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To cite this version:

HAL Id: hal-00768770
https://hal.archives-ouvertes.fr/hal-00768770
Submitted on 23 Dec 2012

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Mathematical model for forecasting and estimating of market demand

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* Nobel prize, 1972, John R.Hicks, Kenneth J.Arrow: „for their pioneering contributions to general economic equilibrium theory and welfare theory“.

Abstract: The scientific study article (a monograph) presents a forecast and estimate the evolution of the market demand.

Key Words: fuzzy sets, forecasting, estimating, statistical extrapolation, market demand.

1 Problem formulation

1.1 Probabilistic Model [10]
The methods operate with certain statistical parameters. The most frequently used is the mean square deviation, namely:

$$\sigma = \sqrt{\frac{\sum_{j=1}^{n} (x_j - \bar{x})^2}{n}} (1)$$

We can use the statistical tables, if we know a certain distribution. Knowing the sequence of the likelihood coefficients k, we determine a probability P(x ≥ \bar{x} + k\sigma), respectively P(x ≤ \bar{x} + k\sigma), if x is maximized and a probability P(x ≥ \bar{x} - k\sigma), respectively P(x ≤ \bar{x} - k\sigma) if x is minimized.

If the distribution function is not known, then we apply Cebâşev's formula:

$$P(\bar{x} - k\sigma \leq x \leq \bar{x} + k\sigma) \geq 1 - \frac{1}{k^2} (2)$$

where k = the likelihood coefficient.

1.2 Fuzzy Model Used to Mean Expanding [3], [13]
There are cases when mean expanding is useful. For instance, it is possible that the mean not to be found in the excellence domain of the variable under research.

This case, after the mean value \(\bar{x}\) is determined, the membership degree [5], [6], \(\mu(\bar{x}) = 1\) is assigned to it. In order to find an expanded mean, we proceed to "expanding" figure 1, for instance, on interval [0.95;1]. We calculate a value \(x'\) with the property that:

$$\mu(x') = 0.95 \quad (3)$$

If this value is single, then the expanded mean is the interval \([x', \bar{x}]\) or \([\bar{x}, x']\). If two values, \(x'\) and \(x''\) would exist, such that:

$$\mu(x') = \mu(x'') = 0.95 \quad (4)$$

then, the expanded mean is given by the interval \([x', \bar{x}] \cup [\bar{x}, x'']\).

1.3 Linguistic Model
For a linguistic procedure to be applied, it is necessary that the continuous parameter x (or the discrete parameter, but with a large number of restricted values) to be subject to a granulation operation. We consider the maximum and the minimum levels and among them, certain levels which are denominated by using words (verbs, adverbs) etc., as for instance: if five levels must be introduced between a minimum weight piece and a maximum weight one, we shall use the following gradations:

- very light - for minimum;
- light - for immediately next level;
- average - for immediately next level;
- heavy - for immediately next level;
very heavy - for immediately next level.

In order to find the average weight, if the frequency of gradations is symmetrical, we allow for the weights: $X_{\min}$ and $X_{\max}$, and we divide the level $[X_{\min}, X_{\max}]$ into equal parts (four intervals exist among the five levels).

An interval $I$ will have the length:

$$I = \frac{X_{\max} - X_{\min}}{4}$$

(5)

In this case, $\bar{x}$ is determined by the relation:

$$\bar{x} = x_{\min} + 2I = x_{\max} - 2I$$

(6)

Practically, $\bar{x}$ is lower rounded, and we obtain for mean $x$ an interval under the form:

$$[\bar{x} - r, \bar{x} + r]$$

(7)

where $r$ = the rounding value.

It is possible that this rounding not to be symmetrical, the obtained interval is:

$$[x_{-r}, x_{+r}]$$

(8)

Where:

$r$ = rounding to left; $r'$ = rounding to right.

If the number of gradations is even, then, the number of intervals is odd, therefore the mean is already represented in an interval. If the frequency of gradations is not symmetrical, then each gradation is weighted with the interval centre (attached to each gradation). Thus, an average gradation representing an interval is obtained.

1.4 Mean Expanding Method

1.4.1 The Truncation Method
The steps of the method are:

Step 1. The value $\bar{x}_i$ is truncated and a minimum $j$-level truncated value is obtained, according to the $j$-level of truncation: $y^m_{ij} = T_j \bar{x}_i$

Step 2. A unit for $j$-level is added, namely $b^j$ (where $b$ is the numeration basis) and the maximum level is obtained.

$y^m_{ij} = y^m + b^j$ (for integer number) and

$y^{-j}_{ij} = y^m + b^{-j}$ (for $j$-rank fractions)

$j$ = decimals number.

1.4.2 The Rounding Method
The method consists in two steps:

Step 1. The number is lower rounded for $n$ sequence, yielding: $x^m_n$

Step 2. The number is upper rounded for the $n$ level, yielding: $x^M_n$

The differences are calculated in the same way:

$$\Delta_n = x^M_n - x^m_n$$

with the property that:

$$\lim_{x\to\infty} \Delta_n = 0$$

2 Models for estimating the evolution of market demand

2.1 Theory of demand
Quantitative theory of demand is based on the following assumptions:

$I_1)$ If a stable income, the demand for certain goods decreases with increasing price, and vice versa.

This assumption allows the formulation of numerical methods with which to predict the increase (decrease) demand for the reduction (increase) the price in a defined proportion.

Graphic illustration where noted:

$q_i = $ the demand volume for the product $I$;

$p_i = $ unit price for product $i$,

(see fig. no.1 a, b, c).

The sensitivity of demand to price changes is illustrated by the elasticity of demand coefficient $c$, of the price $p$, which shows the percentage change (in reverse) application of a good, if its price changes by 1%.

The expression for calculation is:

$$E_{c/p} = \frac{\Delta C}{C} \cdot \frac{\Delta p}{p}$$

(9)

$\Delta C, \Delta p = $ demand growth; change (+/-)-reference price in two periods.
If a variable income, demand for a good increases with income growth and decreases with increasing price. If we assume for each level of income other factors demand function:

\[ q_i = f_i(p_i) \]  

(10)

where possible changes in demand can be represented by several successive demand curves as shown in fig.no.2:

![Fig.no.2.- Evolutions of demand](image)

If the price is kept constant, the application may be described as a function of income:

\[ q_i = f(v) \]  

(11)

The coefficient of elasticity of demand \( c \) to income \( v \), shows the percentage increase in demand when income increases by 1%.

That is:

\[ E_{v/v} = \frac{\Delta C}{C} \cdot \frac{\Delta V}{V} \]  

(12)

Changing prices determines travel demand curves. In the literature, curves expressing the dependence between demand and income are known as Engel curves. Dependence is linear, the demand for different products or product groups known saturation point and is influenced by the interchangeability of products. Demand-income dependence can be expressed primarily through the following types of functions:

a) \( C_1 = m_1 V \) (for strictly necessary goods)  

(13)

b) \( C_2 = m_2 (V - p_2) \) (for everyday consumer goods)  

(14)

c) \( C_3 = m_3 V (V - p_2) \) (for luxury goods)  

(15)

d) \( C_4 = m_4 V - n_3 V^2 \) (for goods that are out of use when a certain level of revenue)  

(16)

The significance of notations are:

\( C = \) demand for the product (or product group) considered;

\( V = \) income;

\( m, n, p = \) econometric parameters.

The interpretation of the four functions, is as follows:

a) \( C_1 \), first function, known as Törnquist function I, show that with increasing income, demand increases at a rate lower, and tends to be capped. The graph is shown in fig.4a:

![Fig.no.4a. -Törnquist I Function](image)

When \( V \) limit \( \to \infty \) is \( m_1 \), so the level toward which \( C_1 \) is \( M_1 \). The first derivative of the function is decreasing, tending to 0, when \( V \to \infty \).

\[ C_1' = \frac{\partial C_1}{\partial V} = \frac{m_1 p_1}{(V + n_1)^2} \]  

(17)

The graph is shown in fig. no.4 b.:

![Fig. no. 4b.-Törnquist I Derivative function](image)

b) \( C_2 \) function (Törnquist II) is similar to \( C_1 \) in terms of demand-income dependence, because it has a threshold of saturation (at \( m_2 \)), which means that the growth of increasingly high on \( V \), \( C_2 \) tend to \( m_2 \), which can be noticed by:

\[ \lim_{V \to \infty} \lim_{V \to \infty} C_2 = \lim_{V \to \infty} \lim_{V \to \infty} \frac{m_2 (V - p_2)}{V + n_2} = m_2 \]  

(18)

The demand for this group of products or services begins to manifest only after \( V > p_2 \), meaning that \( p_2 \) parameter is negative, after calculations.

For \( V = 0 \):

\[ C_2 = \frac{m_2 p_2}{n_2} \]  

(19)

\( C_2 \) value for \( V = 0 \) is not negative, as \( p_2 < 0 \).

The graph is shown in fig. no.4c.:

![Fig. no.4c.-Törnquist II Function](image)

First derivative function is also decreasing (fig.no.4d.):
c) $C_3$ function (Törnquist III) is specified for the products that required a continuous increase with increasing income. The curve presents an oblique asymptote to the branch to $+\infty$.

$$m = \lim_{V \to +\infty} \frac{C_3}{V} = m_3$$

Therefore, $C = m_3V - m_3(p_3 + n_3)$ is the asymptotically wanted. For $n_3 < 0$ demand occurs when $-n_3 < p_3$. For graphic representation, we determine the intersection with the abscissa:

$$C = 0 \Rightarrow V = p_3 + n_3$$

For the function, and

$$C_3 + 0 \Rightarrow m_3V - p_3 = 0 \Rightarrow V = p_3$$

As $n_3 < 0 \Rightarrow p_3 + n_3 < p_3$ and $C_3$ function graph is shown in fig.no. 4e.

![C_3 Function](image)

**d) $C_4$ function** is typical for products or groups of product, which are out of use after a certain level of revenue.

So, the demand for such products are initially increases, reaches a maximum for $V = m_4/2n_4$, then start to decrease reaching zero, which means that for higher incomes than this limit, there is no demand for such products.

### 2 Problem Solution

Consumer demand is treated continuously in connection with the supply of goods, as among them there is a complex interaction.

An approach to the processes occurring on the market require prior research and a proper understanding of the real complexity of the categories of individual and social demand, potential and actual demand.

By research on a type of behavior, we can determine future structure of the supply / demand actually recorded in previous years ($t=1,2,....-N$).

We note:

- $m_{ij}$ = demand of article j of family i;
- $i=\alpha$ (fixed), number of families;
- $j=1,2,....-n$, number of articles of family i (may vary from year to year).

Assortment structure is given by the probability of producing an application of article j in $m_{aj}$ quantity, the quantity demanded would be (26), (27):

$$m_{aj} = \sum_{j=1}^{n} m_j$$

$$P_{aj} = \frac{m_{aj}(t)}{\sum_{j=1}^{n} m_{aj}(t)}$$
Assume that requests for items j form a complete system of events.

For \( t = 1, 2, \ldots, N \) years, it is defined a stochastic matrix:

\[
P_{ij} = \| P_{ij}(t) \|_{xN} : \sum_{j=1}^{N} P_{ij}(t) = 1 \quad (28)
\]

As:

\[
m_{ij}(t) > 0 \Rightarrow (P_{ij}) - \max (\min P_{ij}(t)>0) (29)
\]

\[
t = 1, \ldots, N ;
\]

\[
j = 1, \ldots, n \text{ the matrix is Markov type.}
\]

We consider 2 consecutive years:

\[
\Delta = P_{ij}^{t+1} - P_{ij}^{t}, \quad \text{(P}_{ij}^{t+1} = P_{ij}(t)) \quad (30)
\]

If \( \Delta > 0 \), we note:

\[
\Delta = \Delta^+ \text{ for } j = 1, 2, \ldots, m
\]

\[
\Delta = \Delta^- \text{ for } j = 1, 2, \ldots, n
\]

\[
\Rightarrow \sum_{j=1}^{m} \Delta^+_j = \Delta^+ = -\Delta^- = \sum_{j=m+1}^{n} \Delta^-_j \quad (31)
\]

Starting from the definition of matrix \( |P_{ij}|_{xxN} \), it is defined a transition matrix:

\[
T_g = \| T_{ij} \|_{xxN}, \quad T_{ij} = P_{ij}/ \sum_{j=1}^{n} P_{ij} \quad (32)
\]

Same for the following years \( t = 1, 2, \ldots, N \), we obtain

\[
T_{ij} = \sum_{t=1}^{N} T_{ij} \quad (33)
\]

from which to obtain the matrix:

\[
M = \begin{pmatrix}
T_{ij} \\
\vdots \\
T_{ij}
\end{pmatrix}
\]

from which it can determine the structure of the vector elements of sales next year.

We note (35):

\[
K_i^- = \sum_{j=m+1}^{n} \Delta^-_j = \sum_{j=m+1}^{n} K_j^- = 1
\]

\[
\text{It builds } |P_{ij}|_{xxN} \text{ with elements defined as:}
\]

- if \( i=j \) put \( P_{ij} = \min (P_{ij}, P_{ij}^{t+1}) \)

For \( i, \) for which \( \Delta^-_i \) put \( P_{ij} = \min (\ldots, \ldots) \)

and \( P_{ij} = 0, \) for \( j = 1 \).

For \( i, \) for which \( \Delta^-_i \) to do so:

- if \( j \) comes from elements for which:
  \( \Delta^-_i \) put \( P_{ij} = K_j^- \cdot \Delta^-_j \)

- if \( j \) comes from elements for which:
  \( \Delta^+_i \) put \( P_{ij} = K_j^+ \cdot \Delta^+_j \)

The practical application questions a number of years which is considered to obtain satisfactory results.

For a few years, there is a stochastic matrix (Markov).

From one year to another:

\[
T_{i}^\Sigma = \sum_{i=1}^{N-1} T_{i} \quad (36)
\]

The calculation for year \( N \) of matrix \( T_N \) is done by applying the recurrent relationship:

\[
T_{i}^\Sigma = T_{i} \Sigma + T_{N} \quad (N+1)
\]

\( N \) is determined based on the matrix \( T_2 \), then do calculations to determine the structure vector (N+1) to determine the coefficients \( K_j^- \) and \( K_j^+ \) and determine differences \( \Delta_j \), building on practical application requirements. Observations must cover a sufficient number of years, something that is determined by simulation.

3 Conclusions

Given the dynamic link between demand and income, and its progress, the normal trend is diversification.

Philosophical connections

“Ubi Concordia, ibi Victoria!”

References:


[16] ** Encyclopedia Britanica.