{6a}^{ij} \right) H_a^{\,i} H_a^{\,iT} + \left(\varepsilon_{13b}^{ij} + \varepsilon_{14b}^{ij} + \varepsilon_{15b}^{ij} + \varepsilon_{16b}^{ij} \right) H_b^{\,i} H_b^{\,iT} + \varepsilon_{1c}^{ij} H_c^{\,i} H_c^{\,iT} + \varepsilon_{13d}^{ij} H_d^{\,i} H_d^{\,iT},$$

$$\Gamma_{ij}^{(2,1)} = A_i^* + W_6^{\,j} A_i^T + W_{14}^{\,jT} B_i^T - \sum_{k=1}^{r-1} \phi_k \left(W_6^{\,k} - W_6^{\,r} \right),$$

$$\Gamma_{ij}^{(2,2)} = A_i^* + A_i^{*T} - \sum_{k=1}^{r-1} \phi_k \left(W_6^{\,k} - W_6^{\,r} \right) + \varepsilon_{6c}^{ij} H_c^{\,i} H_c^{\,iT} + \varepsilon_{14d}^{ij} H_d^{\,i} H_d^{\,iT}, \qquad \Gamma_{ij}^{(3,1)} = C_i W_1^{\,j} + D_i W_{13}^{\,j} + W_{15}^{\,jT} B_i^T,$$

$$\Gamma_{ij}^{(3,3)} = D_i W_{15}^{\,j} + W_{15}^{\,jT} D_i^T - W_{11}^{\,j} - W_{11}^{\,jT} + \left(\varepsilon_{15d}^{ij} + \varepsilon_{16d}^{ij} \right) H_d^{\,i} H_d^{\,iT},$$

$$\tilde{Z}_{ij} = \begin{bmatrix} \mathrm{N}_a^{\,i} W_1^{\,j} & 0 & 0 & 0 \\ \mathrm{N}_b^{\,i} W_{13}^{\,j} & 0 & 0 & 0 \\ 0 & \mathrm{N}_a^{\,i} W_6^{\,j} & 0 & 0 \\ 0 & \mathrm{N}_b^{\,i} W_{14}^{\,j} & 0 & 0 \\ 0 & 0 & \mathrm{N}_d^{\,i} W_{15}^{\,j} & 0 \\ 0 & 0 & \mathrm{N}_b^{\,i} W_{15}^{\,j} & 0 \\ 0 & 0 & \mathrm{N}_c^{\,i} W_1^{\,j} & 0 \\ 0 & 0 & \mathrm{N}_d^{\,i} W_{13}^{\,j} & 0 \\ 0 & 0 & \mathrm{N}_c^{\,i} W_6^{\,j} & 0 \\ 0 & 0 & \mathrm{N}_d^{\,i} W_{14}^{\,j} & 0 \\ 0 & 0 & 0 & \mathrm{N}_b^{\,i} W_{16}^{\,j} \\ 0 & 0 & 0 & \mathrm{N}_d^{\,i} W_{16}^{\,j} \end{bmatrix},$$

$$\tilde{\mathrm{P}}_{ij} = -diag\left[\varepsilon_{1a}^{ij} I \quad \varepsilon_{13b}^{ij} I \quad \varepsilon_{6a}^{ij} I \quad \varepsilon_{14b}^{ij} I \quad \varepsilon_{15d}^{ij} I \quad \varepsilon_{15b}^{ij} I \quad \varepsilon_{1c}^{ij} I \quad \varepsilon_{13d}^{ij} I \quad \varepsilon_{6c}^{ij} I \quad \varepsilon_{14d}^{ij} I \quad \varepsilon_{16b}^{ij} I \quad \varepsilon_{16d}^{ij} I \right],$$

and where the scalars ϕ_k are defined as the lower bound of $\dot{h}_k\left(z(t)\right)$ for all $k = 1, 2, \ldots, r$.

Proof:

Let us consider the non quadratic candidate Lyapunov function given by:

$$v\left(x, x^*\right) = \tilde{x}^T(t)\,\tilde{E}\left(\tilde{W}_h\right)^{-1}\tilde{x}(t) \tag{10}$$

The closed-loop system (6) is stable if:

$$\dot{v}\left(x, x^*\right) = \dot{\tilde{x}}^T\tilde{E}\left(\tilde{W}_h\right)^{-1}\tilde{x} + \tilde{x}^T\tilde{E}\left(\tilde{W}_h\right)^{-1}\dot{\tilde{x}} + \tilde{x}^T\tilde{E}\left(\dot{\tilde{W}}_h\right)^{-1}\tilde{x} < 0 \tag{11}$$

Classically for descriptor systems, from (11) one needs:

$$\tilde{E}\left(\tilde{W}_h\right)^{-1} = \left(\tilde{W}_h\right)^{-T}\tilde{E} > 0 \tag{12}$$

Let us consider $\tilde{W}_h = \begin{bmatrix} W_1^h & W_2^h & W_3^h & W_4^h \\ W_5^h & W_6^h & W_7^h & W_8^h \\ W_9^h & W_{10}^h & W_{11}^h & W_{12}^h \\ W_{13}^h & W_{14}^h & W_{15}^h & W_{16}^h \end{bmatrix}$. Multiplying (12), left by $\tilde{W}_h^{\,T}$ and right by $\tilde{W}_h$, one has $\tilde{W}_h^{\,T}\tilde{E} = \tilde{E}\tilde{W}_h > 0$ which leads to $W_1^h = W_1^{hT} > 0$, $W_6^h = W_6^{hT} > 0$, $W_2^h = W_5^{hT}$, $W_3^h = W_4^h = W_7^h = W_8^h = 0$. Considering (6), (11) is obviously satisfied if:

$$\left(\tilde{A}_{h\underline{h}}^{\,T} + \Delta\tilde{A}_h^{\,T}\right)\left(\tilde{W}_h\right)^{-1} + \left(\tilde{W}_h\right)^{-T}\left(\tilde{A}_{h\underline{h}} + \Delta\tilde{A}_h\right) + \tilde{E}\left(\dot{\tilde{W}}_h\right)^{-1} < 0 \tag{13}$$

Multiplying left by $\tilde{W}_h^{\ T}$ and right by $\tilde{W}_h$ and since $\tilde{W}_h^{\ T}\tilde{E} = \tilde{E}\tilde{W}_h > 0$, (13) yields:

$$\tilde{W}_h^{\ T}\left(\tilde{A}_{h\underline{h}}^{\ T} + \Delta\tilde{A}_h^{\ T}\right) + \left(\tilde{A}_{h\underline{h}} + \Delta\tilde{A}_h\right)\tilde{W}_h + \tilde{E}\tilde{W}_h\left(\dot{\tilde{W}}_h\right)^{-1}\tilde{W}_h < 0 \tag{14}$$

It is well-known that $\tilde{W}_h\left(\dot{\tilde{W}}_h\right)^{-1}\tilde{W}_h = -\dot{\tilde{W}}_h$, see e.g. [11]. Thus (14) can be rewritten as:

$$\Psi_{hh\underline{h}} + \Delta\Psi_{hh} - \tilde{E}\dot{\tilde{W}}_h < 0 \tag{15}$$

with $\Psi_{hh\underline{h}} = \tilde{W}_h^{\ T}\tilde{A}_{h\underline{h}}^{\ T} + \tilde{A}_{h\underline{h}}\tilde{W}_h$ and $\Delta\Psi_{hh} = \tilde{W}_h^{\ T}\Delta\tilde{A}_h^{\ T} + \Delta\tilde{A}_h\tilde{W}_h$

Extending $\Psi_{hh\underline{h}}$, it yields

$$\Psi_{hh\underline{h}} = \begin{bmatrix} \Psi_{hh}^{(1,1)} & (*) & (*) & (*) \\ \Psi_{hh\underline{h}}^{(2,1)} & \Psi_{hh\underline{h}}^{(2,2)} & (*) & (*) \\ \Psi_{hh}^{(3,1)} & \Psi_{hh}^{(3,2)} & \Psi_{hh}^{(3,3)} & (*) \\ \Psi_{hh\underline{h}}^{(4,1)} & \Psi_{hh\underline{h}}^{(4,2)} & \Psi_{hh}^{(4,3)} & \Psi_{hh\underline{h}}^{(4,4)} \end{bmatrix} \tag{16}$$

with $\Psi_{hh}^{(1,1)} = A_h W_1^{\ h} + W_1^{\ h} A_h^T + B_h W_{13}^{\ h} + W_{13}^{\ hT} B_h^T$,

$\Psi_{hh\underline{h}}^{(2,1)} = A_h^*\left(W_6^{\ h}\right)^{-1} W_2^{\ hT} + W_2^{\ h} A_h^T + B_h^*\left(W_{11}^{\ h}\right)^{-1} W_9^{\ h} + W_{14}^{\ hT} B_h^T$,

$$\Psi_{hh\underline{h}}^{(2,2)} = A_h^* + A_h^{*T} + B_h^*\left(W_{11}^h\right)^{-1} W_{10}^h + W_{10}^{hT}\left(W_{11}^h\right)^{-T} B_h^{*T},\ \Psi_{hh}^{(3,1)} = C_h W_1^h - W_9^h + D_h W_{13}^h + W_{15}^{hT} B_h^T,$$

$$\Psi_{hh}^{(3,2)} = C_h W_2^h - W_{10}^h + D_h W_{14}^h + B_h^{*T},\ \Psi_{hh}^{(3,3)} = D_h W_{15}^h + W_{15}^{hT} D_h^T - W_{11}^h - W_{11}^{hT},$$

$$\Psi_{hh\underline{h}}^{(4,1)} = C_h^*\left(W_6^h\right)^{-1} W_2^{hT} + D_h^*\left(W_{11}^h\right)^{-1} W_9^h - W_{13}^h + W_{16}^{hT} B_h^T,$$

$$\Psi_{hh\underline{h}}^{(4,2)} = C_h^* - W_{14}^h + D_h^*\left(W_{11}^h\right)^{-1} W_{10}^h + W_{12}^{hT}\left(W_{11}^h\right)^{-T} B_h^{*T},\quad \Psi_{hh}^{(4,3)} = D_h^* + W_{16}^{hT} D_h^T - W_{15}^h - W_{12}^{hT} \quad \text{and}$$

$$\Psi_{hh\underline{h}}^{(4,4)} = D_h^*\left(W_{11}^h\right)^{-1} W_{12}^h + W_{12}^{hT}\left(W_{11}^h\right)^{-T} D_h^{*T} - W_{16}^h - W_{16}^{hT}.$$

Let us recall that, due to the nature of the candidate Lyapunov function (10), $W_9^h, W_{10}^h, \ldots, W_{16}^h$ are slack decision matrices which are free of choice. At a first glance on (16), in order to run to LMI conditions, a solution should be to choose, for instance $W_9^h = W_{10}^h = W_{11}^h = W_{12}^h$. Nevertheless, in that case, the problem remains more restrictive regarding to the considered class of T-S fuzzy systems since $W_9^h \in \mathbb{R}^{q\times n}$, $W_{10}^h \in \mathbb{R}^{q\times n}$, $W_{11}^h \in \mathbb{R}^{q\times q}$ and $W_{12}^h \in \mathbb{R}^{q\times m}$. Indeed, with the latter solution, one has to consider T-S fuzzy systems where the input, output and the state vector have to be casted into the same dimension. Therefore, for the sake of generality, one chooses $W_6^h = W_2^h$, $W_9^h = 0$, $W_{10}^h = 0$ and $W_{12}^h = 0$ which appears as a convenient solution. Thus, (16) becomes:

$$\Upsilon_{hh} = \begin{bmatrix} \Psi_{hh}^{(1,1)} & (*) & (*) & (*) \\ A_h^* + W_6^h A_h^T + W_{14}^{hT} B_h^T & A_h^* + A_h^{*T} & (*) & (*) \\ C_h W_1^h + D_h W_{13}^h + W_{15}^{hT} B_h^T & C_h W_6^h + D_h W_{14}^h + B_h^{*T} & \Psi_{hh}^{(3,3)} & (*) \\ C_h^* - W_{13}^h + W_{16}^{hT} B_h^T & C_h^* - W_{14}^h & D_h^* + W_{16}^{hT} D_h^T - W_{15}^h & -W_{16}^h - W_{16}^{hT} \end{bmatrix} \quad (17)$$

Extending $\Delta\Psi_{hh}$ with $\Delta A_h(t) = H_a^{\bar{h}} f_a(t) \mathrm{N}_a^{\bar{h}}$, $\Delta B_h(t) = H_b^{\bar{h}} f_b(t) \mathrm{N}_b^{\bar{h}}$, $\Delta C_h(t) = H_c^{\bar{h}} f_c(t) \mathrm{N}_c^{\bar{h}}$ and $\Delta D_h(t) = H_d^{\bar{h}} f_d(t) \mathrm{N}_d^{\bar{h}}$, it yields:

$$\Delta\Psi_{hh} = \begin{bmatrix} \Delta\Psi_{hh}^{(1,1)} & (*) & (*) & (*) \\ \Delta\Psi_{hh}^{(2,1)} & 0 & (*) & (*) \\ \Delta\Psi_{hh}^{(3,1)} & H_c^{\bar{h}} f_c \mathrm{N}_c^{\bar{h}} W_6^h + H_d^{\bar{h}} f_d \mathrm{N}_d^{\bar{h}} W_{14}^h & \Delta\Psi_{hh}^{(3,3)} & (*) \\ W_{16}^{hT} \mathrm{N}_b^{\bar{h}T} f_b^T H_b^{\bar{h}T} & 0 & W_{16}^{hT} \mathrm{N}_d^{\bar{h}T} f_d^T H_d^{\bar{h}T} & 0 \end{bmatrix} \quad (18)$$

with $\Delta\Psi_{hh}^{(1,1)} = H_a^{\bar{h}} f_a \mathrm{N}_a^{\bar{h}} W_1^h + H_b^{\bar{h}} f_b \mathrm{N}_b^{\bar{h}} W_{13}^h + W_1^h \mathrm{N}_a^{\bar{h}T} f_a^T H_a^{\bar{h}T} + W_{13}^{hT} \mathrm{N}_b^{\bar{h}T} f_b^T H_b^{\bar{h}T}$,

$\Delta\Psi_{hh}^{(2,1)} = W_6^h \mathrm{N}_a^{\bar{h}T} f_a^T H_a^{\bar{h}T} + W_{14}^{hT} \mathrm{N}_b^{\bar{h}T} f_b^T H_b^{\bar{h}T}$,

$\Delta\Psi_{hh}^{(3,1)} = H_c^{\bar{h}} f_c \mathrm{N}_c^{\bar{h}} W_1^h + H_d^{\bar{h}} f_d \mathrm{N}_d^{\bar{h}} W_{13}^h + W_{15}^{hT} \mathrm{N}_b^{\bar{h}T} f_b^T H_b^{\bar{h}T}$,

and $\Delta\Psi_{hh}^{(3,3)} = H_d^{\bar{h}} f_d \mathrm{N}_d^{\bar{h}} W_{15}^h + W_{15}^{hT} \mathrm{N}_d^{\bar{h}T} f_d^T H_d^{\bar{h}T}$.

Applying lemma 1 on (18), one has $\Delta\Psi_{hh} \le \Delta\bar{\Psi}_{hh} = \begin{bmatrix} \bar{\Psi}_{hh}^{(1,1)} & 0 & 0 & 0 \\ 0 & \bar{\Psi}_{hh}^{(2,2)} & 0 & 0 \\ 0 & 0 & \bar{\Psi}_{hh}^{(3,3)} & 0 \\ 0 & 0 & 0 & \bar{\Psi}_{hh}^{(4,4)} \end{bmatrix}$ where

$$\begin{aligned}\bar{\Psi}_{hh}^{(1,1)} &= \left(\varepsilon_{1a}^{hh} + \varepsilon_{6a}^{hh}\right) H_a^{\bar{h}} H_a^{\bar{h}T} + \left(\varepsilon_{13b}^{hh} + \varepsilon_{14b}^{hh} + \varepsilon_{15b}^{hh} + \varepsilon_{16b}^{hh}\right) H_b^{\bar{h}} H_b^{\bar{h}T} \\ &\quad + \varepsilon_{1c}^{hh} H_c^{\bar{h}} H_c^{\bar{h}T} + \varepsilon_{13d}^{hh} H_d^{\bar{h}} H_d^{\bar{h}T} + \left(\varepsilon_{1a}^{hh}\right)^{-1} W_1^h \mathrm{N}_a^{\bar{h}T} \mathrm{N}_a^{\bar{h}} W_1^h + \left(\varepsilon_{13b}^{hh}\right)^{-1} W_{13}^{hT} \mathrm{N}_b^{\bar{h}T}{}_b \mathrm{N}_b^{\bar{h}} W_{13}^h ,\end{aligned}$$

$$\bar{\Psi}_{hh}^{(2,2)} = \varepsilon_{6c}^{hh} H_c^{\bar{h}} H_c^{\bar{h}T} + \varepsilon_{14d}^{hh} H_d^{\bar{h}} H_d^{\bar{h}T} + \left(\varepsilon_{6a}^{hh}\right)^{-1} W_6^h \mathrm{N}_a^{\bar{h}T} \mathrm{N}_a^{\bar{h}} W_6^h + \left(\varepsilon_{14b}^{hh}\right)^{-1} W_{14}^{hT} \mathrm{N}_b^{\bar{h}T} \mathrm{N}_b^{\bar{h}} W_{14}^h ,$$

$$\begin{aligned}\bar{\Psi}_{hh}^{(3,3)} &= \left(\varepsilon_{15d}^{hh} + \varepsilon_{16d}^{hh}\right) H_d^{\bar{h}} H_d^{\bar{h}T} + \left(\varepsilon_{15d}^{hh}\right)^{-1} W_{15}^{hT} \mathrm{N}_d^{\bar{h}T} \mathrm{N}_d^{\bar{h}} W_{15}^h + \left(\varepsilon_{15b}^{hh}\right)^{-1} W_{15}^{hT} \mathrm{N}_b^{\bar{h}T} \mathrm{N}_b^{\bar{h}} W_{15}^h \\ &\quad + \left(\varepsilon_{1c}^{hh}\right)^{-1} W_1^h \mathrm{N}_c^{\bar{h}T} \mathrm{N}_c^{\bar{h}} W_1^h + \left(\varepsilon_{13d}^{hh}\right)^{-1} W_{13}^{hT} \mathrm{N}_d^{\bar{h}T} \mathrm{N}_d^{\bar{h}} W_{13}^h + \left(\varepsilon_{6c}^{hh}\right)^{-1} W_6^h \mathrm{N}_c^{\bar{h}T} \mathrm{N}_c^{\bar{h}} W_6^h + \left(\varepsilon_{14d}^{hh}\right)^{-1} W_{14}^{hT} \mathrm{N}_d^{\bar{h}T} \mathrm{N}_d^{\bar{h}} W_{14}^h ,\end{aligned}$$

and $\bar{\Psi}_{hh}^{(4,4)} = \varepsilon_{16b}^{hh\ -1} W_{16}^{hT} \mathrm{N}_b^{\bar{h}T} \mathrm{N}_b^{\bar{h}} W_{16}^h + \varepsilon_{16d}^{hh\ -1} W_{16}^{hT} \mathrm{N}_d^{\bar{h}T} \mathrm{N}_d^{\bar{h}} W_{16}^h$

Note that, $\Delta\bar{\Psi}_{hh}$ can be rewritten as:

$$\Delta\bar{\Psi}_{hh} = \tilde{H}_{hh} - \tilde{Z}_{hh}^{T}\left(\tilde{P}_{hh}\right)^{-1}\tilde{Z}_{hh} \tag{19}$$

with $\tilde{H}_{hh} = \begin{bmatrix} \tilde{H}_{hh}^{(1,1)} & 0 & 0 & 0 \\ 0 & \varepsilon_{6c}^{hh} H_c^{\bar{h}} H_c^{\bar{h}T} + \varepsilon_{14d}^{hh} H_d^{\bar{h}} H_d^{\bar{h}T} & 0 & 0 \\ 0 & 0 & \left(\varepsilon_{15d}^{hh} + \varepsilon_{16d}^{hh}\right) H_d^{\bar{h}} H_d^{\bar{h}T} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$,

$\tilde{H}_{hh}^{(1,1)} = \left(\varepsilon_{1a}^{hh} + \varepsilon_{6a}^{hh}\right) H_a^{\bar{h}} H_a^{\bar{h}T} + \left(\varepsilon_{13b}^{hh} + \varepsilon_{14b}^{hh} + \varepsilon_{15b}^{hh} + \varepsilon_{16b}^{hh}\right) H_b^{\bar{h}} H_b^{\bar{h}T} + \varepsilon_{1c}^{hh} H_c^{\bar{h}} H_c^{\bar{h}T} + \varepsilon_{13d}^{hh} H_d^{\bar{h}} H_d^{\bar{h}T}$,

$\tilde{P}_{hh} = -diag\left[\varepsilon_{1a}^{hh} I \quad \varepsilon_{13b}^{hh} I \quad \varepsilon_{6a}^{hh} I \quad \varepsilon_{14b}^{hh} I \quad \varepsilon_{15d}^{hh} I \quad \varepsilon_{15b}^{hh} I \quad \varepsilon_{1c}^{hh} I \quad \varepsilon_{13d}^{hh} I \quad \varepsilon_{6c}^{hh} I \quad \varepsilon_{14d}^{hh} I \quad \varepsilon_{16b}^{hh} I \quad \varepsilon_{16d}^{hh} I\right]$ and

$$\tilde{Z}_{hh} = \begin{bmatrix} N_a^h W_1^h & 0 & 0 & 0 \\ N_b^h W_{13}^h & 0 & 0 & 0 \\ 0 & N_a^i W_6^h & 0 & 0 \\ 0 & N_b^i W_{14}^h & 0 & 0 \\ 0 & 0 & N_d^i W_{15}^h & 0 \\ 0 & 0 & N_b^i W_{15}^h & 0 \\ 0 & 0 & N_c^h W_1^h & 0 \\ 0 & 0 & N_d^h W_{13}^h & 0 \\ 0 & 0 & N_c^i W_6^h & 0 \\ 0 & 0 & N_d^i W_{14}^h & 0 \\ 0 & 0 & 0 & N_b^i W_{16}^h \\ 0 & 0 & 0 & N_d^i W_{16}^h \end{bmatrix}$$

Let us now focus on the term $\tilde{E}\dot{\tilde{W}}_h$ in (15). From the convex property of the membership functions $h_k(z(t))$ one has $\sum_{k=1}^{r} h_k(z(t)) = 1$, so $\dot{h}_r(z(t)) = -\sum_{k=1}^{r-1} \dot{h}_k(z(t))$. Therefore, the following property improves the conservatism of the proposed solutions since it reduces the number of membership function derivates to be taking into account:

$$\tilde{E}\dot{\tilde{W}}_h = \sum_{k=1}^{r-1} \dot{h}_k(z(t))\tilde{E}\tilde{W}_k + \dot{h}_r(z(t))\tilde{E}\tilde{W}_r = \sum_{k=1}^{r-1} \dot{h}_k(z(t))\left(\tilde{E}\tilde{W}_k - \tilde{E}\tilde{W}_r\right) \tag{20}$$

Let us consider for $k = 1,\ldots,r-1$, ϕ_k the lower bounds of $\dot{h}_k(z(t))$. One can write $\tilde{E}\dot{\tilde{W}}_h \geq \sum_{k=1}^{r-1} \phi_k\left(\tilde{E}\tilde{W}_k - \tilde{E}\tilde{W}_r\right)$ with $\tilde{E}\tilde{W}_k - \tilde{E}\tilde{W}_r \geq 0$ for $k = 1,\ldots,r-1$. Thus, considering (17) and (19), (15) holds if:

$$\Upsilon_{hh} + \tilde{H}_{hh} - \tilde{Z}_{hh}^T\left(\tilde{\mathrm{P}}_{hh}\right)^{-1}\tilde{Z}_{hh} - \sum_{k=1}^{r-1} \phi_k\left(\tilde{E}\tilde{W}_k - \tilde{E}\tilde{W}_r\right) < 0 \tag{21}$$

Applying the Schur complement, (21) yields:

$$\Gamma_{hh} = \begin{bmatrix} \Upsilon_{hh} + \tilde{H}_{hh} - \sum_{k=1}^{r-1} \phi_k\left(\tilde{E}\tilde{W}_k - \tilde{E}\tilde{W}_r\right) & (*) \\ \tilde{Z}_{hh} & \tilde{\mathrm{P}}_{hh} \end{bmatrix} < 0 \tag{22}$$

Thus, after rewriting (22) in their extended form and applying lemma 2, the conditions (7), (8) and (9) yield. That ends the proof. ■

Remark 2: For $i = 1,\ldots,r$, $h_i\left(z\left(t\right)\right)$ is required to be at least C^1. This point is satisfied for fuzzy models constructed via a sector nonlinearity approach [36] if the system (2) is at least C^1 or, for instance when membership functions are chosen with a smoothed Gaussian shape.

Remark 3: From (3), W_6^h and W_{11}^h are needed to be non singular. If, for $i = 1,\ldots,r$, W_6^i are solutions of theorem 1, then we have $W_6^i = W_6^{iT} > 0$ imposed by (12). Thus W_6^h is a non singular matrix. Moreover, if (10) is a Lyapunov functional, i.e. (7), (8) and (9) are verified, $\tilde{W}_h$ is a non singular matrix satisfying (11) and $\tilde{W}_h^{-1}$ exists. Recall that $\tilde{W}_h = \begin{bmatrix} W_1^h & W_6^h & 0 & 0 \\ W_6^h & W_6^h & 0 & 0 \\ 0 & 0 & W_{11}^h & 0 \\ W_{13}^h & W_{14}^h & W_{15}^h & W_{16}^h \end{bmatrix}$. Therefore, W_{11}^h is a non singular matrix and so $\left(W_{11}^h\right)^{-1}$ exists.

Remark 4: Introducing the bounds of the time derivative membership functions in (21) with the formulation (20) instead of $\sum_{k=1}^{r} \phi_k \tilde{E}\tilde{W}_k$ allows providing LMI conditions (7), (8) and (9) which obviously include the quadratic case. Thus the proposed fuzzy Lyapunov approach is obviously reducing the conservatism of quadratic approach.

Remark 5: To the best of authors' knowledge, expects our preliminary study [11], theorem 1 is the first result regarding to non quadratic DOFC stabilization for T-S fuzzy models. Moreover, there were no tractable LMI conditions in the previous literature which consider matrices C_i that have not to be common or identity as well as D_i that have not to be zero in equation (2). Only few results exists using the Redheffer product in order to write the closed-loop system dynamics [2][20][46]. Nevertheless, these results are resorting to model transformation, bounding techniques for some cross terms and products between decision variables which are sources of conservatism and ruins tentative to derive non-quadratic LMI conditions. The non quadratic DOFC design methodology depicted in theorem 1 has been obtained thanks to the rewriting of the closed-loop system (6). This has been done using the descriptor redundancy which avoids appearance of crossing terms between the state space matrices and the controller's ones.

4. H∞ BASED DOFC SYNTHESIS

The conditions proposed in Theorem 1 are for $\varphi(t)=0$. This section extend the previous results by the use of an H_∞ criterion. The goal is to stabilize (2) such that the influence of the external disturbance $\varphi(t)$ on the output behavior is minimized. Let us consider the following H_∞ criterion [36][48]:

$$\int_0^\infty \left(y^T(t)\, y(t) - \lambda^2 \varphi^T(t)\, \varphi(t) \right) dt \le 0 \tag{23}$$

Recall that $\tilde{x}(t) = \begin{bmatrix} x^T(t) & x^{*T}(t) & y^T(t) & u^T(t) \end{bmatrix}^T$, thus (23) can be rewritten as:

$$\int_0^\infty \left(\tilde{x}^T(t)\, \tilde{Q}\tilde{x}(t) - \lambda^2 \varphi^T(t)\, \varphi(t) \right) dt \le 0 \tag{24}$$

with $\tilde{Q} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

In that case, the stability of the closed loop-system (6) is guaranteed under the constraint (24) if the LMI conditions summarized in the following theorem hold.

Theorem 2: *The T-S fuzzy model* (2) *is globally asymptotically stable via the non PDC dynamic output feedback compensator* (3) *and guarantee the attenuation level* $\lambda = \sqrt{\eta}$ *if there exist, the matrices* $W_1^i = W_1^{iT} > 0$, $W_6^i = W_6^{iT} > 0$, W_{11}^i, W_{13}^i, W_{14}^i, W_{15}^i, W_{16}^i, A_i^*, B_i^*, C_i^* *and* D_i^*, *for* $i = 1, \ldots, r$, *the scalars* η, ε_{1a}, ε_{6a}, ε_{13b}, ε_{15b}, ε_{16b}, ε_{14b}, ε_{1c}, ε_{6c}, ε_{13d}, ε_{14d}, ε_{15d} *and* ε_{16d} *such that the following LMI conditions are satisfied:*

Minimize $\eta > 0$ such that:

- for $i = 1, 2, \ldots, r$, $\Theta_{ii} < 0$ (25)

- for $i = 1, 2, \ldots, r$ and $1 \le i \ne j \le r$, $\frac{1}{r-1}\Theta_{ii} + \frac{1}{2}\left(\Theta_{ij} + \Theta_{ji}\right) < 0$ (26)

- for $i = 1, 2, \ldots, r-1$, $W_1^i - W_1^r \ge 0$ and $W_6^i - W_6^r \ge 0$ (27)

where $\Theta_{ij} = \begin{bmatrix} & & & & & 0 & (*) \\ & & & & & 0 & 0 \\ & & \Gamma_{ij} & & & (*) & (*) \\ & & & & & 0 & 0 \\ & & & & & 0 & 0 \\ 0 & 0 & W_{11}^{h} & 0 & 0 & -I & 0 \\ F_i^T & 0 & G_i^T & 0 & 0 & 0 & -\eta I \end{bmatrix}$ and with the matrices Γ_{ij} defined in theorem 1.

Proof:

The stability of the closed loop-system (6) is guarantee, under the constraint (24), if:

$$\dot{v}\left(x, x^*\right) + \tilde{x}^T \tilde{Q} \tilde{x} - \lambda^2 \varphi^T \varphi < 0 \tag{28}$$

That is to say if:

$$\tilde{x}^T \left(\left(\tilde{A}_{h\underline{h}}{}^T + \Delta\tilde{A}_h{}^T \right) \tilde{W}_h{}^{-1} + \tilde{W}_h{}^{-T} \left(\tilde{A}_{h\underline{h}} + \Delta\tilde{A}_h \right) + \tilde{E} \dot{\tilde{W}}_h{}^{-1} + \tilde{Q} \right) \tilde{x} + \varphi^T \tilde{F}_h{}^T W_h{}^{-1} \tilde{x} + \tilde{x}^T \tilde{W}_h{}^{-T} \tilde{F}_h \varphi - \lambda^2 \varphi^T \varphi < 0 \tag{29}$$

which is obviously satisfied if:

$$\begin{bmatrix} \left(\tilde{A}_{h\underline{h}}{}^T + \Delta\tilde{A}_h{}^T \right) \tilde{W}_h{}^{-1} + \tilde{W}_h{}^{-T} \left(\tilde{A}_{h\underline{h}} + \Delta\tilde{A}_h \right) + \tilde{E} \dot{\tilde{W}}_h{}^{-1} + \tilde{Q} & (*) \\ \tilde{F}_h{}^T W_h{}^{-1} & -\lambda^2 I \end{bmatrix} < 0 \tag{30}$$

Multiplying left by $\begin{bmatrix} W_h{}^T & 0 \\ 0 & I \end{bmatrix}$ and right by $\begin{bmatrix} W_h & 0 \\ 0 & I \end{bmatrix}$, one has:

$$\begin{bmatrix} W_h{}^T\left(\tilde{A}_{h\underline{h}}{}^T + \Delta\tilde{A}_h{}^T\right) + \left(\tilde{A}_{h\underline{h}} + \Delta\tilde{A}_h\right)W_h + \tilde{E}W_h\dot{\tilde{W}}_h{}^{-1}W_h + W_h{}^T\tilde{Q}W_h & (*) \\ \tilde{F}_h{}^T & -\lambda^2 I \end{bmatrix} < 0 \tag{31}$$

Following the same way as for the proof of theorem 1, with for $k = 1,\ldots,r-1$, $\tilde{E}\tilde{W}_k - \tilde{E}\tilde{W}_r \geq 0$ leads to (27), (31) is satisfied if:

$$\begin{bmatrix} \Upsilon_{hh} + \tilde{H}_{hh} - \tilde{Z}_{hh}^T\tilde{P}_{hh}^{-1}\tilde{Z}_{hh} + W_h{}^T\tilde{Q}W_h - \sum_{k=1}^{r-1}\phi_k\left(\tilde{E}\tilde{W}_k - \tilde{E}\tilde{W}_r\right) & (*) \\ \tilde{F}_h{}^T & -\lambda^2 I \end{bmatrix} < 0 \tag{32}$$

Note that $W_h{}^T\tilde{Q}W_h = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & W_{11}^{hT}W_{11}^{h} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, using the Schur complement and lemma 2, (25) and (26) yield. That ends the proof. ■

Remark 5: The LMI conditions proposed in theorems 1 and 2 are depending on the lower bounds of $\dot{h}_k(z(t))$ for $k = 1,\ldots,r-1$. Even if it is often pointed out as a criticism to fuzzy Lyapunov approach since these parameters may be difficult to choose, a way to obtain these bound has been proposed in [37]. Moreover, let us recall that this approach remains one of the least conservative in terms of LMI based design. In [15], a fuzzy Lyapunov candidate function has been reduced leading to relaxed quadratic stability conditions in the case of descriptor systems. Indeed, some elements in the Lyapunov matrix can be set common in order to make the LMI free of membership function's lower bounds. In the present study, this remains on setting W_1 and W_6 common matrices in the previous theorems and corollaries. Note finally that, obviously, the 'price' to pay for more practical applicability is an increase of the conservatism.

5. SIMULATION RESULTS

Example 1:

In order to illustrate the gain in terms of conservatism regarding the existing results, one compares the feasibility fields obtained from theorem 1 (without uncertainties) with the one obtained from the conditions proposed in [20] (see theorem 2). Note that, as far as we know, there are no new results since [20], excepted our preliminary result [11], dealing with dynamic output feedback stabilization for the general class of T-S systems described by (2), i.e. considering C_i non common and $D_i \neq 0$. Let us consider the following T-S system inspired from [37]:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{2} h_i(z(t))\left[A_i x(t) + B_i u(t)\right] \\ y(t) = \sum_{i=1}^{2} h_i(z(t))\left[C_i x(t) + D_i u(t)\right] \end{cases} \tag{33}$$

with $A_1=\begin{bmatrix}-5\alpha & 10\\ -1 & -2\end{bmatrix}$, $A_2=\begin{bmatrix}-2 & 10\\ 20 & -2\end{bmatrix}$, $B_1=\begin{bmatrix}0\\ 10\end{bmatrix}$, $B_2=\begin{bmatrix}0\\ 3\beta\end{bmatrix}$, $C_1=\begin{bmatrix}1 & 0.5\\ 0 & 1\end{bmatrix}$,

$C_2=\begin{bmatrix}-0.8 & 0\\ 1 & -2\end{bmatrix}$, $D_1=\begin{bmatrix}1\\ -1\end{bmatrix}$ and $D_2=\begin{bmatrix}0.5\\ 1\end{bmatrix}$.

The LMI computation has been done using the Matlab LMI Toolbox [10] and the feasibility has been checked for $-5\le\alpha\le 20$ and $-20\le\beta\le 0$ with ϕ_1 computed for each pair (α,β) as described in [37]. For instance $(\alpha,\beta)=(1,1)$ leads to $\phi_1=-8.08$. As expected, figure 1 shows that the conditions proposed in corollary 1 are less conservative than those proposed in [20].

Example 2:

In this example, the design of a DOFC is considered for an uncertain and disturbed T-S fuzzy model given by:

$$\begin{cases}\dot{x}(t)=\sum_{i=1}^{2}h_i(z(t))\left[(A_i+\Delta A_i(t))x(t)+(B_i+\Delta B_i(t))u(t)+F_i\varphi(t)\right]\\ y(t)=\sum_{i=1}^{2}h_i(z(t))\left[(C_i+\Delta C_i(t))x(t)+(D_i+\Delta D_i(t))u(t)+G_i\varphi(t)\right]\end{cases} \tag{34}$$

with $A_1=\begin{bmatrix}-5 & -4\\ -1 & -2\end{bmatrix}$, $A_2=\begin{bmatrix}-2 & -4\\ 20 & -2\end{bmatrix}$, $B_1=\begin{bmatrix}0\\ 10\end{bmatrix}$, $B_2=\begin{bmatrix}0\\ 3\end{bmatrix}$, $C_1=\begin{bmatrix}2 & -10\\ 5 & -1\end{bmatrix}$, $C_2=\begin{bmatrix}-3 & 20\\ -7 & -2\end{bmatrix}$,

$D_1=\begin{bmatrix}3\\ -1\end{bmatrix}$ and $D_2=\begin{bmatrix}-1\\ 0.5\end{bmatrix}$, $F_1=F_2=\begin{bmatrix}0\\ -0.25\end{bmatrix}$, $G_1=\begin{bmatrix}-0.5\\ 0.5\end{bmatrix}$, $G_2=\begin{bmatrix}0.35\\ 0.5\end{bmatrix}$,

$\Delta A_1(t)=H_a^1 f_a(t)\mathrm{N}_a^1$, $\Delta A_2(t)=H_a^2 f_a(t)\mathrm{N}_a^2$, $\Delta B_1(t)=H_b^1 f_b(t)\mathrm{N}_b^1$, $\Delta B_2(t)=H_b^2 f_b(t)\mathrm{N}_b^2$,

$\Delta C_1(t) = H_c^1 f_c(t) \mathrm{N}_c^1$, $\Delta C_2(t) = H_c^2 f_c(t) \mathrm{N}_c^2$, $\Delta D_1(t) = H_d^1 f_d(t) \mathrm{N}_d^1$ and $\Delta D_2(t) = H_d^2 f_d(t) \mathrm{N}_d^2$ with $H_a^1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $H_a^2 = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$, $H_b^1 = \begin{bmatrix} 0 \\ -2 \end{bmatrix}$, $H_b^2 = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$, $H_c^1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, $H_c^2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$, $H_d^1 = \begin{bmatrix} 0.5 \\ -0.5 \end{bmatrix}$, $H_d^2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$, $N_a^1 = \begin{bmatrix} 1 & 1 \end{bmatrix}$, $N_a^2 = \begin{bmatrix} -1 & 1 \end{bmatrix}$, $N_b^1 = 1$, $N_b^2 = -0.75$, $N_c^1 = \begin{bmatrix} 1 & 1 \end{bmatrix}$, $N_c^2 = \begin{bmatrix} -1 & -1 \end{bmatrix}$, $N_d^1 = -1$, $N_d^2 = 0.5$.

Note that, the lower bound of the membership function derivative can be found for the nominal part of the considered fuzzy system using the approaches proposed in [37], i.e. $\phi_1 = -3.68$. Obviously, the considered model includes some bounded uncertainties and disturbances which are unknown. Thus, even if their effects are attenuated regarding to the state, it is not possible to conclude on the time derivative of the membership function. At least, what can be done for instance is choosing an assumed “greater” value than the one obtained from the nominal part. In the present example we choose $\phi_1 = -7.36$ twice the value of the nominal part is. Let us just point out that there is no solution to this problem and it could be a starting point for future prospects. The solution of Theorem 2 is obtained using the Matlab LMI Toolbox [10]. This provides the DOFC gain matrices given by:

$A_1^* = \begin{bmatrix} \text{-}0.0141 & 0.0079 \\ \text{-}0.0050 & \text{-}0.0053 \end{bmatrix}$, $A_2^* = \begin{bmatrix} \text{-}0.0036 & \text{-}0.0019 \\ 0.0098 & \text{-}0.0181 \end{bmatrix}$, $B_1^* = \begin{bmatrix} 0.0506 & 0.0483 \\ \text{-}0.0575 & \text{-}0.0565 \end{bmatrix}$,

$B_2^* = \begin{bmatrix} \text{-}0.0090 & 0.0106 \\ 0.0435 & 0.0283 \end{bmatrix}$, $C_1^* = 10^{-3}\begin{bmatrix} \text{-}0.6948 & 0.2470 \end{bmatrix}$, $C_2^* = 10^{-3}\begin{bmatrix} 0.3 & \text{-}6.4 \end{bmatrix}$,

$D_1^* = 10^{-3}[-1.2 \quad -1.5]$, $D_2^* = 10^{-3}[-56.6 \quad 12.4]$, $W_6^1 = 10^{-3}\begin{bmatrix} 0.3462 & 0.1732 \\ 0.1732 & 0.1273 \end{bmatrix}$,

$W_6^2 = 10^{-3}\begin{bmatrix} 0.4859 & -0.0179 \\ -0.0179 & 0.0028 \end{bmatrix}$, $W_{11}^1 = W_{11}^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, ensuring the H_∞ performance given by the attenuation level $\lambda = 0.75$.

The closed-loop dynamics has been simulated with the initial values $x_1(0)=1$, $x_2(0)=1$, $x_1^*(0)=0$, $x_2^*(0)=0$. Two cases are considered, the first one is without uncertainties and external disturbances (see the bold solid line in figure 2-5). The second one considers the uncertain function $f_a(t) = f_b(t) = f_c(t) = f_d(t) = \cos(0.01t)$ and $\varphi(t) = \sin(0.001t)$ (thin line). Figures 2 to 5 show respectively the behavior of the state signals [$x_1(t), x_2(t)$], the output signals [$y_1(t), y_2(t)$], the controller's state signal [$x_1^*(t), x_2^*(t)$] and the control signal $u(t)$ for these two cases. Let us point out that, during the simulation, the hypothesis made on the lower bound of the derivative function is verified since $\dot{h}_1(x_1(t)) \geq \phi_1 = -7.36$. Note that, this study deals with the system's state stabilization, i.e. the chosen Lyapunov function is only depending on the system's state. In that case, one can see from figures 2 to 5, that only the system's state signals show robustness regarding to uncertainties. Theoretically, this should be overcame using a Lyapunov function depending on both the state and the output but, in the case of the general class of T-S fuzzy models (2), this means that the Lyapunov should also depends on the input signal and so on, leading to less tractable LMI formulation. One other solution should be rewriting the considered nonlinear model using a convenient diffeomorphism allowing moving all uncertainties in the state equation, that is to say free of uncertainties in the output equations.

6. CONCLUSION

In this paper, the problem of output feedback stabilization of uncertain and disturbed Takagi-Sugeno models has been considered. A non PDC dynamic output feedback compensator has been proposed. The controller was then designed based on a fuzzy Lyapunov approach. Thanks to the descriptor redundancy, strict LMI conditions have been easily obtained. This approach leads to less conservative result and is valuable for uncertain and disturbed T-S fuzzy models through an $H\infty$ criterion. Finally, two academic examples were proposed to show the conservativeness as well as to illustrate the efficiency of the proposed approach.

ACKNOWLEDGEMENT

This work was supported by the French Ministry of Research, the “Département des Ardennes” and the region “Champagne-Ardenne” within the CPER MOSYP. The authors would like to thank Ms Belma Triks for her supporting comments.

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Figures

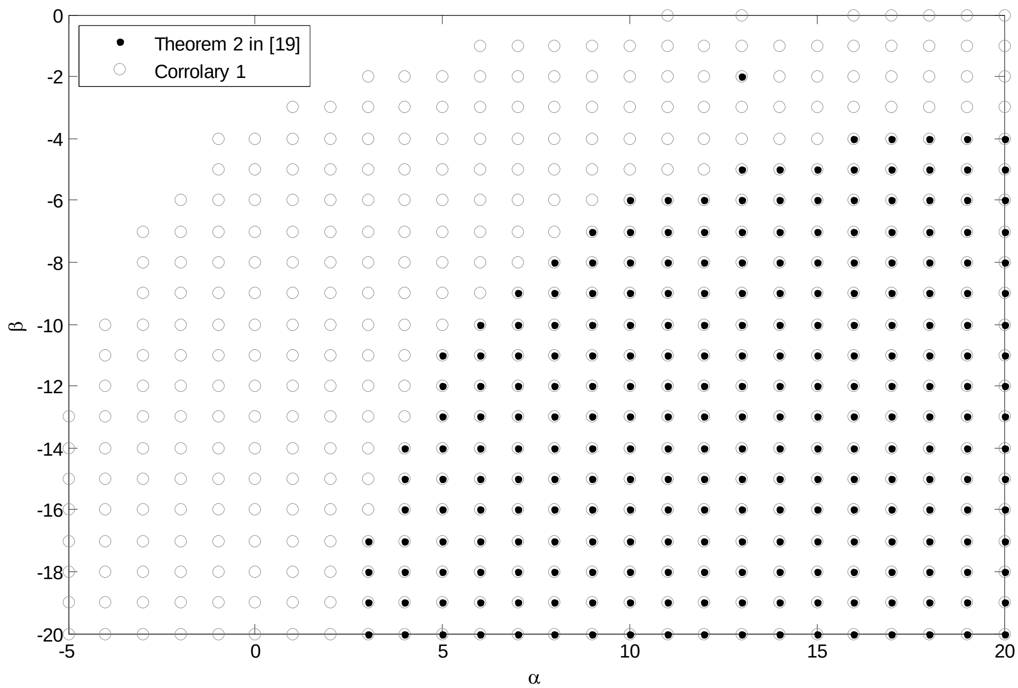

Fig 1: Feasibility fields from theorem 2 an LMI conditions provided in [20].

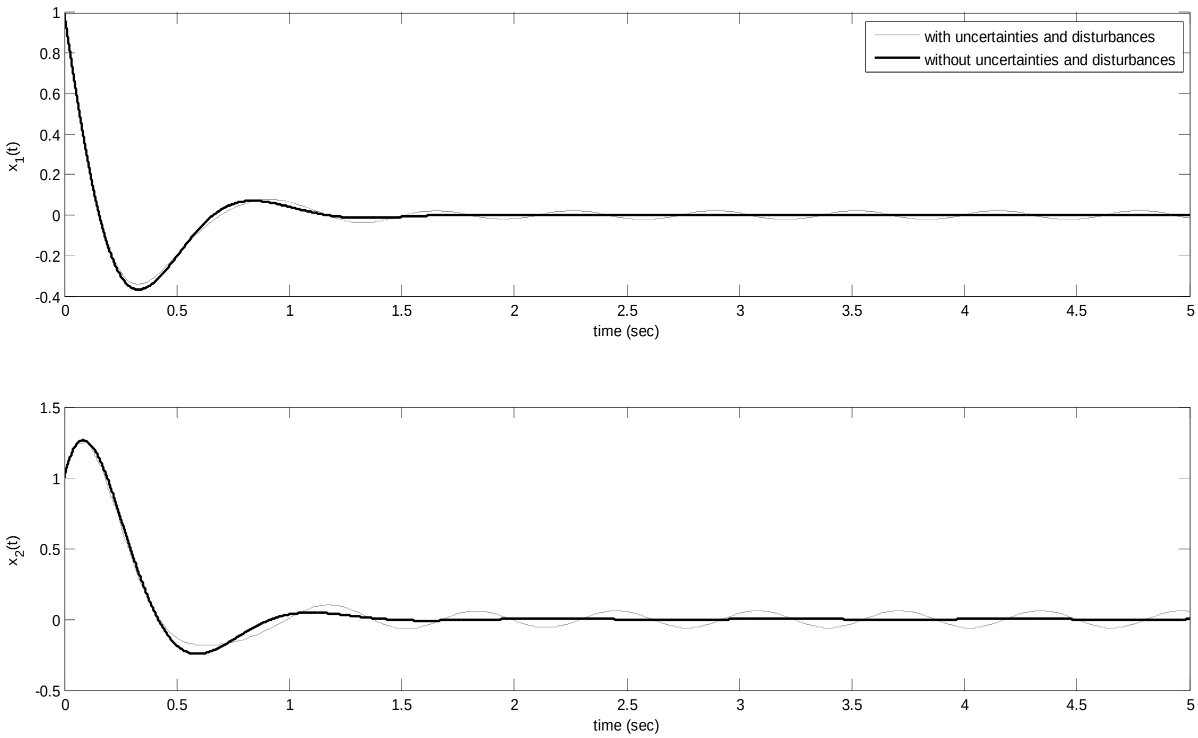

Fig 2: System’s state signals

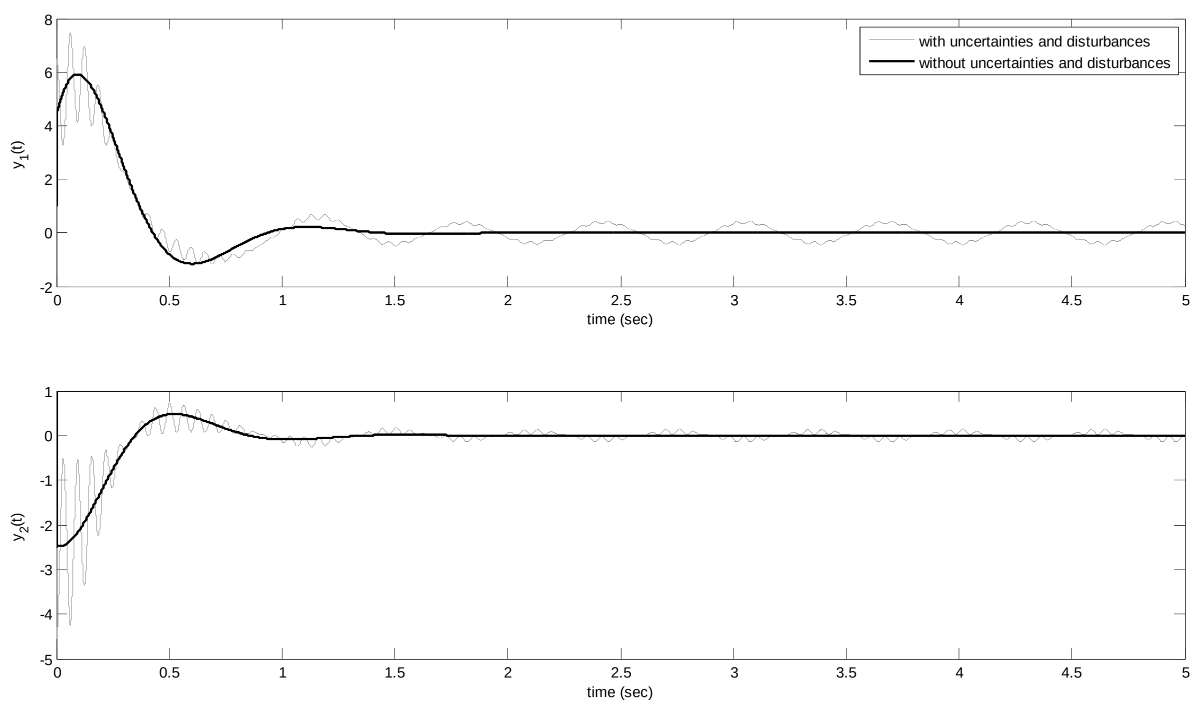

Fig 3: Output signals

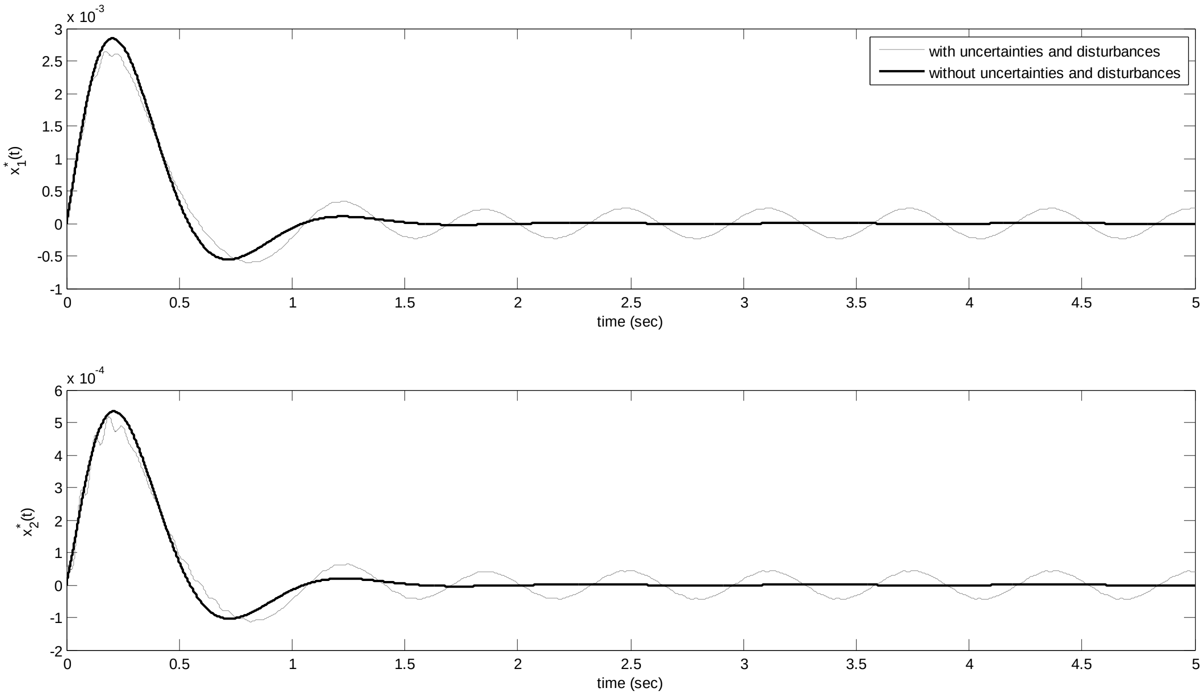

Fig 4: Controller's state signals

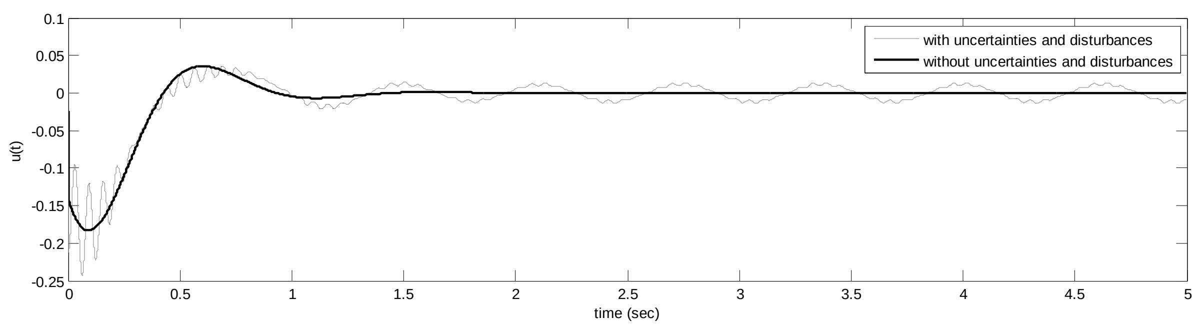

Fig 5: Control signals