Meaningful Thickness Detection on Polygonal Curve
Bertrand Kerautret, Jacques-Olivier Lachaud, Mouhammad Said

To cite this version:

HAL Id: hal-00780710
https://hal.archives-ouvertes.fr/hal-00780710
Submitted on 24 Jan 2013

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers. L’archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d’enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.
Meaningful Thickness Detection on Polygonal Curve

Bertrand Kerautret\textsuperscript{1,2}, Jacques-Olivier Lachaud \textsuperscript{2} and Mouhammed Said \textsuperscript{2}
\textsuperscript{1} LORIA (UMR CNRS 7503) Nancy University, France
\textsuperscript{2} LAMA (UMR CNRS 5127), University of Savoie, France
\textsuperscript{3} LIRIS (UMR CNRS 5205), University of Lyon 2, France
kerautre@loria.fr, Jacques-Olivier.Lachaud@univ-savoie.fr, Mouhammad.Said@univ-lyon2.fr

Keywords: shape analysis, noise detection, meaningful scale, contour representation

Abstract: The notion of meaningful scale was recently introduced to detect the amount of noise present along a digital contour. It relies on the asymptotic properties of the maximal digital straight segment primitive. Even though very useful, the method is restricted to digital contour data and is not able to process other types of geometric data like disconnected set of points. In this work, we propose a solution to overcome this limitation. It exploits another primitive called the Blurred Segment which controls the straight segment recognition precision of disconnected sets of points. The resulting noise detection provides precise results and is also simpler to implement. A first application of contour smoothing demonstrates the efficiency of the proposed method. The algorithms can also be tested online.

1 Introduction

Detecting if a contour is sampled at a meaningful scale and estimate what are the correct scales to analyse it (if they exist) is an important issue in shape analysis. For instance, it makes easier the automated parameterization in geometric shape analysis, contour representation or pattern recognition. In general, the noise is taken into account by a supervised parameter chosen according to the input data quality. The choice of the parameter is largely influential on the quality of the process. For instance in a deformable boundary segmentation technique, the smoothing parameter has a great impact on the result.

Following a principle of perception from the Gestalt theory, Desolneux et al. propose to detect the meaningful edges of a grey level image by using false alarm probability defined on the iso contours of the image (Desolneux et al., 2001). This detection is based on the image gradient and is not directly defined for discrete contour representations. Along the same lines, Cao introduces the notion of meaningful good continuation (Cao, 2003) which relies on geometric properties of the given curve. The false alarm probability was simply approximated by a curvature estimation of the input curve. Other applications of Gestalt theory can be found in a recent article (Desolneux, 2011).

The concept of meaningful scale along digital contour (i.e. boundary of a digital region) has been introduced in a recent work to detect automatically if the current scale is locally significant or not (Kerautret and Lachaud, 2009b). It relies on the study of asymptotic properties of maximal digital straight segment which is a classic tool used to extract geometric parameter on a discrete contour (Lachaud et al., 2005). More precisely the length of discrete maximal segment is analyzed on the contour represented at different scales to determine locally if a point is mean-
Blurred segments were introduced to address noisy data (Debled-Rennesson et al., 2006). We use the following definition (Faure et al., 2009): a set of points is an \( \alpha \)-thick Blurred Segment if and only if its convex hull has an isothetic thickness less than a given real number \( \alpha \). The isothetic thickness of a convex hull is the smallest value between its vertical and its horizontal width (denoted respectively as HW and VW on the upper floating figure). In the same way as previously a maximal \( \alpha \)-thick Blurred Segment can be defined as a segment which can not be extended to the front or to the back.

An illustration is given on Fig. 2 (b). An \( \alpha \)-thick Blurred Segment with \( \alpha = 1 \) is recognized from the point \( A \) by adding alternatively the points \( P_1, Q_1, P_2, Q_2 \) and \( P_3 \) (denoted as 1-thickBS). Neither the points \( P_i \) nor \( Q_3 \) can be added to the maximal 1-thickBS since the resulting isothetic thickness will be greater than \( \alpha = 1 \). Another maximal segment 1-thickBS covering the point \( A \) is illustrated in light color on Fig. 2 (b). For each segment 1-thickBS, its length \( L_1 \) is illustrated in light gray and constitutes an essential property which will be exploited in the definition of meaningful thickness introduced in the next section.

### 3 Meaningful Thickness Detection with Maximal Blurred Segment

Before introducing the new concept of Meaningful Thickness we recall briefly the main idea of the meaningful scale (Kerautret and Lachaud, 2009b) detection and show the main inconvenient.

#### 3.1 Asymptotic property of maximal segments

The meaningful scale detection relies on the analysis of asymptotic property of maximal straight segments. This property is the discrete length \( (L_j^C) \) of a maximal segment belonging to a contour point given at a digitization grid size \( h \). In the following, we will denote by \( D_{\theta h}(S) \) the Gauss digitization process \( (D_{\theta h}(S) = X \cap h\mathbb{Z} \times h\mathbb{Z}) \). From different analysis shown in (Lachaud, 2006; Kerautret and Lachaud, 2009b), several properties can be summed up as follows:

**Property 1.** Let \( S \) be a simply connected shape in \( \mathbb{R}^2 \) with a piecewise \( C^3 \) boundary. Let \( P \) be a point of the
boundary $\partial S$ of $S$. Consider now an open connected neighborhood $U$ of $P$ on $\partial S$. Let $(L_i^j)$ be the digital lengths of the maximal segments along the boundary of $\text{Dig}_h(S)$ and which cover $P$. Then, the asymptotic behaviour of the digital lengths follows these bounds: if $U$ is strictly convex or concave, then
\[ \Omega(1/h^{1/3}) \leq L_i^j \leq O(1/h^{1/2}) \] (1) 
if $U$ has null curvature, then
\[ \Omega(1/h) \leq L_i^j \leq O(1/h) \] (2)

The strategy to exploit this property was to transform the initial discrete contour with several grid sizes $h$ while keeping the point associations and checking the discrete contour consistency. The resulting analysis shows precise and fine noise detection but is however not general for the analysis of other type of non discrete contours.

A natural idea to generalize the analysis to polygonal contour is to consider the primitive of the $\alpha$-thick Blurred Segment described in the previous section which allows to deal with non integer points and not necessary connected. The primitive presents another advantage with its thickness parameter $\alpha$ that can be used as a scale parameter.

3.2 Thickness asymptotic properties of Blurred Segments

To define the notion of Meaningful Thickness with the $\alpha$-thick Blurred Segment, we need first to focus on the asymptotic properties of the blurred segments in the multi-thickness decomposition of a given contour. The Euclidean length $L$ will replace the digital length $L_i^j$ used in the previous Property 1. $L$ is defined as the length of the bounding box obtained from the $\alpha$-thick Blurred Segment convex hull. Fig. 2 (b) illustrates such a bounding box with the length of two $1$-thick Blurred Segments covering the point $A$ ($1$-thick $\text{BS}_0$ and $1$-thick $\text{BS}_1$). Their bounding boxes are given respectively by the points $P_1, Q_1, Q_2, Q_3$ and $Q_4, P_2, P_3, P_4, Q_1$.

When the Euclidean lengths of blurred segments around a point of a polygonal contour is computed, we observe an increasing sequence of lengths for the increasing sequence of real thicknesses $t_i = ik\sqrt{2}$ where $k$ is the mean distance between consecutive polygon vertices. When plotted in logscale, its slope is related to the localization of the point in a flat or curved zone. More precisely, letting $(L_i^j)$ be the Euclidean lengths of the blurred segments along the digital contour and covering a point, we have observed experimentally the following behavior:

Property 2. (Multi-thickness). The plots of the lengths $L_i^j/t_i$ in log-scale are approximately affine with negative slopes as specified besides:

<table>
<thead>
<tr>
<th>plot</th>
<th>$t_i$</th>
<th>expected slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(\log(t_i),\log(\max_j L_i^j/t_i))$</td>
<td>$\approx -\frac{1}{2}$</td>
<td>$\approx -1$</td>
</tr>
<tr>
<td>$(\log(t_i),\log(\min_j L_i^j/t_i))$</td>
<td>$\approx -\frac{3}{4}$</td>
<td>$\approx -1$</td>
</tr>
</tbody>
</table>

Fig. 4 and Fig. 3 illustrates such a behaviour on an ellipse shape represented by a disconnected set of points with some missing parts. The set of $\alpha$-thick Blurred Segments covering a specific point $P$ ($l_i$ segments cover $P$) is represented on Fig. 4 with four different thicknesses $(t = \sqrt{2}, 2\sqrt{2}, 3\sqrt{2}, 4\sqrt{2})$. For each thickness $t_i$, the lengths $(L_i^j)_{j=1...\ell_i}$ are represented on the plot of Fig. 4. On this simple example we can check that the segment length verifies the previous Property 2 with their $\min$ and $\max$ values near the slope $-\frac{1}{2}$ and $-\frac{3}{4}$ which correspond to the hypothesis of a curved contour part. Other measures on different shapes are given in (Said, 2010).

As in the multi-scale computation (Kerautret and Lachaud, 2009b), the multi-thickness results allows...
We analyze now the local geometry of some point \( P \) on a polygonal curve \( C \) having a mean distance between vertices equals to \( k \). For various values of thickness \( t_i = ik\sqrt{2} \), \( i = 1..n \), we compute the Euclidean lengths \( L_j^t \) of the blurred segments of \( C \). For a given thickness \( ik\sqrt{2} \), the average length of all blurred segments covering a point \( P \) is denoted as \( L^t = \frac{1}{k} \sum_j L_j^t \), where \( l_i \) represents the number of blurred segments containing \( P \). Fig. 3 illustrates the blurred segments covering a point, obtained on the contour with several values of thickness \( t_i \).

We define the **multi-thickness profile** \( s_n(P) \) of a point \( P \) as the graph \((\log(t_i), \log(L_i^t/t_i))_{i=1,...,n}\). We also define the **ideal multi-thickness criterion** \( p_n(P) \) of a point \( P \) on the boundary of a digital object as the slope coefficient of the simple linear regression of \( s_n(P) \). Property 2 indicates that \( p_n(P) \) should be around -1 if \( P \) is in flat zone, whereas it should be within \([-1/2, -1/3]\) if \( P \) is in a strictly convex or concave zone.

**Detecting noise and local meaningful thickness.**

The multi-thickness profile can be used to detect noisy polygonal curves. We show on Fig. 5 (b) the multi-thickness profile of a point \( P_i \) located on a perfectly digitized curved zone and the multi-thickness profiles of the points \( P_{11}, P_{12} \) and \( P_{10} \) located in noisy zones (image (a)). On the first profile, the decreasing affine relation is immediately visible. On the latter profiles, it is increasing at fine resolution and then falls back on a decreasing affine profile after a given thickness. We apply also the multi-thickness profiles on the polygonal curve with flat and curved areas (Fig. 5 (c,d)). The difference between them is the slope of the affine relation of the profiles (slope near \(-\frac{1}{2}\) for the plots of points \( P_{12}, P_{10} \) and \( P_{11} \) near -1 for the plots of points \( P_{10} \) and \( P_{12} \)).

We then introduce a **noise threshold** \( T_m \) which discriminates between a curved zone and a noisy zone. This threshold should be somewhere between \([-\frac{1}{2}, 0]\). However after several experiments on noisy shapes it appears that the use of the upper threshold value \( T_{m=0} \) gives best results both on curved or flat noisy parts.

A **meaningful thickness** of a multi-thickness profile \((X_{ij}, Y_{ij})_{1\leq i\leq n}\) is then a pair \((i_1, i_2)\), \(1 \leq i_1 < i_2 \leq n\), such that for all \( i \), \( i_1 \leq i < i_2 \), \( \frac{Y(j+1)}{Y(j)} - \frac{Y(j)}{Y(j-1)} \leq T_m \), and the property is not true for \( i_1 - 1 \) and \( i_2 \). The first meaningful thickness \((i_1)\) of a point \( P \) can be considered as a noise level and is denoted as \( t_n(P) = i_1 \). Note that this noise level definition is the independant to
meaningful thickness profile and the second one is the parameter used in the method. The first parameter is the maximal thickness \( t_{\text{max}} \) used to create the meaningful thickness profile and the second one is the minimal slope to consider a point as noise (parameter \( T_m \)).

The first experiment of Fig. 6 (a-c) shows that the parameter \( t_{\text{max}} \) does not change the quality of the detection. The images (b,c) show quite similar noise levels. For the first experiment (a) the pixels drawn in red show that no meaningful thickness was found since the maximal value \( t_{\text{max}} = 5 \sqrt{2} \) was too small and the meaningful thickness is in fact greater than \( t_{\text{max}} \). The stability for the other parameter \( T_m \) was also experimented on Fig. 6 (d-f). The default value of \( T_m \) set to 0 was experimented as giving best results but we can see that a large change of this parameter does not really change the noise detection quality. Other experiments\(^1\) confirm that the proposed method can be considered as parameter free. Note that for all other experiments these parameters were set to \( t_{\text{max}} = 15 \) and \( T_m = 0 \) (and also in the online demonstration).

\section{Experiment of meaningful thickness detection}

\begin{figure}[ht]
\centering
\includegraphics[width=0.8\textwidth]{figure6}
\caption{Evaluation of the independence of the meaningful thickness detection from the different parameters. The size of the blue boxes represents for each point the obtained meaningful scale or thickness \( \alpha_c \). The first row presents the evaluation by varying the maximal thickness used to define the \textit{multi-thickness} profile \( t_{\text{max}} \). The red color indicates present on image (a) indicates that there exists no meaningful thickness less than \( t_{\text{max}} \). The second row shows the stability by the change of the noise threshold parameter \( T_m \).}
\end{figure}

\^1\text{Other experiments can be done online (Kerautret et al., 2011)}
the polygon which tends to be detected as noise with the method based on the meaningful thickness (see close up view of image (e) and (f)). Note that the meaningful scale detection appears to be a little more dynamic than the meaningful thickness. From a computational point of view, the meaningful scale method is faster (76 ms and 87 ms for respectively (b) and (e)) than the meaningful thickness approach (542 ms and 485 ms. for respectively (c) and (f) on a Mac OS X 2.8 Ghz Core 2 Duo), but the thickness detection uses an $O(n^2)$ version of the blurred segment detection while all maximal straight segments are computed in linear time according to the number of contour point. More objective time comparisons are let to future works.

**Experiment on polygonal curves.** The new possibility to detect the meaningful thickness on polygonal curves was experimented in Fig. 8. The first experiment applies the detection on a non uniformly sampled contour (contour of Fig. 5 (a)). The resulting meaningful thickness is everywhere 1 as expected (Fig. 8 (a)). By adding noise on different quadrants of the previous contour, the detector consistently increases the meaningful thickness (image (c)). The other experiment applied on ellipse also show nice meaningful thickness detection (d).

To apply comparisons with other comparable approach, we have experimented the method of the meaningful good continuation of Cao (Cao, 2003). As briefly described in the introduction and contrary to our method this approach has a parameter $\varepsilon$ which can be tuned to adjust the level of what can be considered as meaningful or not. On results presented on Fig. 8 (e,f) we can see that the meaningful contour parts are well identified and are not in contradiction with the meaningful thickness detection but our approach does not need to set any parameter and can also give directly the meaningful thickness (if it exists).

**Application to extract meaningful contours in im-**
Figure 9: Application to meaningful contours extraction (image (c)) using all iso level contours (image (b)). The straight parts obtained from meaningful multi-thickness profile are represented in (d).

4.2 Simple application for contour smoothing

This meaningful thickness detection can be used in numerous applications (in particular, in most of the algorithms which use the $\alpha$-thick Blurred Segment primitive). We present here a simple potential application of contour smoothing by taking the meaningful thickness as a constraint for a curve reconstruction. The reconstruction method is an iterative process that computes the new points as a weighted average of its neighbors, constrained by the meaningful thickness (displayed in light blue on Fig. 10 (b,e,h)). Note that these constraints were defined from the meaningful thickness on all non meaningful edges by using linear interpolation between the vertex of the polygon.

The resulting reconstruction visible on images Fig.10 (c,f,i) show very fine polygonal contours where noise are no more visible. Moreover all initial contour parts with no noise are well preserved after the reconstruction. Another interesting quality is visible with the preservation of all discontinuities in particular for the open contour of Fig. 10 (g-i).

5 Conclusion

A new concept of meaningful thickness was presented. The proposed method can be considered as parameter free and can be applied both on discrete or polygonal contour. The results are very promising and open the door to new unsupervised applications. The simple contour smoothing application is a first application which already shows fine results without the need to set any particular parameter. The proposed method is simple to implement and the user can test the algorithm with their own data (Kerautret et al., 2011). The source code is also already available from the ImuGene library (Ima, 2011) and is planned to be integrated as a module in the new DGtal library (DGt, 2011).

REFERENCES

Figure 10: Application of contour smoothing using the meaningful thickness on several shapes. The initial noisy source contour are given in image (a,d,g). The blue areas represent the noise constraints (b,e,h) and the resulting smoothed contour is represented in red (c,f,i).


