Linear aggregation of conditional extreme-value index estimators

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Abstract

When an extreme phenomena depends on a covariate, one has to deal with Conditional Extreme Value Analysis. In this paper, we propose a linear aggregation technique to estimate the conditional extreme-value index.

Extreme value analysis (EVA) is a branch of statistics dealing with the extreme deviations from the mean of probability distributions. It seeks to assess the probability of events that are more extreme than any observed prior. Extreme value analysis is widely used in many disciplines, ranging from structural engineering, finance, earth sciences, traffic prediction, geological engineering, etc. For example, EVA might be used in the field of hydrology to estimate the value an unusually large flooding event, such as the 100-year flood. Similarly, for the design of a breakwater, a coastal engineer would seek to estimate the 50-year wave and design the structure accordingly.

Let X be a random variable and $F(x) = P(X \leq x)$ its distribution function. We can define a new random variable, Y_n, as the maximum of n copies of the random variable X: $Y_n = \max\{X_1, X_2, ..., X_n\}$. Y_n is the n-sample maximum of the random variable X. If the events generating the realizations of X are independent, the cumulative distribution of Y_n may be expressed as $[F(y)]^n$. Upon definition of a renormalized variable $S_n = (Y_n - b_n)/a_n$, the extreme value theorem establishes that, if

$$\lim_{n\to\infty} P(S_n < s) = \lim_{n\to\infty} F^n(a_n s + b_n) = H(s) \tag{1}$$

where $a_n > 0$ and b_n are normalization constants, then the function $H(s)$ in Eq. (1) must be one of the three following types:

- EV1 or Gumbel: $H(s) = \exp(-\exp(-s))$
- EV2 or Fréchet: $H(s) = \exp(-s^{-\alpha})$
- EV3 or Weibull: $H(s) = \exp(-|s|^{\alpha})$

The three asymptotic types, EV1-EV3, can be thought of as special cases of a single Generalized Extreme Value distribution (GEV) :

$$H_{GEV}(s) = \exp\left\{ -\left(1 + \gamma\frac{s-\mu}{\sigma}\right)_{+}^{-1/\gamma} \right\} \tag{2}$$

where $(.)_+ = \max(., 0)$, μ is the location parameter, $\sigma > 0$ is the scale parameter, and γ is a shape parameter. The limit $\gamma = 0$ corresponds to the EV1 distribution, $\gamma > 0$ to the EV2 distribution (with $\alpha = 1/\gamma$) and $\gamma < 0$ to the EV3 distribution (with $\alpha = -1/\gamma$).

When the studied phenomena depends on a covariate, one has to deal with Conditional Extreme Value Analysis (CEVA). This branch of statistics has become very active these past ten years, the main contributions to this domain are listed below:

- Theoretical issues: [11, 15, 9, 14, 16, 13, 4, 12, 7, 18, 10, 3, 1, 25, 24, 21]
- Quantile regression: [19, 8, 22]
- Application to finance: [5, 2, 17, 6, 20, 23]

Our mail result is the following:

Theorem *Let* $\hat{\gamma}_1(x), \ldots, \hat{\gamma}_K(x)$ *be* K *consistent estimators of the conditional extreme-value index* $\gamma(x)$*. Let* $w_1, \ldots, w_K$ *be* K *weights summing to one. Then,*

$$\hat{\gamma}(x) := \sum_{i=1}^{K} w_i \hat{\gamma}_i(x)$$

is a consistent estimator of the conditional extreme-value index $\gamma(x)$*.*

Proof. The proof is based on the continuity of the sum function and on the property that $\sum_{i=1}^{K} w_i = 1$. Let us highlight that the positivity of the weights in not required. Moreover, this result is valid whatever the sign of $\gamma(x)$.

References

[1] B. Abdous, A.L. Fougères, and K. Ghoudi. Extreme behaviour for bivariate elliptical distributions. *Canadian Journal of Statistics*, 33(3):317–334, 2005.

[2] T.G. Bali and D. Weinbaum. A conditional extreme value volatility estimator based on high-frequency returns. *Journal of Economic Dynamics and Control*, 31(2):361–397, 2007.

[3] J. Beirlant, T. De Wet, and Y. Goegebeur. Nonparametric estimation of extreme conditional quantiles. *Journal of statistical computation and simulation*, 74(8):567–580, 2004.

[4] J. Beirlant and Y. Goegebeur. Local polynomial maximum likelihood estimation for pareto-type distributions. *Journal of Multivariate Analysis*, 89(1):97–118, 2004.

[5] H.N.E. Byström. Managing extreme risks in tranquil and volatile markets using conditional extreme value theory. *International Review of Financial Analysis*, 13(2):133–152, 2004.

[6] H.N.E. Byström. Extreme value theory and extremely large electricity price changes. *International Review of Economics & Finance*, 14(1):41–55, 2005.

[7] V. Chavez-Demoulin and A.C. Davison. Generalized additive modelling of sample extremes. *Journal of the Royal Statistical Society: Series C (Applied Statistics)*, 54(1):207–222, 2005.

[8] A. Daouia, L. Gardes, and S. Girard. On kernel smoothing for extremal quantile regression. *Bernoulli*, 2012. DOI: 10.3150/12-BEJ466.

[9] A. Daouia, L. Gardes, S. Girard, and A. Lekina. Kernel estimators of extreme level curves. *Test*, 20(2):311–333, 2011.

[10] B. Das and S.I. Resnick. Conditioning on an extreme component: Model consistency with regular variation on cones. *Bernoulli*, 17(1):226–252, 2011.

[11] B. Das and S.I. Resnick. Detecting a conditional extreme value model. *Extremes*, 14(1):29–61, 2011.

[12] AC Davison and NI Ramesh. Local likelihood smoothing of sample extremes. *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, 62(1):191–208, 2000.

[13] L. Gardes and S. Girard. A moving window approach for nonparametric estimation of the conditional tail index. *Journal of Multivariate Analysis*, 99(10):2368–2388, 2008.

[14] L. Gardes and S. Girard. Conditional extremes from heavy-tailed distributions: An application to the estimation of extreme rainfall return levels. *Extremes*, 13(2):177–204, 2010.

[15] L. Gardes and S. Girard. Functional kernel estimators of large conditional quantiles. *Electronic Journal of Statistics*, 6:1715–1744, 2012.

[16] L. Gardes, S. Girard, and A. Lekina. Functional nonparametric estimation of conditional extreme quantiles. *Journal of Multivariate Analysis*, 101(2):419–433, 2010.

[17] A. Ghorbel and A. Trabelsi. Predictive performance of conditional extreme value theory in value-at-risk estimation. *International Journal of Monetary Economics and Finance*, 1(2):121–148, 2008.

[18] P. Hall and N. Tajvidi. Nonparametric analysis of temporal trend when fitting parametric models to extreme-value data. *Statistical Science*, pages 153–167, 2000.

[19] R. Koenker and K. Hallock. Quantile regression: An introduction. *Journal of Economic Perspectives*, 15(4):43–56, 2001.

[20] V. Marimoutou, B. Raggad, and A. Trabelsi. Extreme value theory and value at risk: application to oil market. *Energy Economics*, 31(4):519–530, 2009.

[21] S. Rao. A review on conditional extreme value analysis. http://hal.archives-ouvertes.fr/hal-00770546, 2013.

[22] O. Rosen and A. Cohen. Extreme percentile regression. In *Statistical Theory and Computational Aspects of Smoothing: Proceedings of the COMPSTAT94 satellite meeting held in Semmering, Austria*, pages 27–28, 1994.

[23] C. Wang, H. Zhuang, Z. Fang, and T. Lu. A model of conditional var of high frequency extreme value based on generalized extreme value distribution [j]. *Journal of Systems & Management*, 3:003, 2008.

[24] H. Wang and C.L. Tsai. Tail index regression. *Journal of the American Statistical Association*, 104(487):1233–1240, 2009.

[25] H.J. Wang, D. Li, and X. He. Estimation of high conditional quantiles for heavy-tailed distributions. *Journal of the American Statistical Association*, (to appear), 2012.