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## CALIBRATION OF OPEN CHANNEL FLOW MODELS: A SYSTEM ANALYSIS AND CONTROL ENGINEERING APPROACH

### *Calage de modèles hydrauliques à surface libre : une approche par la théorie de l'analyse des systèmes et de l'automatique*

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#### KEY WORDS

Sensitivity, Bode plot,  $\ell_1$  norm, Worst-case perturbation, Input scenario

#### ABSTRACT

*Any project using a model should include a calibration phase. For a 1D hydrodynamic model based on Saint-Venant's equations, this often means adjusting roughness coefficients, discharge coefficients at cross and lateral devices, and seepage. Several methods are described in the literature, based on minimization approaches. Whatever algorithm selected, there is nevertheless an important question to address first, which is seldom studied in the literature: "is it possible to identify the parameters I want from the measurements I have?" This first question is linked to what is called "sensitivity". This depends on the relationship between parameters and outputs of the model, but also on the desired precision on the parameters and on the available one on the measurements. Some people introduced the concept of "equifinality" stating that, in some cases, several different sets of parameters give the same model outputs within given uncertainties. This concept is somehow peculiar in the sense that it is used to criticize the model itself, whereas it may also be attached to the set of data used for the calibration. An important question is therefore "what type of input scenario would be best to be able to calibrate the model?" This second question can be addressed under some assumptions using the "worst-case" concept. The paper provides methodologies for answering these 2 questions, and is applied to an example taken from the literature. This shows that having a minimization algorithm is maybe useful, but being able to answer the 2 above questions is even more important.*

#### RESUME

*Calage de modèles hydrauliques à surface libre : une approche par la théorie de l'analyse des systèmes et de l'automatique.*

*Tout projet mettant en œuvre un modèle nécessite une phase de calage. Pour un modèle hydrodynamique basé sur les équations de Saint-Venant, cela signifie ajuster des coefficients de frottement, des coefficients de débits aux ouvrages frontaux et latéraux, et des termes d'infiltration. Plusieurs méthodes basées sur la minimisation d'un critère existent. Quelque soit la méthode retenue, il y a une première question importante mais souvent négligée à se poser : "est-il possible d'identifier les paramètres désirés à partir des données disponibles ?" Cette première question peut être reliée à la notion de "sensibilité". Elle dépend du lien reliant les paramètres aux sorties, mais aussi des précisions souhaitées sur ces paramètres et de la précision disponible sur les mesures. Le concept d'équifinalité souligne le fait que, dans certains cas, plusieurs jeux de paramètres peuvent fournir les mêmes sorties d'un modèle, modulo des incertitudes*

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*données. Ce concept est sujet à caution, car il est utilisé pour critiquer un modèle, intrinsèquement, alors qu'il est tout autant lié au jeu de données utilisé pour le calage. Une question importante est donc "quel serait le jeu de données optimal pour le calage ?" Le concept de "pire cas" peut apporter des éléments de réponse à cette question, sous certaines hypothèses. L'article présente des méthodologies permettant de répondre à ces questions, et les illustre sur un exemple de la littérature. Cela montre qu'un algorithme de minimisation peut être utile, mais que savoir répondre aux 2 questions posées l'est tout autant.*

## 1. INTRODUCTION

The calibration of an open channel flow model is a question arising for any project using such tool. We could argue on how good this calibration must be, for being confident on the validity of the conclusions drawn from its use. Experienced modelers have ideas on the range of errors we could reasonably accept from such models. We will limit our discussion to 1D hydraulic numerical models, but some of the concepts developed in this paper are also valid for 1.5D and 2D hydraulic numerical models. We will focus on hydrodynamic (transient) models, but we will also address some issues dealing with steady flow models (either fully steady flow models, or unsteady flow models used with steady flow scenarios).

We could roughly say that, for irrigation canal, where cross section geometry is usually simple (e.g. trapezoidal) and possibly measured with a good precision, the water level precision of a corresponding hydraulic model can be expected to be around a few centimeters. For the same type of systems, the discharge precision could be around 5 to 10% of the nominal discharge. If these criteria are satisfied, we can reasonably assess that the numerical model is correctly set up and calibrated. If the error is larger at some locations (e.g.: > 10cm), it can be an indication that some features of the system have been forgotten (e.g.: weir, important section), that some model parameters are wrong (e.g.: cross section geometry, roughness coefficient, device headloss coefficient) or some measurements are erroneous (e.g.: longitudinal land leveling references). The numerical parameters of the model (e.g.: space step, time step, implicitation coefficient) can also have a part of this responsibility [1], [2]. In natural systems, such as rivers, these precisions can be degraded due to the higher imprecision in the geometry data, local additional head losses, wind effects, or difficulty in modeling the friction phenomenon (e.g.: composite beds, aquatic plants). In these cases, the 1D assumption can also be questionable at some locations such as meanders. For this type of systems, the water level precision could be around several tenths of centimeters and the discharge precision several tenths of percents of the nominal discharge. It also depends a lot on the complexity of the cross sections and on the hydraulic conditions (low or high flows).

When these hydrodynamic models are used for the design of automatic controllers, such as for operating the gates of irrigation canals, their calibration is an important issue for several reasons. The transient dynamics must be correctly represented over a range of frequencies covering the domain of solicitation of the system. The static equilibriums must be well calculated to correctly model static gain and have correct nominal water state conditions. Automatic controller design methodologies must verify a posteriori, or impose a priori [3], robustness conditions allowing some discrepancies of the model(s) used for their design. Nevertheless, being able to assess these model uncertainties and trying to reduce them is always important, although seldom done in practice.

We do agree on the warnings given by hydraulics experts on a "blind" "automatic" calibration of physically-based hydraulic models, suggesting even to remove this "calibration" phase in favor of a more in-depth "validation" one [15]. But, at the same time, we can think that the development of "clever" calibration tools can be useful for several reasons: some engineers using hydraulic models are not 30-year experienced experts; some of them are not participating in the survey along the system to assess by observation the roughness coefficients; roughness coefficients are difficult to assess in some cases (aquatic plants, natural complex sections with rocks, trees, undergrowth, etc); roughness coefficients must also account for numerical characteristics [17]. The amount of sensors and measurements on systems dramatically increased recently and will continue to do so. This will provide lots of data that could be useful for clever calibration procedures. Hydraulic models are also used for automatic control purposes where abundance of measurements are already available, where it is easy to generate new sets of data for any desired input scenario and where "optimal calibration" may be interesting (e.g.: Gignac canal, SCP canal, Rhône River).

The question is then: "how to calibrate a model?" For a 1D hydrodynamic model based on Saint-Venant's equations, this often means adjusting Manning-Strickler roughness coefficients, discharge coefficients at cross and lateral devices, and seepage values. The literature proposes several methods, based on minimization techniques approaches ([4], [5], [6], [7]). Recent works are using Monte Carlo or Kalman

filtering for reconstructing unknown inflows [8] and Strickler roughness coefficients [9]. Some of these methods are programmed in hydraulic software such as SIC software (Cemagref) or Dassflow software [7].

Whatever the algorithm selected for this calibration, there is nevertheless an important question to address first, which is seldom studied in the literature, and rapidly skipped by modelers: "is it possible to identify the parameters I want from the measurements I have?" This first question is linked to what is called "sensitivity". This depends on the relationship between parameters and outputs of the model, but also on the desired precision on the parameters and on the available one on the measurements. Control engineers also defined the concept of "observability" leading to an algebraic criteria indicating if internal states can be reconstructed from the measurement of outputs during a certain time horizon. This concept is quite generic, since internal states can include parameters and unknown inputs [3], [8], [9]. It is linked to the number, location, type and time horizon of these measurements. In this paper we will focus our studies on the "sensitivity" concept using some tools coming from the control engineering world.

Some people introduced the concept of "equifinality" [2] stating that, in some cases, several different sets of parameters give the same model outputs within given uncertainties. This concept is somehow peculiar in the sense that it is used to criticize the model itself, whereas it may also be attached to the set of data used for the calibration or validation. An important question is therefore "what type of input scenario would be best to be able to calibrate the model?" A reverse question, if the input scenario is imposed, is "are the characteristics of my input scenario rich enough in order to calibrate some or all of the parameters of my model?" After a presentation of the calibration framework, the following paper presents methodologies and tools addressing such questions. Illustrations will be given on an example taken from the literature [5].

## 2. CALIBRATION APPROACHES

### 2.1 Problem setup

A hydraulic model can be considered, on a mathematic point of view, as an operator  $\Phi$ :

$$\begin{aligned} \Phi : E^n &\rightarrow E^m \\ u &\mapsto y = \Phi(u) \\ \text{with } y(t) &= f(u(t), x(t), p, t) \end{aligned} \tag{1}$$

Where  $E^l$  is a space of dimension  $l$  (usually  $\ell_\infty(Z_+)$ , space of all bounded  $(\infty)$  vector-valued real sequence  $(\ell^l)$  on positive integers  $(Z_+)$  — representing the time instants —, of dimension  $l$ ),  $u$  is the vector (dimension  $n$ ) of the inputs of the model (e.g.: upstream node(s) discharge(s), downstream node(s) water elevation(s), intermediate gate or weir positions), and  $y$  is the vector (dimension  $m$ ) of the outputs of the model (e.g.: water levels and discharges at some locations). This operator, in the general case, can be defined using a function  $f$  of several additional variables and parameters such as:  $x$  a vector of internal states (e.g.: water levels and discharges at all calculation cross sections, distant by a space step  $\Delta x$ ),  $p$  a vector of parameters, and  $t$  the time (usually discrete values at time step  $\Delta t$ ). The variables  $y$ ,  $u$  and  $x$  are functions of time  $t$ . Parameters vector  $p$  is supposed to be constant over time. The space step  $\Delta x$  is usually varying along the system, and can also be time dependent. The time step  $\Delta t$  is usually constant along the system, but can also vary depending on the time  $t$ . The function  $f$  is, in our case, a non-linear function obtained in numerical form after the discretization of the Saint-Venant equations with the Preissmann implicit scheme [10]. Many other schemes are available and the following discussions remain valid.

The parameters vector  $p$  is usually composed of many different things:

- $p_n$ : the numerical parameters (e.g.: implicit coefficient of the numerical scheme, relaxation coefficient of a Newton-Raphson algorithm, convergence test of a bisection algorithm),
- $p_g$ : the geometry parameters (e.g.: cross sections, gates or weirs dimensions),
- $p_h$ : the hydraulic parameters (e.g.: roughness coefficient(s) at each calculation cross section, gate or weir discharge coefficient(s)).

If measurements of the outputs  $y_m$  are available at several time instants over a time horizon  $[T_1, T_2]$  (including the particular case  $T_1 = T_2$ ), it is then possible to try to calibrate the model. Without loss of generality we can assume that all outputs  $y$  are measured, since the function  $f$  can be adjusted in this purpose. Calibrating means finding a vector of parameters  $p$  minimizing the model output error criteria:

$$p = \text{Arg min } J(y, y_m), \quad (2)$$

where  $J$  is a criteria over the output vector  $y$  as defined in (1) and the corresponding measurement vector  $y_m$ . We can, for example, select a classical quadratic criteria:

$$J = \sum_{t \in [T_1, T_2]} \|y - y_m\|^2 \quad (3)$$

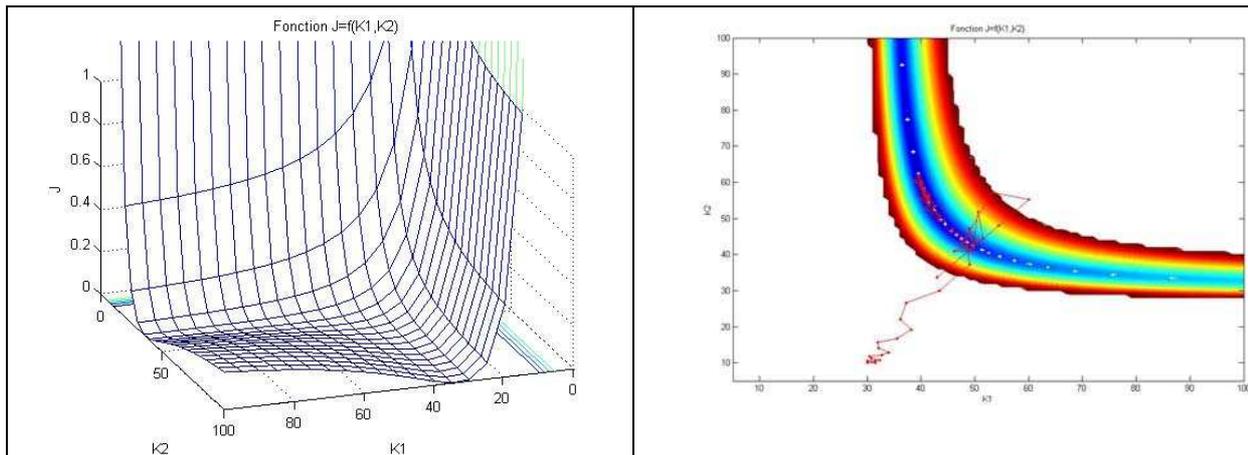
This calibration procedure could be, ideally, done over the entire vector  $p$ . In general the  $p_n$  parameters are fixed for given software (although some authors play on them in calibration studies [2]). The  $p_g$  parameters are usually supposed to be measured on the field, or obtained from design documents. They are therefore defined as reifiable quantities. The calibration procedure will therefore focus only on the  $p_h$  parameters. The advantage (and often necessity) of such an approach is to reduce the number of parameters to adjust. In case of difficulties to get a precise calibration over  $p_h$ , the examination of the  $p_g$  parameters can be done and can help to find an explanation (e.g.: geometry data mistakes). In some cases where the  $p_g$  parameters are difficult to be measured (e.g.: large flat natural rivers), part of them and  $p_h$  parameters can be calibrated altogether [7].

Depending on the type of model, size of the different vectors ( $u, y, x$ ), location and number of the measurements  $y_m$ , duration of the time horizon  $[T_1, T_2]$ , characteristics of the input  $u$  and outputs  $y$  (e.g.: magnitude and frequency) this procedure can be simple or very complex. In some cases, the minimum of the criteria  $J_{min}$  is nil, meaning that the model outputs fit perfectly the measurements. In most cases, this minimum is  $J_{min} > 0$ . This can be linked to poor measurements ( $u, y_m$ ), to bad assumption on the parameters that were supposed to be known (e.g.:  $p_g$ ) or to the model structure itself. In some cases, there exist one unique global minimum (e.g.: a unique roughness coefficient over the entire system obtained for one set of steady flow measurements). But, in most cases, several sets of parameters give the same minimum  $J_{min}$  (especially if we take into account the limited precision of the measurements).

## 2.2 Algorithms

### 2.2.1 Steady flow

Several calibration algorithms have been designed by model developers, and described in software documentations or Journal papers. For steady flow measurements, an iterative procedure based on the gradient method has been implemented into the SIC software and gives satisfactory results [4]. In this method, one unique roughness coefficient  $p$  (or a proportional ratio over the initial ones) can be calibrated for all cross sections located between 2 water level measurement points ( $y_{m \text{ up}}$  and  $y_{m \text{ down}}$ ) corresponding to one steady flow scenario ( $u = Q_{up}$  is also supposed to be know). In this case, the function  $g(p) = y - y_{m \text{ up}} = f(p) - y_{m \text{ up}}$  is a monotonous increasing function of the Manning coefficient  $p$ . If  $y_{m \text{ up}}$  belongs to  $[f(p_{min}), f(p_{max})]$ , there exist a unique solution  $p_{opt}$  such that  $J(p_{opt}) = 0$ . In other cases, the  $p_{opt}$  parameter is calculated so as to minimize the criteria  $J$ . This steady flow calibration problem has also been solved for slightly more complex situations with looped channel networks [6], using a projected augmented Lagrangian optimization method. We can also extent the calibration procedure in steady flow conditions using several different nominal discharges and sets of corresponding measurements. Using  $n$  sets of data ( $y_{m \text{ up}}, y_{m \text{ down}}, Q_{up}$ )<sub>1,...,n</sub> we can even try to calibrate  $n$  different parameters  $p_{1,...,n}$  for  $n$  different zones in the studied canal or river. We have indeed  $n$  equations and therefore we can try to search for  $n$  unknown variables. In this simple case, we can already wonder in which case the data sets will be "rich enough" so as to allow a correct determination of the  $n$  parameters  $p_{1,...,n}$ . This procedure has been tested using the SIC software, but is not detailed in this paper. We can guess that, if the  $n$  sets of data correspond to input conditions sufficiently different (i.e.  $|Q_{up \ i} - Q_{up \ j}|$  "large enough" for  $i, j \in [1, n], i \neq j$ ), the shape of the functional  $J$  (Figure 1a, for the simple case  $n = 2$ ) is such that a simplex Nelder-Mead method allows to find a (global?) optimum set of ( $p_{1,...,n}$ ) parameters (Figure 1.b) minimizing the criteria  $J$ . We will address this issue in more details for the unsteady flow case. In particular we will try to quantify the "rich enough" and "large enough" concepts used here.



**Figure 1:** (a) Plot of the quadratic criteria  $J$  as a function of the 2 parameters  $p_1 = K_1$  and  $p_2 = K_2$  (Strickler coefficients at the first half and second half of a 3km long canal). (b) Search path of a simplex Nelder-Mead algorithm finding a global minimum  $K_1 = 40$ ,  $K_2 = 60$  starting from an initial guess of  $K_1 = 30$ ,  $K_2 = 10$ ). Blue color corresponds to small values of  $J$  and red color to high values of  $J$ .

Steady flow calibration has several advantages, if steady flow conditions can be obtained on the system to be calibrated. Measurements can be taken by few people traveling along the system, in few minutes, hours or days depending on the size of the system. The algorithm used can be quite simple, especially if only one roughness coefficient has to be calibrated for every portion of the system having a measurement data set ( $y_{m\ up}$ ,  $y_{m\ down}$ ,  $Q_{up}$ ). Other algorithms are available for more complex situations, but we can reach observability limits as for unsteady flow approaches detailed in the following sections. Steady flow calibration has also many limitations. First, it is often difficult to have real steady state flow conditions over large natural river or irrigation canals. Natural rivers have inputs that cannot be controlled, and it is difficult to ask a canal manager not to move any gate during couple of days. Second, the corresponding calibration procedures do not use dynamical information (e.g.: time lags, oscillation modes) that we could think useful in order to get a good hydrodynamic model. As a consequence, even if we can calibrate very well a model in steady state flow conditions, we have no proof that the model will also have good transient representation of the real system. A classical procedure is therefore to calibrate a model in steady state, and to check if the model matches the real system for one or several unsteady flow conditions. In case it does not, what to do? Unsteady flow calibration may be an option!

### 2.2.2 Unsteady flow

Several authors have suggested and tested algorithms to solve this unsteady flow calibration problem. In the following chapters we will use the example described in [5]. In this paper the authors calibrate the roughness Manning coefficients at the 7 cross sections of a 20 km long river reach, using upstream and downstream water levels measurements. The input of the model is the upstream discharge. The reach has a longitudinal slope of 0.002, and a Manning coefficient increasing linearly from 0.02 for  $X = 0$  to 0.03 for  $X = 20$  km. The section is trapezoidal, with bank slope of 4, and bottom width of 30 m. The nominal discharge is not specified (we took 200 m<sup>3</sup>/s). The downstream boundary condition is not specified (we took a uniform flow rating curve condition for the tail Manning coefficient 0.03). The input scenario is not specified, except that its duration is 50 minutes.

## 3. SENSITIVITY ANALYSIS AND INPUT SCENARIO

When a calibration method claims to be able to calibrate  $n$  parameters  $p_{1,\dots,n}$  (e.g.: in our case  $n = 7$ ) altogether using a given data set (e.g.: in our case  $(y_{m\ up}, y_{m\ down}, Q_{up})_{[T_1, T_2]}$ ), it should also be able to calibrate only 1 of them ( $p_i$ ), if the  $n-1$  others are supposed to be already known. Before running any calibration program, we should first verify that the model outputs ( $y_{up}$ ,  $y_{down}$ ) are sensitive to this parameter  $p_i$  [11]. Classical approaches to do so are to make 2 simulation runs with 2 different values of  $p_i$  ( $p_i$  and  $p_i + \Delta p_i$ ) and see if the outputs are different. For those who want to stick to the GLUE method they can run millions of simulations [2] using Monte Carlo approach with randomly selected parameters in the interval  $[p_i, p_i + \Delta p_i]$ . We know

that, in most cases (unless the reach is long enough so that the uniform flow is attained, or the critical depth is attained, etc.), this sensitivity will do exist, and the problem of the calibration of  $p_i$  will be easy to solve, since it can even been done with a simple steady flow data set.

We can think of a slightly more complex case, when 2 parameters ( $p_i$  and  $p_j$ ) are unknown and therefore to be calibrated altogether. In this case, a single steady flow data set cannot allow solving the problem (2 unknowns for only 1 equation), and there exist an infinite number of solutions (or no solution). In our example from [5], the Strickler coefficients at section 4 and 6 with nominal values ( $K(4) = 40$ ,  $K(6) = 35.2939$ ) and alternative values ( $K(4) = 31.62891$ ,  $K(6) = 65.2939$ ) give exactly the same ( $y_{up}$ ,  $y_{down}$ ) output values for an input discharge  $Q = 200\text{m}^3/\text{s}$  at better than  $10^{-5}$  m precision. In this case, we can think of solving the problem with an unsteady flow data set (another option would have been with several steady flow data sets). The questions are then:

- Is it possible to detect from measurements data set  $(y_{m\ up}, y_{m\ down})_{[T_1, T_2]}$  the correct nominal combination of  $K(4)$  and  $K(6)$ ?
- Is it possible to quantify how easy this detection is (e.g.: in case we want to take into account the finite precision we have on the measurements)?
- What is the best scenario  $(Q_{up})_{[T_1, T_2]}$  that would maximize the sensitivity of  $(p_i, p_j)$  over the measurements data set?

Global sensitivity analysis methodologies with multi-parameters do exist (e.g.: Latin hypercubes, Morris, Sobol, Metamodel approaches, etc.). But, for transient models, the choice of the input scenario is very important and cannot be easily added (due to the number of additional variables) as other parameters in a sensitivity analysis. Since we want to focus on the influence of the input scenario (along with the sensitivity over  $p$ ) we will rather study the difference operator  $\Phi_d$  obtained as the difference between the two operators  $\Phi_1$  and  $\Phi_2$ :  $\Phi_d = \Phi_2 - \Phi_1$ , for a given  $\Delta p$  difference on the parameter vector  $p$ , with more precisely:

$$\begin{aligned} \Phi_1 : u(t) &\mapsto y_1(t) = f(u(t), x_1(t), p, t) \\ \Phi_2 : u(t) &\mapsto y_2(t) = f(u(t), x_2(t), p + \Delta p, t) \end{aligned} \quad (4)$$

In order to be able to use the powerful system analysis tools, we will approximate these operators by their LTI (Linear Time Invariant) discrete state-space form:

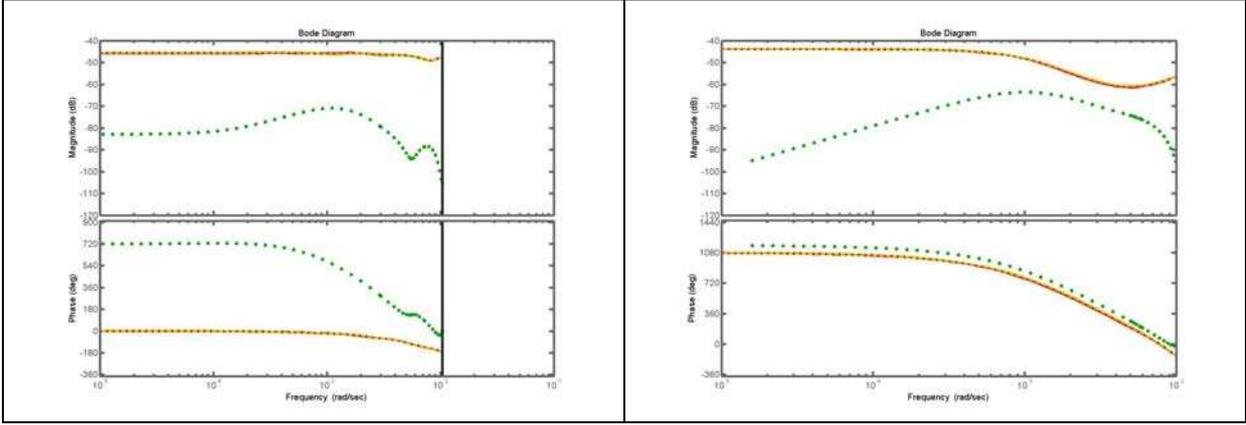
$$\begin{aligned} \Phi_1 : &\begin{cases} x_1^+ = A_1 x_1 + B_1 (u - u_{1_0}) \\ y_1 = y_{1_0} + C_1 x_1 + D_1 (u - u_{1_0}) \end{cases} \\ \Phi_2 : &\begin{cases} x_2^+ = A_2 x_2 + B_2 (u - u_{2_0}) \\ y_2 = y_{2_0} + C_2 x_2 + D_2 (u - u_{2_0}) \end{cases} \end{aligned} \quad (5)$$

with  $u_{i_0}$  the initial reference input vector at time 0,  $y_{i_0}$  the corresponding initial output vector for the operator  $\Phi_i$  ( $i=1,2$ ). In our simple above described example we have  $u_{1_0} = u_{2_0}$  and  $y_{1_0} = y_{2_0}$ .  $A_i$ ,  $B_i$ ,  $C_i$  and  $D_i$  are matrices of appropriate dimensions and depend (non-linearly) on the  $p$  parameter vector. These matrices are obtained from the linearization of the function  $f$  (Equation 1) around a nominal state  $x_{i_0}$ . At this stage, and in the following chapters, we suppose that the  $A_i$ ,  $B_i$ ,  $C_i$  and  $D_i$  are constant in time (LTI framework). This simplification is valid in our example, due to the simple geometry of the system. In addition, the discrepancy introduced by the non-linear effects that we neglect in doing this simplification, usually adds some damping which will emphasize our conclusions instead of invalidating them.

We can now study the sensitivity of the  $p$  parameter vector by studying the  $\Phi_d$  operator.

#### 4. FREQUENCY ANALYSIS (BODE PLOT)

We can study the frequency characteristic of the  $\Phi_d$  operator through the Bode plot (Figure 2). This plot shows the magnitude and phase of the output as a function of the frequency of a sinusoidal input of magnitude 1 ( $\text{m}^3/\text{s}$  in our case, since the input is the upstream discharge).



**Figure 2:** Bode plots of the model operators  $\Phi_1$  (red dotted),  $\Phi_2$  (yellow line) and  $\Phi_d$  (green dotted) for the outputs corresponding to the upstream water elevation (left figure) and downstream water elevation (right figure).

Location	$\Delta Z_{\text{static}}$ (dB)	$\Delta Z_{\text{static}}$ (mm)	$\Delta Z_{\text{max}}$ (dB)	$\Delta Z_{\text{max}}$ (mm)	Freq. for $\Delta Z_{\text{max}}$ (rad/s)	Ts for $\Delta Z_{\text{max}}$ (s)
Upstream	-82.91	0.07	-71.02	0.28	$1.11 \cdot 10^{-3}$	5646
Downstream	$-\infty$	0	-63.62	0.66	$1.00 \cdot 10^{-3}$	6283

**Table 1:** Key values of the Bode plot.

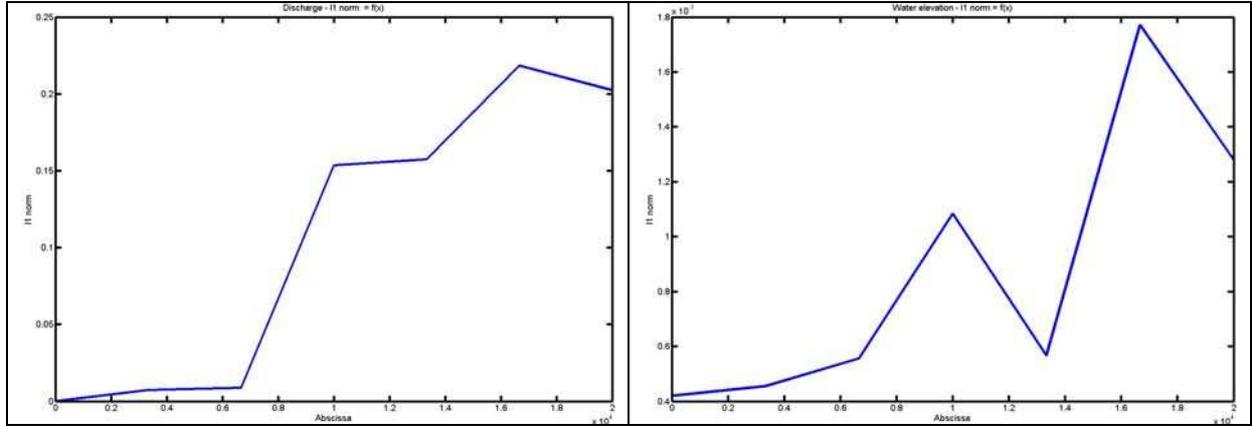
From the above analysis we can state that, in order to calibrate Stricklers coefficients at cross section 4 and 6 altogether, for a sinusoidal type input scenario, it would be optimal to select a frequency around  $F_{\text{opt\_up}} = 1.11 \cdot 10^{-3}$  rad/s (corresponding to a time period of 5646 s) as far as the upstream water level measurement is used for this calibration, and  $F_{\text{opt\_down}} = 1.00 \cdot 10^{-3}$  rad/s (corresponding to a time period of 6283 s) for the downstream water level measurement. Since we use both water level measurements we must better use an input scenario with sufficient energy in those frequencies. We checked that this frequency is linked to the position of Stricklers 4 and 6 but not to the change  $\Delta p$  we selected. We can also observe in Figure 2 and Table 1 that the maximum gain (-63.62 dB) is obtained for the downstream water level measurement at frequency  $F_{\text{opt\_down}}$ . As said previously, we do not know the scenario used by the authors of the paper [5]. But since it lasts for 3000 s, we can imagine its basic frequency can be  $\pi/(2 \cdot 3000) = 5.24 \cdot 10^{-4}$  (rad/s) (for a quarter of sinusoid cycle) to  $2\pi/3000 = 2.09 \cdot 10^{-3}$  (rad/s) (for a complete sinusoid cycle). If the authors have selected this input scenario by chance they have been lucky since these frequencies are close to the optimal ones. Another question can be what must be the magnitude of the input scenario. A gain of -63.62 dB corresponds to  $\Delta Z = 10^{-63.62/20} = 0.66$  mm of maximum deviation (of the downstream water level) between the 2 models for an optimum input scenario (sinusoidal of amplitude  $1 \text{ m}^3/\text{s}$ ) at frequency  $F_{\text{opt\_down}}$ . If the measurement precision is 1 cm, the amplitude of the input scenario should therefore be at least  $15 \text{ m}^3/\text{s}$  ( $0.01/0.00066$ ) to be able to detect a difference between  $\Phi_1$  and  $\Phi_2$ . This is correct if the neglected non-linear effects are not introducing additional damping, which has been verified. With non-linear damping, this amplitude will have to be further increased. With other choices on the input scenario, we can state that it will not be possible to calibrate the Strickler at section 4 and Strickler at section 6 at the same time with a better precision than the error  $\Delta p$  we introduced on the parameters ( $\Delta K(4) = 8.4$ ,  $\Delta K(6) = 30$ ).

## 5. TIME DOMAIN ANALYSIS ( $\ell_1$ NORM)

### 5.1 Calculation of the $\ell_1$ norm

The above study using the Bode plot is based on input scenarios being sinusoidal functions. We may wonder what would be the maximum deviations of the outputs of the operator  $\Phi_d$  for any input scenario, with no constraint on its shape nor frequency (its only constraint is that it is a bounded scenario). After normalizing  $\Phi_d$  we can write this constraint as  $\|\mathbf{u}\|_{\ell_\infty} \leq 1$ . If we look for the maximum deviations of the

outputs  $\|y\|_{\ell_\infty}$  under the previous constraint, we look by definition for the induced  $\ell_\infty - \ell_\infty$  norm of the  $\Phi_d$  operator. It can be proved easily that this induced norm is the  $\ell_1$  norm [3]. In Figure 3 we plot this norm for every cross section along the system, in both discharge and water elevation terms.



**Figure 3:**  $\ell_1$  norm of the difference model operator  $\Phi_d$  along the system. This norm is calculated for the outputs corresponding to discharges (left figure) and water elevations (right figure).

Location	$\ell_1$ norm on $\Delta Q$ (m <sup>3</sup> /s)	$\ell_1$ norm on $\Delta Z$ (mm)
Upstream	0	0.42
Downstream	0.202	1.28

**Table 2:** Key values of the  $\ell_1$  norm analysis.

We can verify that the sensitivity of the (K(4),K(6)) parameters is stronger towards the downstream water level rather than towards the upstream water level. We can also observe that the stronger sensitivity is obtained at section n°6. We can also observe that there exist a  $\ell_1$  worst-case scenario with better sensitivity than a sinusoidal one since the norm is 1.28 mm compared to 0.66 mm for  $\Delta Z_{dn}$  (and 0.42 mm compared to 0.28 mm for  $\Delta Z_{up}$ ).

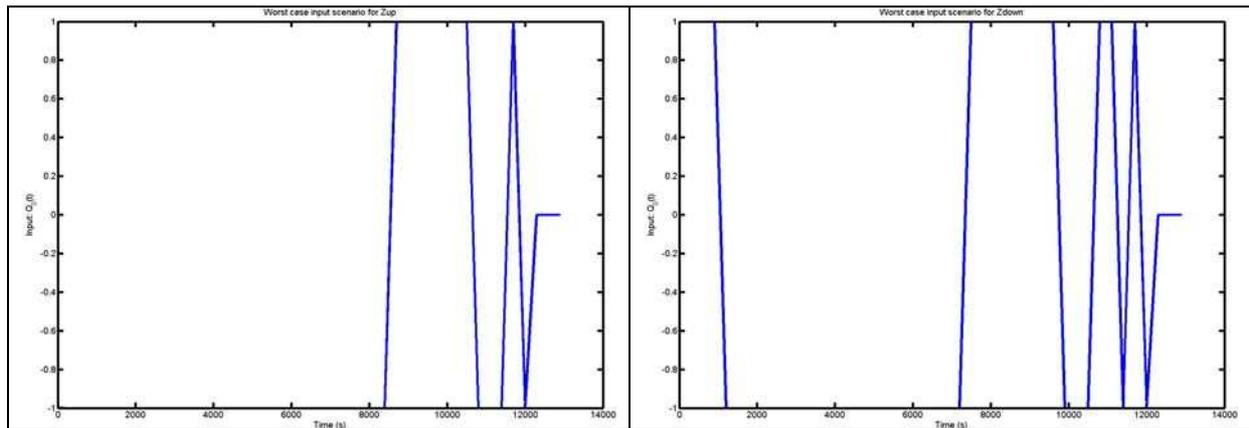
## 5.2 Calculation of the worst-case scenario

An interesting feature of the  $\ell_1$  norm is that it is easy to calculate its corresponding worst-case scenario [3], or more realistic  $\varepsilon$ -worst-case scenario [12]. When calculated for the  $\Phi_d$  operator this gives the input  $u = (Q_{up})_{[T_1, T_2]}$  (with  $\ell_\infty$  norm = 1) giving the maximum output  $y = (y_{up}, y_{down})_{[T_1, T_2]}$  (in terms of  $\ell_\infty$  norm). Due to the definition of  $\Phi_d = \Phi_2 - \Phi_1$  this gives the best input scenario allowing to discriminate between  $\Phi_1$  and  $\Phi_2$ , i.e. detecting the parameters vector offset  $\Delta p$  we introduced, from the measurement  $(y_{m up}, y_{m down})_{[T_1, T_2]}$  (Figure 4).

## 6. CONCLUSIONS

This paper does not present a new algorithm to calibrate roughness coefficients, neither in steady flow, nor in unsteady flow situations. Rather, this paper presents a methodology for the analysis of a system to be calibrated, based on system analysis and control theory. Two different complementary approaches have been presented: a frequency one based on the Bode plot, and a time domain one based on the  $\ell_1$  norm. The first one can give indications on the sensitivity of some parameters of the model, on the optimum magnitude and frequency of a sinusoidal input scenario that can be best selected for the calibration of any parameter of the model. The second one can give additional indications such as the best location for the measurements used for the calibration, the magnitude of an input scenario that can be selected for the calibration, or a worst-case

scenario that can be more complex than a sinusoidal one with a better discriminating effect for a same maximum magnitude. It can be seen as a way to quantify the equifinality concept. In addition, this latter methodology is generic for multivariable systems, in case the input scenario can be applied on several inputs  $u$  (e.g.: upstream discharge, lateral inflow and/or gate positions along the system). In case a desired input scenario cannot be imposed on a real system, the methodology can also evaluate the quality of an input scenario, from a magnitude and frequency analysis. Many applications of this methodology can be foreseen beyond the simple example used in this paper: evaluation of the influence of numerical parameters of numerical schemes, importance of the different terms of the Saint-Venant equations (e.g. inertia terms), dynamical behavior as a function of the nominal state, etc.



**Figure 4:** Worst-case input scenario  $Q_0(t)$  for the  $\ell_1$  norm of the difference model operator  $\Phi_d$ . This scenario is calculated for the outputs corresponding to the upstream water elevation (left figure) and downstream water elevation (right figure).

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