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Qualitative analysis of a BDMP by Finite Automaton

Pierre-Yves Chaux 1,2 & Jean-Marc Roussel 1 & Jean-Jacques Lesage 1 & Gilles Deleuze 2 & Marc Bouissou 2

1 LURPA - 61, av. du président Wilson 94235 Cachan cedex, France
{pierre-yves.chaux, jean-jacques.lesage, jean-marc.roussel}@lurpa.ens-cachan.fr

2 EDF R&D - 1, av. du Général de Gaulle 92140 Clamart, France
{pierre-yves.chaux, marc.bouissou, gilles.deleuze}@edf.fr

ABSTRACT: The Boolean Driven Markov Processes (BDMPs) were developed by EDF to conduct predictive risk modelling and assessment on critical systems. A BDMP model is a description of the combinations of failures that makes a system to fail like it is done with Fault tree models. The calculation of all the sequences of events that conduct to the global failure is complex as they are implicitly represented by a BDMP that generates a large state space. This paper provides a semantic analysis of BDMP models conducted within the languages and finite automaton theories in order to develop a systematic way to obtain an extensive representation of all scenarios contained in a BDMP. This representation is obtained by using an algorithm given in this paper which constructs an equivalent finite automaton of a BDMP. This FA representation leads to the possibility of extracting all the minimal scenarios of failure implicitly described by a BDMP.

1 INTRODUCTION

Many studies which have been carried out on predictive risk modelling and assessment target two complementary objectives (Henley and Kumamoto 1981). On the one hand, quantitative analyses aim at calculating the failure rate of the modelled system or of one of its subsystems. On the other hand the qualitative analyses aim at determining the scenarios which lead to the failure of the whole system. While those scenarios are in most cases only constituted by basic components failures, they may also include repairs of these components.

The Boolean Driven Markov Processes (BDMP) were created to include all the Electricité De France expertise in the construction and analysis of reliability models. While staying close to Static Fault Tree models (Stamatelatos, Vesely, Dugan, Fragola, Minarick, and Railsback 2002), BDMPs extend the fault tree capacities by allowing to model both the failures and repairs of basic components. This extension also enables the description of the redundancies between components and between complex sub-systems constituted by a number of components, which can be redundant one from another.

The main goal of this study is to conduct a qualitative analysis of a BDMP while setting aside all its capacities to model and conduct quantitative analysis on the reliability of a system. The qualitative analysis consists in enumerating all the sequences of repair and failure events that are implicitly described by a BDMP. To explicitly describe those combinations, this study is conducted within the languages and finite automata (FA) theories. For that, each scenario of failures and repairs is translated into a sequence of events. The set constituted by those sequences (or words) is a language.

Because the language generated by a BDMP is regular, it can be represented by a FA. This paper describes an algorithm which is used to construct the FA "equivalent" to a BDMP in the way that this FA generates the same language as the one which is implicitly described in the BDMP. The qualitative studies can now be conducted on the FA rather than on the BDMP itself. This allows us to gain all the benefits of handling a formal model on which many results were published.

The section 2 will present a short example on the safety modelling of a system using the BDMP in order to describe its modelling capacities. In section 3, we will describe how the scenarios that are implicitly described by a BDMP can represented by the generated language of a finite automaton. After proposing a semantical analysis of the BDMP model in section 4,
Section 5 will describe the generation mechanism of this automaton.

2 MODELLING A SYSTEM SAFETY USING A BDMP

In order to illustrate our works and describe how the scenarios conducting to the global failure are described in a BDMP model, the example of Figure 1 has been chosen. The main function of the system is to provide the power supply to the components which support the cooling and the control functions of a nuclear reactor core. The distribution board DBA1 is one of the two distribution boards that achieve this function. The modelling of the failure scenarios of the system combines several difficulties that are present in most reliability assessment studies.

- All the components can be repaired.
- Two cold redundancies have to be modelled, one between the two possible supplies for the DBA1 board (DBB1 and the diesel generator) and another between the supplies of the DBB1 board (electric transformers 1 and 2). If DBB1 can’t be supplied by transformer Tr1 then transformer Tr2 is supplying DBB1. If DBA1 can’t be supplied by DBB1 through the circuit breaker CB1 then the diesel generator is started and the circuit breaker CB2 is closed.
- As most components can only fail during their operation (active mode), the diesel generator can also fail while in stand-by (dormant mode).

BDMPs were developed by M. Bouissou to offer to the engineers a formalism to model complex failure mechanisms while using a user-friendly environment (Bouissou and Bon 2003). Based on the Fault Tree model (Stamatelatos, Vesely, Dugan, Fragola, Minarick, and Railsback 2002), the BDMP offer the same capacities to model redundancies as the Spare gates from the Dynamic Fault tree (Dugan, Bavuso, and Boyd 1992). But it also expend their capacities:

- For the modelling of the redundancies between complex sub-systems (not only between components);
- For the modelling of the repair events of components where the DFT are modelling non-repairable components only, even if an extension of DFT for the modelling of repair procedures can be found in (Bobbio and Raiteri 2004).

The BDMP shown in Figure 2 models the scenarios of failure and repairs events of the power supply system as:

- The Top event (TE) at the root of the tree represents the global failure of the system (main function is faulty: the LHA distribution board is powering the safety components of the core);
- the Fault Tree structure describes the static combinations of components faults that make the whole system to fail by using AND and OR logical gates;
- the triggers describe the redundancy mechanisms: A trigger is a link between the failure state of a subsystem and the mode of another. On the example, the trigger between the transformers Tr1 and Tr2 indicates that if the transformer Tr1 (which is the primary component) fails then the transformer Tr2 (the secondary system at the destination of the trigger) is activated. When the transformer Tr1 is repaired, the system returns into its initial configuration where the function is provided by Tr1.

The system is faulty when LHA is faulty OR when LHA is not powered. The power supply of LHA failed if LGD AND the power supply from the diesel generator failed.
3 QUALITATIVE ANALYSIS OF A BDMP

The qualitative analysis of a BDMP is mostly like those done on Dynamic Fault Trees (DFT). On both cases, the goal is to extract sequences of failure and repair events that lead to the whole system faillure, those sequences are called Cut sequences. This extraction is to be done on models that implicitly define sequential relations between the occurrences of failure events (and repair events for BDMP models). Many methods for the quantitative analysis of DFT have been developed and published. Most of these methods are translating the DFT model into formalisms which can be more or less effective to conduct this kind of analysis.

As (Dugan, Bavuso, and Boyd 1992), (Coppit, Sullivan, and Dugan 2000) are developing translations of a DFT into Markov processes, (Boudali and Dugan 2005) presented a translation into continuous time Bayesian networks, (Bobbio and Raiteri 2004) one into Well Formed Petri Nets and more recently one into Interactive I/O Markov chains (Boudali, Crouzen, and Stoelinga 2007). The main goal of those translations is to conduct quantitative analysis. Among those, only the one using Well Formed Petri Nets seems to be able to support qualitative studies but they are making the generation of the marking graph too complex to extract the cut sequences.

Specifically designed to allow the qualitative studies of a DFT, (Tang and Dugan 2004) developed a method based on the separation between an equivalent static fault tree and the sequential constrains generated by the dynamic gates of a DFT. The cut sets of the equivalent static fault tree are calculated using Zero-suppressed Boolean decision diagrams then the sequential constrains (which represent the dynamic behavior of a DFT) are applied on the cuts sets to obtain a set of cut sequences. This method, which is not fully automated, return a superset of the cut sequences of a DFT, then it is an approximation even if the results are conservatives.

(Merle, Roussel, and Lesage 2011) recently developed a fully algebraical approach to conduct both qualitative and quantitative analyses on the DFT model. this method is based on the preliminary calculation of the structural function. This calculation, which is fully algebraical, is then used to determine a canonical form of the structural function, from it, the cut sequences are automatically extracted.

All those works are conducted under the hypothesis that basic components cannot be repaired. Because a BDMP models both the failure and repair mechanisms, a specific translation method is to be developed. To do that, the scenarios of failure and repair events implicitly described by a BDMP will be seen as sequences from a language as this language can be represented by a finite automaton.

All scenarios implicitly described by a BDMP are constituted by the failure and repair events of the leaves and can be represented by a language \( \mathcal{L} \) constituted by word on an alphabet \( \Sigma \). The \( \Sigma \) alphabet is the union of the failure alphabet \( \Sigma_f \) and the repair alphabet \( \Sigma_r \). Then \( \mathcal{L} \) contains all the failure and repair sequences described by a BDMP. As all those sequences are described by the BDMP only a subset of those actually contribute to the global failure of the system (the main function of the system is lost) while the others are non-critical (the main function of the system is still up).

Theorem 1. The language \( \mathcal{L} \) containing all scenarios described by a BDMP is regular.

Proof. Let \( \mathcal{L}_1 = \{ u | u \in \Sigma \} \) be the language describing the BDMP scenarios after only one occurrence of an event. As \( \mathcal{L}_1 \) is a union of singletons, by definition \( \mathcal{L}_1 \) is regular. Let \( \mathcal{L}_n = \{ u | |u| \leq n, u = va \} \) with \( |v| \leq (n - 1), a \in \Sigma \).

Suppose that \( \mathcal{L}_n \) is regular: \( \mathcal{L}_{n+1} = \{ u | |u| \leq n, u = va \} \) with \( v \in \mathcal{L}_n, a \in \Sigma \). \( \mathcal{L}_{n+1} \) is regular because it is constituted by the union of words of \( \mathcal{L}_n \) with words which are concatenations between words of \( \mathcal{L}_n \) and words of \( \mathcal{L}_1 \).

Theorem 1 makes the usage of the Kleene theorem possible: as \( \mathcal{L} \) is regular, there is a finite automaton (FA) generating this language.

4 SEMANTIC ANALYSIS OF A BDMP

A BDMP can be defined by a 4-tuple

\[ < L, G, T, te > \] where:

\[ L = \{ L_i \} \] is the set of leaves. Those can be of two type: the \( F \) leaves which allow the description of a failure which can only occur in an active mode are in \( L_F \) while \( SF \) leaves allow the description of a failures which can occur in both active and dormant mode are in the \( L_{SF} \) set. The \( L \) set can then be decomposed as \( L = L_F \cup L_{SF} \).

\[ G = \{ G_i \} \] is the set of static gates. That can be of kinds \( OR \) or \( AND \) respectively placed in \( G_{OR} \) and \( G_{AND} \) (\( G = G_{OR} \cup G_{AND} \)).

Using the \( L \) and \( G \) sets, the node set of a BDMP is defined as \( N = L \cup G \) the union of leaves and gates. An extended node \( N^* \) set is also defined by adding the Top Event \( te \) to the node set \( N \) (\( N^* = N \cup \{ te \} \)). The state of a node is defined using a 2-tuple of booleans \( \{ F_{N_i}, M_{N_i} \} \) where \( F_{N_i} \) represents the failure of the node \( (True \) if \( N_i \) is faulty) and \( M_{N_i} \) represents the mode of the node \( (True \) if in an active mode, \( False \) if in a dormant mode).

Each gate \( G_i \) possesses at least two inputs which are defined by \( Inputs(G_i) \subseteq N \). A function
function is used to calculate the set of gate that are using a node $N_i$ as an input: $UpStream(N_i) = \{ G_j \in (G \cup \{te\}) | N_i \in Inputs(G_j) \}$

$T = \{T_i\}$ is the set of the triggers of a BDMP. A trigger $T_i$ has an origin which is denoted $Orig(T_i)$ and a destination denoted $Dest(T_i)$. As both of these elements are node from $N$, a trigger $T_i$ is defined by a 2-tuple $(Orig(T_i), Dest(T_i)) \in N^2$. To calculate the mode of each node, a function $UpTrigger(N_i) \subseteq N$ is also needed, this functions returns the set of node that are at the origin of the triggers that have $N_i$ as their destination: $UpTrigger(N_i) = \{ Orig(T_j) \in N | T_j \in T, Dest(T_j) = N_i \}$

The Top event $te$ is the representation of the failure of the modelled system, it has only one input and no outputs ($Inputs(te) \subseteq N, Card(Inputs(te)) = 1$).

On the example of figure 2:

- $L = \{ L_{DBA}, L_{CB}, L_{DBB}, L_{Tr}, L_{CB}, L_{Diesel} \}$
- $G = \{ G_{Fail_{DBA}}, G_{Fail_{PS_{DBB}}}, G_{Fail_{PS_{Diesel}}}, G_{Fail_{DBB}} \}$
- $T = \{ T_1, T_2 \}$

$T_1 = (G_{Fail_{PS_{DBB}}}, G_{Fail_{PS_{Diesel}}})$

$T_2 = (L_{Tr}, L_{Tr})$

While the section 4.1 is describing how the fault of each node is calculated in the fault tree, the section 4.2 will describe the calculation of the mode and the section 4.3 will describe the behavior of the leaves.

4.1 Logical description of the fault tree structure

The role of the static fault tree structure, which is constructed using the leaves as basic components, is to define the relation between $F_{te}$ the global failure boolean and the $F_{Li}$ the failures of the leaves. This relation is defined using AND and OR gates. To calculate this relation all the failure of the gates must be known as the failure of the leaves are result of their behavior.

For each gate $G_i \in G$ the failure $F_{Gi}$ can be calculated knowing the gate type :

- For an OR gate, $F_{Gi} = \bigvee_{N_j \in Inputs(G_i)} F_{N_j}$
- For an AND gate, $F_{Gi} = \bigwedge_{N_j \in Inputs(G_i)} F_{N_j}$

As the fault tree structure is acyclic, the failure booleans can be calculated at each level, going from the leaves to the root level. The failure boolean at the root level $F_{te}$ is representing the top event of the tree.

On the example of the figure 2:

$F_{te} = F_{DBA} \lor F_{DBB} \lor (F_{CB} \lor F_{DBB1} \lor (F_{Tr} \land F_{Tr2}) \land (F_{CB2} \lor F_{Diesel})$
• $M_{Tr2} = F_{T1}$
• $M_{CB2} = (F_{CB1} \lor F_{DBB1} \lor (F_{Tr1} \land F_{Tr2}))$
• $M_{Diesel} = (F_{CB1} \lor F_{DBB1} \lor (F_{Tr1} \land F_{Tr2}))$

4.3 Event based description of a leaf

To define the mechanism described by leaf $L_i$ of a BDMP, the leaf is seen in this study as a automaton which can evolve using two kind of events. On the one hand the events used to describe the failures and repair mechanism are in the alphabet $\Sigma_f \cup \Sigma_r$, on the other hand those used for switching mode are in the alphabet $\Sigma_m$. The failure and repair events are generated by the leaf when the component they represent sustains a fault or a repair while the mode switching events are generated by the BDMP structure. The automaton representing the generic behavior for both kind of leaves is shown in figure 3.

$$F_i = \text{False} \quad F_i = \text{True}$$

![figure 3: Generic behavior of a leaf $L_i$](image)

Four states can be identified from the operation state of a component (faulty, non-faulty) and its mode (active, dormant). The initial state is chosen between $Q_{00}$ and $Q_{10}$ as at the initialization of the model all leaves are non-faulty. $Q_{00}$ is the initial state if the leaf is initially in a dormant mode and $Q_{10}$ is the initial state if the leaf is initially in an active mode. Some evolutions between those four states are generated by the events of failures and repairs occurrences, where:

- $f_{id}$ is the failure event while in a dormant mode.
- $f_{ia}$ if the failure event while in an active mode.
- $r_i$ is the repair event of the leaf which can occur during active and dormant mode.

All the other evolution are conducted by the mode switching events which are:

- $a_i$ if the event that make the leaf swich from a dormant mode to an active mode.
- $d_i$ is the event that make the leaf swich from an active mode to a dormant mode.

Another way to describe a state of a leaf $i$ is to associate a tuple of two Boolean values $(M_i, F_i)$ . $M_i$ represents the mode of the leaf ($T_rue$ if the leaf is in its active mode) while $F_i$ is set to $T_rue$ when the leaf is faulty. While describing the behavior of a SF leaf, the automaton of figure 3 is able to represent the behavior of a F leaf by removing the transition from $Q_{00}$ to $Q_{01}$ on the occurrence of event $f_{di}$.

5 GENERATING THE "EQUIVALENT" AUTOMATON OF A BDMP

5.1 The Finite State Automata formalism used to describe a BDMP

A Finite Automaton (FA) is defined by the tuple $< \Sigma, Q, q_0, Q_M, \delta >$ where:

- $\Sigma$ is the entry alphabet of the automaton,
- $Q$ is the state set of the automaton,
- $q_0$ is the initial state,
- $Q_M$ is the set of marked states of the automaton ($Q_M \in Q$),
- $\delta$ is the transition set:

$$\delta = \{ < q_i, u, q_j > | (q_i, q_j) \in Q^2, u \in \Sigma \}$$

The entry alphabet $\Sigma$ is the set containing all the failure events ($f_{ia}, f_{id}$) and all the repair events ($r_i$) from the leaves $L_i \in L$ of the BDMP.

Each state $q \in Q$ is a state of the automaton which is associated with a state of the BDMP. A BDMP state is defined by the vector $F_q$, containing all the fault level from the leaves. As the maximum number of BDMP states is $n = 2^{\text{Card}(L)}$ where $L$ is the leaf set. The maximum number of states in the automaton is also limited by $n$.

When a BDMP is initialized all its leaves are non-faulty, this BDMP state will be associated with a state of the automaton $q_0$. The marked state set $Q_M$ is chosen to represent the set of BDMP states where the system has failed. This choice allows to calculate the sequences of failure defined by a BDMP as the marked language of the automaton.

The transition function $\delta$ represents the evolutions between two states of the automaton on the occurrences of an event from the alphabet $\Sigma$.

As $\Sigma$ and $q_0$ are easily deducted from a given BDMP, $Q, Q_M$ and $\delta$ must be calculated. In order to automatically generate the FA which represents the BDMP scenarios, a specific algorithm is developed in the following sections of this paper.
5.2 The generation mechanism

The FA generation mostly consists in generating the state set \( Q_0 \), the marked state set \( Q_M \) and the transition function \( \delta \). This generation starts by using the initial state \( q_0 \) and then exploring the BDMP states to generate the automaton states. This is done by Algorithm 1.

Algorithm 1 Generate states and transitions of the "equivalent" FSA of a BDMP

Require: \( \Sigma = \Sigma_r \cup \Sigma_r \cup \Sigma_r \) : Symbol alphabet, \( L = \{ L_i \} \): leaf dictionary (contain leaf type), \( \{ M_{Li} \} \) Boolean mode function of leaf \( L_i \), \( \Delta \): Top event boolean function, \( \Delta : (SF, \Sigma) \to SF \): function representing the event effects on a failure vector.

1: \# Generating initial state
2: \( F_{q_0} \leftarrow [F_{Li} = Falalse | L_i \in L] \)
3: \( SF \leftarrow F_{q_0} \)
4: \( Q \leftarrow q_0 \)
5: \# Step 1: Evaluate \( M_{L_i} \)
6: \# Step 2: generate sensible events \( SE \) in state \( q_n \)
7: \( SE = \text{Sensible_events}(L, F_{q_n}, M_{q_n}) \)
8: \# Step 3: Generate next state \( q_n \)
9: \# Generating next state \( q_n \)
10: \( q_n \leftarrow \Delta(F_{q_n}, u) \)
11: \( \text{if } F_{q_n} \notin SF \text{ then} \)
12: \( \text{if } F_{q_n} \notin SF \text{ then} \)
13: \text{if } F_{q_n} \notin SF \text{ then} \)
14: \text{if } F_{q_n} \notin SF \text{ then} \)
15: \text{if } F_{q_n} \notin SF \text{ then} \)
16: \text{if } F_{q_n} \notin SF \text{ then} \)
17: \text{if } F_{q_n} \notin SF \text{ then} \)
18: \text{if } F_{q_n} \notin SF \text{ then} \)
19: \text{if } F_{q_n} \notin SF \text{ then} \)
20: \text{if } F_{q_n} \notin SF \text{ then} \)
21: \text{if } F_{q_n} \notin SF \text{ then} \)
22: \text{if } F_{q_n} \notin SF \text{ then} \)
23: \text{return} \( \Sigma, Q, q_0, Q_M, \delta \)

A BDMP state is completely defined by the fault states (faulty, non-faulty) of all its leaves and is associated with an automaton state \( q_i \in Q \). It will be represented by a Boolean vector \( F_{q_i} \) of length \( Card(L) \). Each component of this vector is the value of the fault Boolean \( F_{L_i} \) of each leaf \( L_i \in L \). All those BDMP states are placed in the set \( SF \) representing the state set of a BDMP. The faulty states of a BDMP \((TE = True)\) are associated with marked states of the automaton, those automaton states are placed in the marked state set \( Q_M \in Q \).

In order to apply the evolution associated with the occurrence of an event from \( \Sigma \) on a BDMP state, a \( \Delta : (SF, \Sigma) \to SF \) function need to be defined. \( \Delta \) is a representation of the effect of a failure or repair event on the leaves according the the automaton representation of a leaf proposed in section 4.3.

A mode vector \( M_{q_i} \) is also defined. This Boolean vector contains all the mode \( M_{Li} \) from the leaves \( L_i \in L \) and is associated with a automaton state \( q_i \in Q \). This vector \( M_{q_i} \) is obtained by updating the mode of each leaf using the boolean function associated with each \( M_{Li} \) and the fault values in the vector \( F_{q_i} \) associated with \( q_i \).

Algorithm 2 Sensible_events \((L, F_{q_i}, M_{q_i})\)

Require: \( L = \{ L_i \} \): leaf dictionary (contain leaf type), \( F = [F_{Li}] \) failure vector, \( M = [M_{Li}] \) mode vector.

1: \# Generating initial state
2: \# Step 1: Evaluate \( M_{L_i} \)
3: \text{if } F_{Li} \in L do \text{ then} \)
4: \text{if } F_{Li} \in L do \text{ then} \)
5: \text{if } F_{Li} \in L do \text{ then} \)
6: \text{if } F_{Li} \in L do \text{ then} \)
7: \text{if } F_{Li} \in L do \text{ then} \)
8: \text{if } F_{Li} \in L do \text{ then} \)
9: \text{if } F_{Li} \in L do \text{ then} \)
10: \text{if } F_{Li} \in L do \text{ then} \)
11: \text{if } F_{Li} \in L do \text{ then} \)
12: \text{if } F_{Li} \in L do \text{ then} \)
13: \text{if } F_{Li} \in L do \text{ then} \)
14: \text{if } F_{Li} \in L do \text{ then} \)
15: \text{if } F_{Li} \in L do \text{ then} \)
16: \text{if } F_{Li} \in L do \text{ then} \)
17: \text{if } F_{Li} \in L do \text{ then} \)
18: \text{if } F_{Li} \in L do \text{ then} \)
19: \text{if } F_{Li} \in L do \text{ then} \)
20: \text{if } F_{Li} \in L do \text{ then} \)
21: \text{if } F_{Li} \in L do \text{ then} \)
22: \text{if } F_{Li} \in L do \text{ then} \)
23: \text{if } F_{Li} \in L do \text{ then} \)

After the generation of initial state \( q_0 \) associated with the BDMP state \( F_{q_0} \) where all fault Boolean are \( False \) (lines 1-4), the algorithm generates the next states from the current state \( q_i \in Q \) using the "FOR" loop between lines 5 and 22. The first step is to update the mode vector \( M_{q_i} \) associated with current state (line 7). As both the failure \( F_{q_i} \) and mode vectors \( (F_{q_i}) \) are known, all the current states of the leaves are known. In order to calculate \( SE \), the \( \Sigma \) subset that contains the events the BDMP is sensible to, the algorithm 2 is used in line 9. This second algorithm generate \( SE \) by seeking the events each leaf is sensible to according to their automaton model shown in section 4.3. When the \( SE \) set is known the "FOR" loop between lines 11 and 21 calculate for each tuple \( (q_i, u \in SE) \) the next state \( q_n \). Using the \( \Delta \) function in the line 12 of the algorithm 1, the next BDMP state \( F_{q_n} \) is calculated. In order to know if \( q_n \) is already present in \( Q \) (line 13), this new BDMP state is compared to the existing ones which are present in the \( SF \) set. If \( q_n \) is a new automaton state then \( q_n \) is placed in \( Q \) (line 15) and \( F_{q_n} \) in \( SF \) (line 14). By updating the top event function \( F_{te} \) at the new state, the global fault state is known. If \( q_n \) is a state where the top event function is \( True \) (the system is faulty) then \( q_n \) is also placed in the marked state set \( Q_m \) (lines 16-18). Even if \( q_n \) was already known it is necessary to place the transition \( (q_i, u, q_n) > \) in the transition function \( \delta \) (line 20). When the algorithm has completed the generation, the five elements of the tuple describing this automaton are returned. The convergence of
the algorithm is satisfied by a finite maximum number of explored states ($2^{\text{Card}(L)}$).

### 5.3 The "equivalent" FSA of the studied example

In order to generate the "equivalent" automaton of a BDMP several elements must be known. The $L = \{L_i\}$ set of the leaves of the BDMP presented in figure 2, each leaf must also be associated with its type, depending on its behavior:

<table>
<thead>
<tr>
<th>Leaf</th>
<th>Type</th>
<th>associated events</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_{DBA1}$</td>
<td>$F$</td>
<td>$f_{0a}, r_0$</td>
</tr>
<tr>
<td>$L_{CB1}$</td>
<td>$F$</td>
<td>$f_{1a}, r_1$</td>
</tr>
<tr>
<td>$L_{DBB1}$</td>
<td>$F$</td>
<td>$f_{2a}, r_2$</td>
</tr>
<tr>
<td>$L_{Tr1}$</td>
<td>$F$</td>
<td>$f_{3a}, r_3$</td>
</tr>
<tr>
<td>$L_{Tr2}$</td>
<td>$F$</td>
<td>$f_{4a}, r_4$</td>
</tr>
<tr>
<td>$L_{CB2}$</td>
<td>$F$</td>
<td>$f_{5a}, r_5$</td>
</tr>
<tr>
<td>$L_{Diesel}$</td>
<td>$SF$</td>
<td>$f_{6d}, f_{6a}, r_6$</td>
</tr>
</tbody>
</table>

From these informations extracted from the BDMP, the algorithm 1 generates the "equivalent" automaton. This FA is composed by 128 states and 852 transitions. Among those states, 103 are marked and represents all the situations where the whole system failed. As the number of state is exactly equal to $2^n$ where $n = \text{Card}(L) = 7$ is the number of leaves in the BDMP, all possible situations are reachable from the initial state.

A formal verification of the semantical consistency between the FA and the BDMP is not possible but some simple characteristics of the BDMP behavior can be easily verified on the FA. Figure 4 shows the initial state and the first next states of the equivalent FA. From the initial state only 4 components can fail, those are the DBA1 board, the CB1 circuit breaker, the DBB1 board and the Tr1 transformer in their active mode and the diesel generator in its dormant mode, moreover all repairs can only occur after the associated component failed.

Figure 5: The first states of the 'equivalent' FA of the studied system

Figure 5: FA extracted from the equivalent FA of the studied example

The first usage of this equivalent automaton is to extract the minimal cut sequence of the BDMP model. Another usage can be to assist the choice of a maintenance strategy. To illustrate this usage, an extracted FA from the equivalent one is used. This FA, shown in figure 5, associates the number of failures which are critical for the system with the failure state of the transformer Tr1 ($f_{3a}, r_3$ events) and the diesel generator ($f_{6d}, r_6$ events). Considering the scenario where both transformer Tr1 and the diesel generator failed, the current state after both failures is the state denoted D in figure 5. In this state, where 4 components are critical for the system, two repair events can occur. As the repair of the transformer Tr1 is setting the system in the C state where there are still 3 critical components, the repair of the diesel generator is setting the system in the B state where only one component is critical. So if there is only one maintenance team available, the priority must always be given to the repair of the diesel generator even if the team already stated the corrective maintenance on the transformer Tr1.

### 6 CONCLUSIONS

In this paper a method to generate automatically the equivalent finite automaton to a BDMP was described. This equivalent automaton generates a language that contains all the possible scenarios of failure and repair events that are implicitly described by a BDMP. In order to do so, a complete study of the semantic of a BDMP was necessary.

This FA makes many kinds of qualitative analysis possible on the BDMP model. This paper presents one example of those by giving a way to elaborate
a corrective maintenance strategy between two components by looking which repair makes the system in a more secure state.

Our current work focuses on:

- giving the minimality criteria needed to define what are the cut sequences on the BDMP model. This definition is particularly important to conduct quantitative studies as they were formally defined in the literature.

- formalizing the leaves that describe a failure mechanism that can only append where the leaf is switching mode. This kind of leaf (that was ignored in this paper) are allowing the BDMP model to describe much more complex failure mechanisms.

- developing and testing the usage of techniques adapted to handle large state spaces. The model-checking techniques may be used to obtain the cut sequences from a BDMP with more leaves.

REFERENCES


