Formal Expression of Sensitivity and Energy Relationship in the Context of the Coupling Matrix

Monica Martinez-Mendoza, Fabien Seyfert, Christoph Ernst, Alejandro Alvarez-Melcon

Formal Expression of Sensitivity and Energy Relationship in the Context of the Coupling Matrix

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***Abstract*—Precise formulas to express the formal relationship between the time average stored energy in the resonators of a lowpass filter network and the sensitivity of the reflection S parameter with respect to the coupling matrix terms are demonstrated in this paper, considering the normalized frequency axis. These relationships are found in the modern context of the $N + 2$ coupling matrix, and for both diagonal and general non-diagonal coupling elements of the matrix. The results are valid for any type of coupling topology represented by the $N + 2$ coupling matrix. Different examples are included as validation. Furthermore, important implications and applications derived from the new relationships are highlighted.**

I. Introduction

The relationship between the stored energy in the elements of an LC N-port network and the sensitivity of the S-Parameters with respect to each L or C element was first discussed in [1]. In that paper, the sensitivity was defined as the variation of the S-Parameters with respect to independent variations in the L or C elements of the network under evaluation.

Later, coupling matrices representing lowpass filter networks became fashionable and started to be widely used. The coupling matrix concept was first introduced by Atia and Williams [2] in the early 70s, where it was applied to symmetric waveguide filters. The representation of microwave filters in matrix form is specially useful because it provides a very precise model of the network and simultaneously allows to operate on the coupling matrix in a very simple way. These operations may be useful to simplify the synthesis process, to simulate complex networks in a simpler way or to reconfigure the filter topology into another configuration more suitable for the specific practical implementation. Another advantage of the coupling matrix is that it represents some of the real physical properties of the circuit elements. Each coupling matrix element is related to a specific circuit element in the final prototype. The non-diagonal matrix elements represent the couplings between the resonators of the network, and their values provide information about the magnitude of these couplings. On the other hand, the diagonal coupling elements are related to the differences in the resonant frequencies of the resonators with respect to the center frequency of the filter.

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Although the initial matrices were NxN matrices, where N is the order of the filter, $N+2$ coupling matrix forms were introduced some years later [3] . The $N+2$ coupling matrix form includes information about the input and the output port and is widely used in the microwave community, since it presents several advantages with respect to the NxN matrix form. Among these advantages are the possibility to implement fully canonical filters with a nonzero direct coupling term, and the possibility to include frequency invariant reactance elements (FIR) in the network, allowing to synthesize symmetric or asymmetric responses.

Although the $N+2$ coupling matrix can be a representation of an LC network in certain cases (synchronously tuned filters), it is important to highlight that there is no direct equivalence between the *variation* of an element in an LC network and the *variation* of an element in the $N+2$ coupling matrix. There are two reasons for this observation. First, a coupling is related to two resonators, thereby if a coupling term M_{pq} is varied, two elements are being affected in the LC network represented by the coupling matrix. Second, the LC network presented in [1] does not consider the possibility to include FIR elements. Thereby, a variation in a FIR element, which is expressed as a variation in a diagonal coupling term M_{pp}, cannot be translated into the variation of an element of the LC network with no FIR elements.

In this context, although there is no direct equivalence between isolated variations in L or C elements and variations in the coupling matrix terms M_{pp} and M_{pq}, a close relationship between the stored energy in the elements of the network and the sensitivity of the reflection S-Parameter with respect to the diagonal elements of the $N + 2$ coupling matrix was already noticed in [4], [5], [6]. Nevertheless, precise formulas to express this relationship were not known and the obtained results were just based on observations at that time.

This paper presents precise formulas to express the formal relationship between the time average stored energy in the resonators of a lowpass filter network, and the sensitivity of the reflection S-Parameter with respect to the coupling matrix terms considering the normalized frequency axis. Sensitivity in the real passband domain can be obtained by using the standard lowpass to bandpass transformation. These relationships are found in the modern context of the $N+2$ coupling matrix, and for both diagonal and general non-diagonal coupling elements of the $N + 2$ coupling matrix. It is important to remark that the obtained results are based on the $N + 2$ coupling matrix, thus they are only valid for transfer functions fulfilling the narrow band approximation. Different examples will be used to illustrate the implications and to highlight applications derived from the novel relationships.

II. Sensitivity of the S-parameters

In this section, following the same ideas as [7], [8], we derive formulas for the sensitivities of the S-parameters with respect to the couplings.

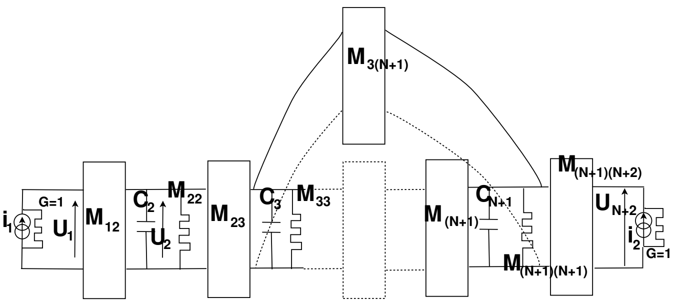

Fig. 1. General low pass circuit related to the $(N+2) \times (N+2)$ coupling matrix framework (for simpler notations, the capacitors have been numbered starting with $C_2, C_3 \ldots$ and ending with C_{N+1}).

The lowpass circuit we consider is detailed in Figure 1: all non-diagonal couplings are here frequency invariant admittance inverters (FIR), the diagonal ones constant susceptances modelling the frequency shift of each resonator (i.e. the FIR elements mentioned previously), the input and output loads as well as all capacitors are normalized to one. At a given frequency ω, we define the sensitivity of the scattering parameter $S_{i,j}$ with respect to $M_{l,k}$ as:

$$\frac{\partial S_{i,j}(\omega)}{\partial M_{l,k}} \tag{1}$$

The nodal equations ruling the behaviour of circuit in Fig. 1 are expressed in terms of the voltage vector U as follows:

$$[G + j\omega I + jM][U] = A[U] = [i] \tag{2}$$

where:

- $[G]$: Admittance matrix with all elements zero except for $G_{1,1} = 1$ and $G_{N+2,N+2} = 1$. Size $(N+2)$ x $(N+2)$.
- $[I]$: Identity matrix up to $I_{1,1} = 0$ and $I_{N+2,N+2} = 0$. Size $(N+2)$ x $(N+2)$.
- $[M]$: Coupling matrix. Size $(N+2)$ x $(N+2)$.
- $[U]$ is the voltage vector, and $[i] = [i_1, 0, \ldots, i_2]^t$ is the excitation vector.
- $A = [G + j\omega I + jM]$ by definition.

From this we compute the S-parameters as:

$$S_{1,1} = 2[A^{-1}]_{1,1} - 1, \; S_{2,2} = 2[A^{-1}]_{n+2,n+2} - 1$$
$$S_{1,2} = 2[A^{-1}]_{1,n+2}. \tag{3}$$

where a similar notation as in [7], [8] has been used, and $[A^{-1}]_{i,j}$ is the i-th element of the solution vector of the system shown in (2), when the excitation is placed in the j-th element of the excitation vector $[i]$. If we set $W_{i,j} = [A^{-1}]_{i,j}$, some differential calculus yields for the sensitivities:

$$l = k, \quad \frac{\partial S_{1,1}}{\partial M_{l,l}} = -2W_{1,l}^2, \quad \frac{\partial S_{2,2}(\omega)}{\partial M_{l,l}} = -2W_{N+2,l}^2 \tag{4}$$

$$l \neq k, \quad \frac{\partial S_{1,1}}{\partial M_{l,k}} = -4W_{1,l}W_{1,k},$$
$$\frac{\partial S_{2,2}(\omega)}{\partial M_{l,k}} = -4W_{N+2,l}W_{N+2,k} \tag{5}$$

$$l = k, \; \frac{\partial S_{1,2}}{\partial M_{l,l}} = -2W_{1,l}W_{N+2,l} \tag{6}$$

$$l \neq k, \; \frac{\partial S_{1,2}}{\partial M_{l,k}} = -2(W_{1,l}W_{N+2,k} + W_{1,k}W_{N+2,l}) \tag{7}$$

Latter formulas are, up to sign changes, equivalent to those found in [8] where a dual circuit based on impedance inverters, and inductive series resonators was implicitly used.

III. Relationship between sensitivities and reactive energy

In this section we come to the main goal of this paper, namely the derivation of relationships between sensitivities and the reactive energy stored in the resonators. We suppose that the network is excited by only one source at a time. When source (i_1) is active (i.e. $i_2 = 0$) we define the generic parameter $T_{k,1} = 1/2|U_k|^2$, while when source (i_2) is active (i.e. $i_1 = 0$) we define in a similar manner $T_{k,2} = 1/2|U_k|^2$. Depending on which voltage (k) is considered, these quantities have different physical interpretations:

- For $1 < k < n+2$, the expression $1/2|U_k|^2 = 1/2|U_k|^2/C_k$ represents the average reactive energy stored in capacitor (k).
- For $k = 1, N+2$, the expression $1/2|U_k|^2$ represents the average power dissipated in the unit admittance at the access ports.

After some manipulations we find,

$$T_{k,1} = \frac{1}{2}|U_k|^2 \tag{8}$$
$$= \frac{1}{2}|W_{1,k}\, i_1|^2 \tag{9}$$
$$= \frac{1}{2}|W_{1,k}\, 2\, a_1|^2 \tag{10}$$
$$= \left|\frac{\partial S_{1,1}}{\partial M_{k,k}}\right| |a_1|^2 \tag{11}$$

where a_1 is the amplitude of the incident wave. Equation (9) is obtained from the the definition of $W_{i,j}$ and the circuital equation (2), and (11) is a direct consequence of (4). We obtain in the same manner the relation between $T_{k,2}$ and the sensitivity of $S_{2,2}$. For the rest of the paper we will suppose that the input powers $|a_1|^2, |a_2|^2$ are normalized to 1 Watt, which yields following normalized equations

$$T_{k,1} = 2|W_{1,k}|^2 = \left|\frac{\partial S_{1,1}}{\partial M_{k,k}}\right| \tag{12}$$

$$T_{k,2} = 2|W_{N+2,k}|^2 = \left|\frac{\partial S_{2,2}}{\partial M_{k,k}}\right| \tag{13}$$

Combining equations (12),(13) with (5),(6),(7) completes the correspondence between sensitivities (in fact their modulus) and the reactive stored energies (or dissipated power at the loads).

$$l \neq k, \; \left|\frac{\partial S_{1,1}}{\partial M_{l,k}}\right| = 2\sqrt{T_{l,1}T_{k,1}}, \; \left|\frac{\partial S_{2,2}}{\partial M_{l,k}}\right| = 2\sqrt{T_{l,2}T_{k,2}} \tag{14}$$

$$l = k, \; \left|\frac{\partial S_{1,2}}{\partial M_{l,l}}\right| = \sqrt{T_{l,1}T_{l,2}} \tag{15}$$

$$l \neq k, \; \left|\frac{\partial S_{1,2}}{\partial M_{l,k}}\right| = \sqrt{T_{l,1}T_{k,2}} + \sqrt{T_{k,1}T_{l,2}} \tag{16}$$

IV. Implications and Applications

An important implication of the equations derived in previous section is to yield bounds on the sensitivities in terms of physical quantities: the reactive stored energies $T_{k,l}$ for $1 < k < n+2$ or dissipated powers for $k = 1, N+2$. To handle the sensitivity of the modulus of $S_{1,1} = |S_{1,1}|e^{j\,\theta_{S_{11}}}$ with respect to diagonal couplings (away from reflexion zeros) remark that

$$\left|\frac{\partial S_{1,1}}{\partial M_{k,k}}\right|^2 = \left|\frac{\partial |S_{1,1}|}{\partial M_{k,k}}\right|^2 + |S_{1,1}|^2 \left|\frac{\partial Arg(S_{1,1})}{\partial M_{k,k}}\right|^2 \tag{17}$$

which yields

$$\left|\frac{\partial |S_{1,1}|}{\partial M_{k,k}}\right| \leq T_{k,1} \tag{18}$$

This is an important result, since it shows that the time average stored energy of a resonator $T_{k,1}$ is a maximum boundary of the sensitivity of the $|S_{11}|$ parameter. Thus, the time average stored energy in the resonators of a filter can be used to predict a maximum deviation in the reflection parameter of the filter for a specific manufacturing tolerance affecting the resonator dimensions. This was already intuitively noticed in [4], [5], but a mathematical proof was not available at that time.

Another interesting interpretation cast some light on the global structure of the sensitivity parameters. The reactive energy derives from an energy function verifying the edge port conservation rule, provided the considered network is loss-less: the total reactive energy of a loss-less network can therefore be computed from its edge port parameters, here the S-parameters. Precisely we have (see [1], [9], [10]),

$$T_1 = \sum_{k=2}^{N+1} T_{k,1} = -|S_{1,1}|^2 \frac{\partial Arg(S_{1,1})}{\partial \omega} - |S_{1,2}|^2 \frac{\partial Arg(S_{1,2})}{\partial \omega} \tag{19}$$

$$T_2 = \sum_{k=2}^{N+1} T_{k,2} = -|S_{2,2}|^2 \frac{\partial Arg(S_{2,2})}{\partial \omega} - |S_{2,1}|^2 \frac{\partial Arg(S_{2,1})}{\partial \omega} \tag{20}$$

and both expressions are equal to the group delay $\frac{-\partial Arg(S_{2,1})}{\partial \omega}$ for auto-reciprocal characteristics ($S_{1,1} = \pm S_{2,2}$). In view of (12) and (13) this shows that the global sensitivities, that is $\sum_1^{N+1} \left|\frac{\partial S_{1,1}}{\partial M_{k,k}}\right|$ and $\sum_1^{N+1} \left|\frac{\partial S_{2,2}}{\partial M_{k,k}}\right|$, depend only on the filter's response. In particular, they do not depend on the coupling matrix topology chosen to realize the filter. In other words, the filtering characteristic sets the global sensitivity, whereas the repartition of the latter in each single resonator is driven by the coupling topology. Equations (12) to (16) reveals that, coupling schemes with an equally distributed time average stored energy among the resonators, will exhibit a lower sensitivity boundary of the reflexion S-Parameters. In this sense, works targeted to equally distribute the time average stored energy in the resonators of a filter with the aim of improving the power handling capability of the structure [11], [12], can be employed to find structures with low sensitivity.

V. Validation and Remarks

In this section, the results previously derived will be illustrated through several simple examples. The examples will deal with different transfer functions and different filter topologies.

The first objective of the study is to show the coincidence between the absolute value of the reflection S-Parameter variation with respect to diagonal coupling matrix elements and the time average stored energy in the resonators related to that diagonal elements. In order to to this, a second order Chebyshev transfer function with 25 dB return losses has been synthesized. The transfer function is labelled as TF_1 in Fig. 2. If the filter network representing TF_1 is synthesized into

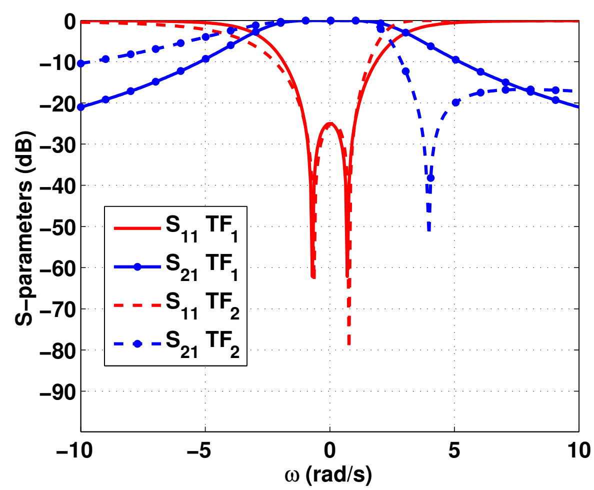

Fig. 2. Second order Chebyshev transfer function without finite transmission zeros (TF_1) and with one transmission zero above the passband (TF_2)

an inline configuration (see Fig. 3(a)), the coupling matrix results:

$$M_1 = \begin{pmatrix} 0 & 1.4312 & 0 & 0 \\ 1.4312 & 0 & 2.1670 & 0 \\ 0 & 2.1670 & 0 & 1.4312 \\ 0 & 0 & 1.4312 & 0 \end{pmatrix} \tag{21}$$

On the other hand, if the filter network representing TF_1 is synthesized into a transversal configuration (see Fig. 3(b)), the coupling matrix results:

$$M_2 = \begin{pmatrix} 0 & -1.0120 & 1.0120 & 0 \\ -1.0120 & -2.1670 & 0 & 1.0120 \\ 1.0120 & 0 & 2.1670 & 1.0120 \\ 0 & 1.0120 & 1.0120 & 0 \end{pmatrix} \tag{22}$$

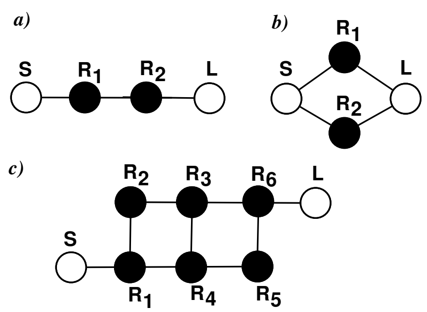

Fig. 3. Filter topologies: a) Inline, b) Transversal, c) Extended Box.

The time average stored energy in the resonators of the inline and the transversal networks, as well as the absolute value of the reflection S-Parameter variation with respect to diagonal coupling matrix elements M_{11} and M_{22}, are shown in Fig. 4 (inline topology) and Fig. 5 (transversal topology). As predicted by (12), Fig. 4 and Fig. 5 show that the sensitivity of S_{11} with respect to each diagonal coupling matrix element ($Sens_{k,k}$) and the time average stored energy ($T_{K,1}$) in its associated resonator are exactly equal.

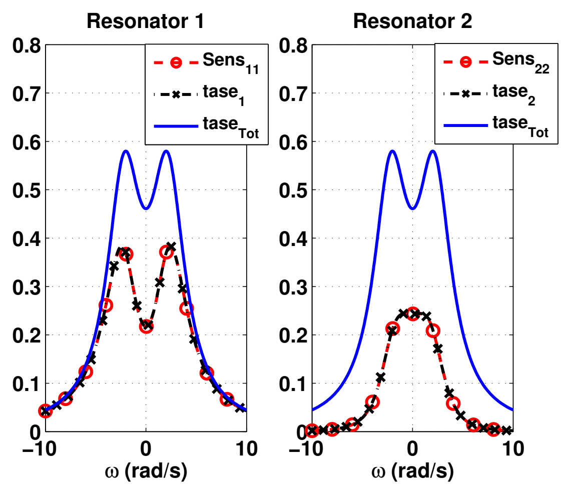

Fig. 4. Absolute value of the S_{11} variation with respect to each diagonal coupling matrix element M_{kk} ($Sens_{kk}$) and time average stored energy (measured in Joules) in its associated resonator ($tase_k$). Transfer function TF_1 (Fig. 2) in inline configuration (Fig. 3(a)).

As stated by (19) the total stored energy $tase_{TOT}$ is the same in both topologies (see Fig. 4 and Fig. 5). This total energy is however better balanced between resonators in the in-line topology as compared to the transversal one. The maximal sensitivity (across all diagonal couplings) is therefore, for most frequencies, lower in the in-line topology.

The second target is to show that the total sensitivity of S_{11} with respect to the diagonal coupling matrix terms is different for different transfer functions, independently of their filter topology. Note that the total sensitivity of S_{11} with respect to the diagonal coupling matrix terms equals the sum of the stored energy in the resonators of the networks, which is known to be constant for a given transfer function [11], [12].

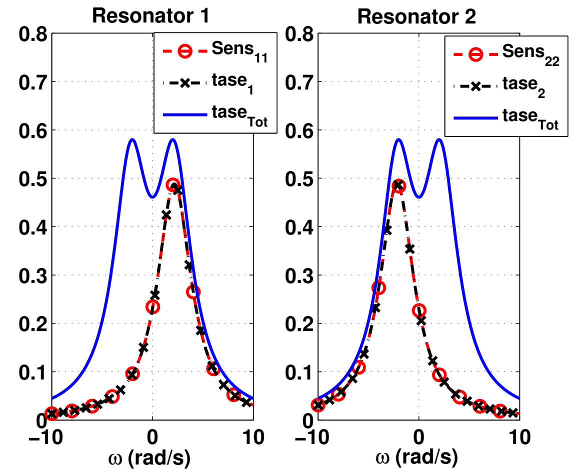

Fig. 5. Absolute value of the S_{11} variation with respect to each diagonal coupling matrix element M_{kk} ($Sens_{kk}$) and time average stored energy (measured in Joules) in its associated resonator ($tase_k$). Transfer function TF_1 (Fig. 2) in transversal configuration (Fig. 3(b)).

A different transfer function to the one used in the previous examples (see TF_1 in Fig. 2) have been synthesized in transversal configuration. This new transfer function exhibits a transmission zero at $\omega = j \cdot 4$ rad/s, and presents 25 dB of return losses. It is labelled as TF_2 in Fig. 2, and its coupling matrix results:

$$M_3 = \begin{pmatrix} 0 & 0.7299 & -1.2926 & 0 \\ 0.7299 & -2.0783 & 0 & 0.7299 \\ -1.2926 & 0 & 1.9501 & 1.2926 \\ 0 & 0.7299 & 1.2926 & 0 \end{pmatrix} \quad (23)$$

The total sensitivity of S_{11} with respect to the diagonal coupling matrix terms ($Sens_{11} + Sens_{22}$) for both transfer functions TF_1 and TF_2 of Fig. 2, is shown in Fig. 6.

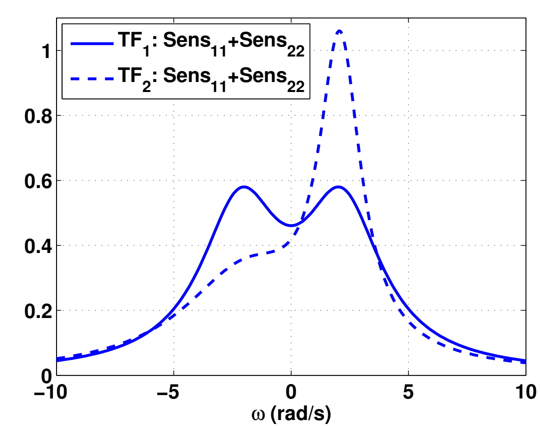

Fig. 6. Total sensitivity of S_{11} with respect to the diagonal coupling matrix terms for TF_1 and TF_2 shown in Fig. 2.

Note that the total sensitivity with respect to diagonal terms equals the total time averaged stored energy in the network, and thus it also equals the group delay of the transfer function under study if the available power is $a_1 = 1$ [11], [12]. It is known that the time averaged stored energy in a network changes if the transfer function represented by the network is changed. Thus, it is expected that the total sensitivity of S_{11} with respect to the diagonal coupling matrix terms varies for different transfer functions. As expected, we notice that the sensitivity of TF_2 is clearly higher than the sensitivity of TF_1 in the frequency range of the passband closer to the transmission zero of TF_2, but lower in the other region.

The next target is to show that the sum of the coupling sensitivities is different for different topologies implementing the same transfer function. In order to do that, the total sensitivity of S_{11} with respect to the general coupling matrix elements (non diagonal) Sen_{Tot} for TF_1 of Fig. 2 implemented in inline and transversal configurations, is shown in Fig. 7. It is observed that both traces are different, being larger the total sensitivity Sen_{Tot} with respect to the couplings in the transversal network. This was intuitively expected, since the number of couplings in both topologies is different (the transversal topology implements four main couplings, whereas the inline topology is formed with just three main couplings). To sum up, when including couplings in the global sensitivity, the sensitivity becomes dependant on the topology. Besides, there is no longer equality with the group delay, although an upper bound involving the group delay could still be computed for each specific topology.

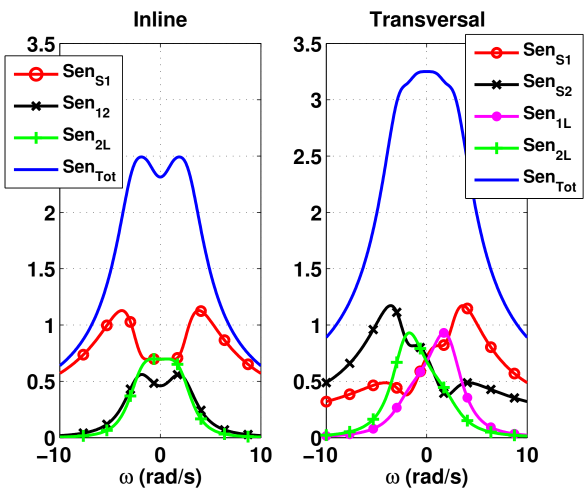

Fig. 7. Sensitivity of S_{11} with respect to the general coupling matrix elements for transfer function TF_1 of Fig. 2 in inline and transversal configurations.

Note that the sensitivity of S_{11} with respect to the individual general coupling matrix elements for transfer function TF_1 of Fig. 2 in inline and transversal configurations are also plotted in Fig. 7. Similar plots have been obtained from the time average stored energy and the dissipated power in the ports by applying (14), (15), (16), although they are not included in Fig. 7 because exactly the same results are obtained.

Finally, the last target is to compare the total sensitivity with respect to the general coupling matrix elements (i.e. non-diagonal) of two different implementations of the same transfer function with the same topology. This will allow to choose the coupling matrix with the lowest sensitivity. Note that the total sensitivity with respect to the diagonal elements will be the same in both cases, since as demonstrated in the previous section, it equals to the group delay of the response.

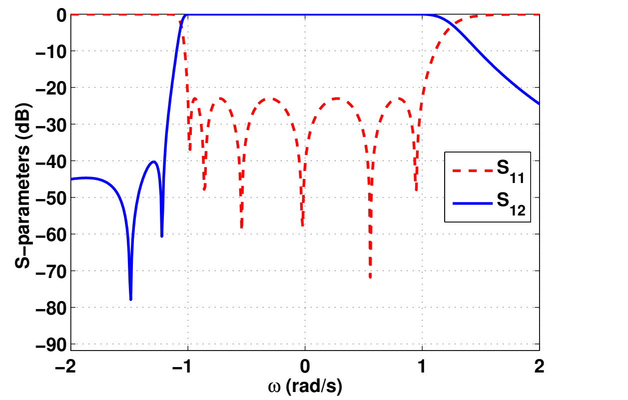

Fig. 8. Sixth order Chebyshev transfer function with two asymmetric finite transmission zeros below the passband.

In order to compare two different implementations of the same transfer function with the same topology, the sixth order transfer function shown in Fig. 8 will be synthesized. The selected topology is the extended box (see Fig. 3(c)). The extended box filter configuration was introduced in [13], and it is known to have several solutions for a given coupling matrix synthesis problem [14]. This offers some flexibility in order to select the solution with the best sensitivity behavior. Two different coupling matrices M^4 and M^5 have been synthesized using the software package Dedale-HF [15].

Figure. 9 shows, for both solutions, the stored energies parameters $T_{3,1} \ldots T_{6,1}$ with respect to the normalized frequency ω ($T_{2,1}$ and $T_{7,1}$ are the same for both solutions, hence not plotted). It is observed that the M^5 solution is extremely sensitive with respect to the resonance frequency of the second resonator, and this on the lower border of the pass-band (see $T_{3,1}$ in Figure.9). This is confirmed by the value of its frequency shift $M^5_{3,3} = 1.0014$, which indicates that resonator 2 is heavily de-tuned with respect to the central frequency. It suggests that solution M^4 achieves here a better balance of the total stored energy. This in turn will result, as shown in the paper, in a lower global maximal sensitivity, obtained here in both solutions for the $M_{3,3}$ element around $\omega = -1$. Due to formula (14) solution M^4 will also outperform M^5 when considering maximal sensitivities (for example over the frequency range $[-1.5, 1.5]$) with respect to the couplings $M_{2,3}$ and $M_{3,4}$. Of course, due to the invariance of T_1, the solution M^5 performs "better" than M^4 when looking at the sensitivities related to the fourth and fifth resonators.

VI. Conclusions

Precise formulas to express the formal relationship between the time average stored energy in the resonators of a lowpass filter network and the sensitivity of the reflection S parameter with respect to the coupling matrix terms have been presented in this paper. The relationships have been described in the modern context of the N+2 coupling matrix for both diagonal and general coupling elements of the matrix. Furthermore, important implications have been derived from the new relationships found in the paper, allowing to establish maximum sensitivity boundaries. The latter were obtained from energy

$$M^4 = \begin{pmatrix} 0 & 1.0638 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1.0638 & -0.0441 & 0.1604 & 0 & 0.8873 & 0 & 0 & 0 \\ 0 & 0.1604 & 0.7700 & 0.4839 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.4839 & -0.3035 & -0.5668 & 0 & 0.7702 & 0 \\ 0 & 0.8873 & 0 & -0.5668 & -0.0832 & 0.4117 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.4117 & 0.6113 & 0.4688 & 0 \\ 0 & 0 & 0 & 0.7702 & 0 & 0.4688 & -0.0441 & 1.0638 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.0638 & 0 \end{pmatrix}$$

$$M^5 = \begin{pmatrix} 0 & 1.0638 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1.0638 & -0.0441 & -0.2806 & 0 & 0.8569 & 0 & 0 & 0 \\ 0 & -0.2806 & 1.0014 & 0.0867 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.0867 & 0.7066 & 0.3146 & 0 & 0.5630 & 0 \\ 0 & 0.8569 & 0 & 0.3146 & -0.1696 & 0.4879 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.4879 & -0.5437 & -0.7042 & 0 \\ 0 & 0 & 0 & 0.5630 & 0 & -0.7042 & -0.0441 & 1.0638 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.0638 & 0 \end{pmatrix}$$

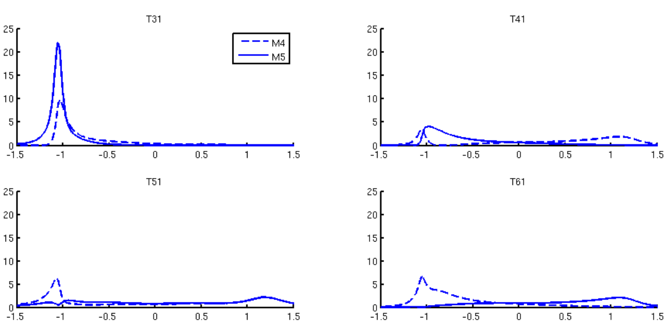

Fig. 9. Stored energies for second to forth resonator and for both solution M^4 and M^5

considerations for the special case of the classical lowpass $(N+2 \times N+2)$ prototype yielding some insight about how to perform practically with the computation of the reactive energies. Another, but more abstract way to go is certainly to extend, in all generality, Kishi's energy theory to networks with constant inverters, and FIR elements.

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