Distributed MIMO coding scheme with low decoding complexity for future mobile TV broadcasting
Ming Liu, Maryline Héland, Matthieu Crussière, Jean-François Héland

To cite this version:

HAL Id: hal-00763445
https://hal.archives-ouvertes.fr/hal-00763445
Submitted on 10 Dec 2012
A new proposal for \( \text{n} \)-\( \text{MIMO} \) coding: Among them, distributed \( \text{MIMO} \) coding. Fig. 1 shows the distribution of the \( \text{MIMO} \) scenario considered in this work where \( \text{MIMO} \) coding is carried out not only among the same cell (intra-cell coding) but also over the neighboring cells (inter-cell coding); the receiver equips two antennas forming a 4 × 2 \( \text{MIMO} \) transmission. Due to the difference between the propagation distances \( (d_1 \text{ and } d_2) \), the signals coming from different cells has different power, which affects the performance of the distributed \( \text{MIMO} \) coding.

Some space-time block codes (STBCs) such as 3D code, Jafarkhani code, L2 code, D4ABBA code, Biglieri-Hong-Viterbo (BHV) code and Sinrath-Sr Jain code which adapt to this scenario have been proposed in the literatures [1, 2, 3] and references therein. However, all of them are either too complex for practical use or not robust to power imbalance of the received signal. In this work, we propose a novel STBC which is efficient, robust and simple for the distributed \( \text{MIMO} \) broadcasting scenario.

A new proposal for \( 4 \times 2 \) \( \text{MIMO} \) transmission: All the aforementioned STBCs form codeword over four channel uses (i.e. \( T = 4 \)). To reach full-rate, eight information symbols (i.e. \( Q = 8 \)) should be stacked in a codeword, which will result in high decoding complexity. To reduce the complexity, we propose to encode four information symbols (i.e. \( Q = 4 \)) over two channel uses (i.e. \( T = 2 \)). As proved later, this proposal guarantees full code rate while limiting the decoding complexity. Recall that the Golden code [4] possesses high diversity with full rate in a 2 × 2 \( \text{MIMO} \) transmission. Its encoding matrix is:

\[
C_{\text{CC}} = \begin{bmatrix}
X_1(1) \\
X_2(1) \\
X_1(2) \\
X_2(2)
\end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix}
\alpha \left( s_1 + s_2 \theta \right) \\
\alpha \left( s_2 + s_1 \theta \right) \\
\alpha \left( s_2 + s_1 \theta \right) \\
\alpha \left( s_1 + s_2 \theta \right)
\end{bmatrix},
\]

where \( \theta = \frac{i}{1+i} \). For example, \( \alpha = 1 + i \theta \) and \( \bar{\alpha} = 1 - i \theta \) with \( i = \sqrt{-1} \). Stacking the four elements of the codeword in a vector \( \mathbf{x} = [X_1(1), X_2(1), X_1(2), X_2(2)]^T \), the encoding can be further expressed by a linear transform (denoted by a generator matrix \( G \)) to information symbols \( s = [s_1, s_2, s_3, s_4]^T \):

\[
\begin{bmatrix}
X_1(1) \\
X_2(1) \\
X_1(2) \\
X_2(2)
\end{bmatrix} = \begin{bmatrix}
\cos \varphi & \sin \varphi & 0 & 0 \\
\cos \varphi & \sin \varphi & 0 & 0 \\
0 & 0 & \cos \varphi & \sin \varphi \\
0 & 0 & \cos \varphi & \sin \varphi
\end{bmatrix} \begin{bmatrix}
s_1 \\
s_2 \\
s_3 \\
s_4
\end{bmatrix},
\]

where \( \varphi = \frac{1}{1 + i} \). \( \cos \varphi = \frac{\theta}{1 + \theta^2} \) with \( \tan \varphi = -\frac{1}{\theta} = \theta \) and \( \varphi_1 = e^{\varphi s_1} \), \( \varphi_2 = e^{\varphi s_2} \), \( \varphi_3 = e^{\varphi s_3} \), \( \varphi_4 = e^{\varphi s_4} \) with \( \varphi = \text{atan} (\theta) \) and \( \varphi_4 = \text{atan} (\bar{\theta}) \). The generator matrix is a unitary, block diagonal matrix. Denote the two blocks as \( G_1 \) and \( G_2 \), respectively.

We know that Alamouti code is the most robust scheme in presence of imbalance of received signal power [1]. With this knowledge and exploiting the property of Golden code, we propose a new STBC:

\[
C = \frac{1}{\sqrt{2}} \begin{bmatrix}
X_1(1) - X_2(2) \\
X_2(1) - X_1(2) \\
\alpha (X_1(1) + \alpha^* X_2(2)) + \bar{\alpha} (X_2(1) + \alpha^* X_1(2)) \\
\alpha (X_1(1) + \alpha^* X_2(2)) - \bar{\alpha} (X_2(1) + \alpha^* X_1(2))
\end{bmatrix},
\]

where \( \cdot^* \) is the complex conjugate. We note that, being different from the 3D code [1] which hierarchically encodes two Golden codewords with an Alamouti structure, the new code arranges one single Golden codeword in a similar manner for the sake of low decoding complexity while keeping the merit of robustness to power imbalance. Since there are only 4 information symbols in one codeword, the search space of the new code using ML decoder is \( O(M^4) \). We will highlight some important properties of the proposed code and give concise proofs as follows.

**Property 1 (Full Rate):** The proposed STBC in (3) achieves full rate.

Proof: The code rate of the proposed code is \( R = Q/T = 2 \) which is equal to the number of received antennas \( N_r \). Hence the code is full rate.

**Property 2 (Rank of Codeword Difference Matrix):** The rank of the codeword difference matrix \( D = \hat{C} - \bar{C} \) is always equal to 2 for all pairs of distinct codewords \( C \) and \( \bar{C} \), \( C \neq \bar{C} \).

Proof: Suppose two distinct codewords \( C \) and \( \bar{C} \) are generated from two information symbol groups \( s = [s_1, s_2, s_3, s_4]^T \) and \( \bar{s} = [s_1, s_2, s_3, s_4]^T \) in which the corresponding symbols \( s_j \) and \( \bar{s}_j \) are not exactly the same, i.e. \( s_j \neq \bar{s}_j \) being nonzero for at least one location \( j \). At least one encoded symbol \( X_{m}(n) \) is different from the corresponding \( \bar{X}_{m}(n) \), \( \forall m, n \in \{ 1, 2 \} \). Hence, none of the two columns of matrix \( D \) is a zero vector. With this fact and considering that the two columns of matrix \( D \) are orthogonal, these two columns are necessarily linearily independent. Therefore, the column rank of the codeword difference matrix \( D \) is 2.

Note that this is the maximum rank that can be expected from a \( 4 \times 2 \) codeword matrix.

**Property 3 (Diversity Gain):** The diversity gain provided by the proposed code is \( 2N_r \).

Proof: The achievable diversity gain of a STBC is \( r N_r \), given \( N_r \) the number of receive antennas and \( r \) the minimum rank of codeword difference matrix over all pairs of distinct codewords [5]. With this fact and taking the result of Property 2, it achieves Property 3.

**Property 4 (Coding Gain):** The proposed code achieves a coding gain of \( d_{\text{min}} \), where \( d_{\text{min}} \) is the minimum Euclidean distance of the constellation adopted by the information symbols.

Proof: The coding gain [5] provided by STBC code can be computed as \( (\prod_{i=1}^{1} \lambda_i)^{\frac{1}{2}} \), where \( \lambda_i, i = 1, 2, \ldots, r \), are the nonzero eigenvalues of the codeword distance matrix \( E \). Taking into account the value of \( D \) with the proposed code, the two nonzero eigenvalue are \( \lambda_1 = \lambda_2 = \sum_{i=1}^{2} \sum_{j=1}^{2} |X_{m}(n) - \bar{X}_{m}(n)|^{2} \) which is equal to the squared norm the first column of matrix \( D \) (denoted by \( d \)). Considering that the \( X \) is linearly coded from \( [s_1, s_2, s_3, s_4]^T \), \( d \) can be expressed as \( d = G(s - \bar{s}) \). Its squared norm is equal to \( |d|^2 \), which is minimized when there is only one information symbol \( s_j \) being different from the corresponding one \( \bar{s}_j \) and these two symbols are the closest to each other among all combinations of constellation points. Therefore, the resulting minimum value of eigenvalues is \( d_{\text{min}} \), the minimum Euclidean distance of the constellation adopted by the information symbols. Taking into account the minimum rank of the proposed code \( r \), the resulting coding gain is equal to \( d_{\text{min}}^{r-2} \).

**Low-complexity decoding algorithm:** Assuming that the channel remains constant over two time slots, the received signal \( y \) can be written as:

\[
y = \begin{bmatrix}
y_1(1) \\
y_1(2) \\
y_2(1) \\
y_2(2)
\end{bmatrix} + \begin{bmatrix}
W_1(1) \\
W_1(2) \\
W_2(1) \\
W_2(2)
\end{bmatrix},
\]

where \( W_i(1) \) and \( W_i(2) \) are the channel estimates for the two time slots. The problem of the decoding algorithm is to find the codeword \( \mathbf{s} \) that maximizes the log-likelihood ratio (LLR) of each information symbol. This can be achieved by computing the maximum a posteriori (MAP) probability of each symbol, which is given by:

\[
P(s_k = 1 | y) = \frac{P(y | s_k = 1) P(s_k = 1)}{P(y)},
\]

where \( P(y | s_k = 1) \) is the likelihood of the received signal given that \( s_k = 1 \), and \( P(s_k = 1) \) is the prior probability of \( s_k \). The MAP estimate of \( s_k \) is the symbol that maximizes this probability.
Table 1: Worst case decoding complexity of STBCs

<table>
<thead>
<tr>
<th>STBC scheme</th>
<th>Code rate</th>
<th>Decoding complexity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed code</td>
<td>2</td>
<td>O(M^{1.5}) [ML] or O(M^{2}) (conditional ML)</td>
</tr>
<tr>
<td>3D code</td>
<td>2</td>
<td>O(M^{2})</td>
</tr>
<tr>
<td>BHV code</td>
<td>2</td>
<td>O(M^{2.5})</td>
</tr>
<tr>
<td>DABB code</td>
<td>2</td>
<td>O(M^{3})</td>
</tr>
<tr>
<td>Srinath-Rajan code</td>
<td>2</td>
<td>O(M^{3.5})</td>
</tr>
<tr>
<td>Jaharkani code</td>
<td>1</td>
<td>O(M^{1.5})</td>
</tr>
<tr>
<td>L2 code</td>
<td>1</td>
<td>O(M^{2})</td>
</tr>
</tbody>
</table>

where \( w \sim \mathcal{C}N(0, 2\sigma^2 I_4) \) is complex white Gaussian noise vector and \( H \) is the channel matrix. The \((m, n)\)th element of \( H \) is the channel gain from the \( m \)th transmit antenna to the \( n \)th receive antenna, modeled as independent and identically distributed (i.i.d) circular symmetric Gaussian distribution. Dividing the channel matrix into two parts i.e. \( H = [H_1 \ H_2] \) with \( H_1 \) and \( H_2 \) being the first and last two columns, respectively, and considering that the encoding matrix of the Golden code (2) is block-wise diagonal, the received signal in (4) can be written in block form:

\[
y = [H_1 \ H_2] \begin{bmatrix} G_1 & 0 \\ 0 & G_2 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} + w = F_1 u + F_2 v + w,
\]

where \( u = [s_1, s_2]^T \), \( v = [s_3, s_4]^T \), \( F_j = H_j G_j \) with \( j \in [1, 2] \). The likelihood function of information symbols \( u \) and \( v \) provided is \( y \) [6]:

\[
p(y|u, v) \propto \exp \left( - \frac{1}{2\sigma^2} \| y - F_1 u - F_2 v \|^2 \right)
\]

\[
= \exp \left( - \frac{1}{2\sigma^2} \left( y - F_1 u \right) \left( I - F_2 F_2^H \right) \left( y - F_1 u \right)^H \right)
\]

\[
\times \exp \left( - \frac{1}{2\sigma^2} \left( v - \bar{v}(u) \right) \left( I - F_2^H F_2 \right) \left( v - \bar{v}(u) \right)^H \right).
\]

It is a zero-forcing solution of \( v \). Hence, the likelihood function (6) can be solved by a conditional ML searching in two steps: 1) finding \( \bar{v} \) that maximizes (6) with respect to a specific \( u \); 2) finding the best choice of \( u \) and \( \bar{v} \) that maximizes (6) by repeating the trial over all possibilities of \( u \). The optimal solution can be obtained as:

\[
\hat{u} = \arg \min_{u \in \mathbb{C}^2} \| y - F_1 u - F_2 \bar{v}(u) \|^2,
\]

\[
\bar{v} = Q(\bar{v}(\hat{u})),
\]

where \( Q(\cdot) \) is the hard decision function and \( \bar{v}(u) \) is given in (8). Since the estimation of \( v \) is achieved by linear processing, the proposed code is decoded through an exhaustive search over \( M^2 \) combinations of information symbols. Therefore, the overall computational complexity is \( O(M^4) \). The exhaustive search can be replaced by the sphere decoder to further reduce the complexity [6]. This conditional ML searching offers a trade-off between the ML and linear decoding methods.

Comparison with the state-of-the-art STBCs: We compare the proposed code with some state-of-the-art four-transmit-antenna STBCs, namely Jafarkhani code, DjABB code, Key, Srinath-Rajan code (c.f. [3] and references therein), L2 code [2] and 3D code [1]. The associated worst-case decoding complexities using sphere decoder are presented in Table 1. The proposed code needs the lowest complexity \( O(M^{1.5}) \) among all rate-two codes to obtain the ML solution. Note that the rate-one codes should use \( M^2 \)-QAM to achieve the same throughput as the rate-two ones with \( M \)-QAM. Therefore, the proposed code requires the same decoding complexity as rate-one codes to support the same system throughputs. Moreover, it needs less complexity \( O(M^2) \) among all STBCs considered, if the conditional ML searching is employed.

Fig. 2 shows the bit error rate (BER) performance of STBCs in different geographical locations using real DVB-NGH configurations. The proposed code exhibits a strong resistance to the power imbalance. It suffers less than 0.2 dB performance loss in high power imbalance case using ML decoding. In the contrary, most of the existing STBCs are not robust in distributed MIMO scenarios. Interestingly, when the power imbalance is high, namely greater than 15 dB, the proposed code outperforms BHV, Srinath-Rajan and DjABB codes and approaches 3D code which however requires much higher complexity. The proposed conditional ML searching introduces negligible performance degradation with real system specifications while significantly reducing the complexity.