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A multi-objective model for the hazardous materials transportation problem based on lane reservation

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Abstract—This paper presents an application of the lane reservation strategy in the hazardous materials transportation. Once an accident of hazardous materials transportation happens, its effect is significant. Lane reservation can reduce the hazardous materials transportation risk enormously; however, it will also impact on the normal traffic. The proposed problem is to reserve lanes on the network and select the path for each hazardous materials shipment among the reserved lanes in order to make a trade-off between the impact on the normal traffic and the total risk of the hazardous materials transportation. We first develop a multi-objective integer linear programming model for the problem with multiple types of hazardous materials and multiple origin-destination pairs. Then we adopt the ε-constraint method to solve the multi-objective model. The performance of the method is evaluated on a large set of randomly generated instances. Computational results show that the ε-constraint method can not only solve the proposed problems within reasonable time but also generate candidate solutions that can shape an efficient Pareto front for the multi-objective optimization problem.

Keywords—hazardous materials transportation; multi-objective optimization; lane reservation; risk; ε-constraint

I. INTRODUCTION

In the countries and areas with the developed industry, large quantities of hazardous substances, such as raw materials, intermediate or final products and wastes, are shipped through the transportation networks by different vehicles, like road, rail and inland waterways. For example, in the United States, approximately 800,000 shipments of hazardous materials are transported each day in bulk and in smaller shipment configurations [1].

Hazardous materials include explosives, gases, flammable liquids and solids, oxidizing substances, poisonous and infectious substances, corrosive substances, and hazardous wastes [2]. Although rare, accidental releases of hazardous materials do occur during transportation, and these events often have very undesirable consequences, including fatalities [2]. Many researchers have indicated that the risks related to hazardous materials transportation can be of the same magnitude as those arising from fixed installations according to the accident history [3][4]. Sometimes the transportation network goes through areas of high population density and areas which are only crossed by transportation routes without having plants, so that, in case of an accidental spill, a large number of persons could be affected. According to U.S. Department of Transportation statistics, 156,442 hazardous materials transportation accidents occurred from 1995 to 2004, resulting in a total of 221 fatalities and 3143 injuries [5]. In European countries, a good many events occurred during the transportation on roads and railways. Specially, the chemical accident in Seveso, Italy in 1976 caused the contamination of a large population by 2, 3, 7, 8-tetrachlorodibenzo-p-dioxin (TCDD). After twenty years later, possible long-term effects examined through mortality and cancer incidence studies indicates that the effect of hazardous material transportation accident is enormous and far-reaching [6]. Furthermore, hazardous materials releases during transportation may occur in environmentally sensitive areas exposed to high risk value, such as the natural and historical interesting places.

Historical evidences have shown that the hazardous materials transportation risk is often tremendous and thus need to be taken into account in order to keep it under control and reduce it. Generally, we divide the operations research problems closely related to hazardous materials transportation into two main subproblems, as follows.

1) Hazardous materials transportation network design problem

Kara and Verter [7] [8] gave the definition of hazardous materials transportation network design problem; that is, given an existing road network, the hazardous network design problem involves selecting the road segments that should be closed to hazardous materials transportation so as to minimize total risk. They also addressed a bi-level integer linear programming model for the problem where ones can select minimum risk routs as a subset of the transportation network and the carriers select minimum cost on the available subset. Since then, the hazardous materials transportation network design problem has received attention of other researchers, see for Bianco et al [9], Erkut and Gizara [10], Erkut and Alp [11].

2) Location and routing problem for Hazardous materials transportation

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In recent three decades, many researchers have focused on the location and routing problem for hazardous materials transportation. As we known, the location and routing problem (LRP) is NP-hard. Problems integrating location and routing decisions pertaining to hazardous materials transportation seem much more complicated, because besides the transportation cost and facility cost in the traditional LRP, at least two types of risk are necessarily to be considered: transport risk and facility risk and models for hazardous materials LRP should be multi-objective by nature [7].

The earliest hazardous materials LRP work was done by Shobrys [12] on locating the storage facilities and selecting routes for the spent fuel shipments so as to simultaneously minimize the total cost and total transportation risk. Jacobs and Warmerdam [13] modeled the hazardous materials LRP as a continuous network flow problem with minimizing the linear combination of cost and risk. It is likely that List and Mirchandani [14] firstly considered risk equity as one of the objectives of their model. Current and Ratick [15] followed them. Stowers and Palekar [16] introduced a bi-objective LRP model with a single facility and a single commodity, whose two objectives were different from others, namely minimizing the total exposure and minimizing the maximum exposure.

All the hazardous materials transportation problems listed above doesn’t involve in lane reservation strategy. In this study, we address the problem to select an optimal route for hazardous materials transportation shipments based on lane reservation strategy. The essence of the lane reservation problem is to optimally select the lanes and/or the time intervals to reserve for only special tasks so as to satisfy the condition of time or safety and simultaneously minimize the impact on the normal traffic. Lane reservation strategy on the transportation network is a flexible and economic option for special event or situation, such as great sports meeting and great emergencies in the city contingency management. In fact, lane reservation strategy has been successfully applied in true-life by some large sportive games organizers. For example, John Black [17] gave a brief description of strategic planning, master planning and operational planning for the 2000 Sydney Olympics, and mentioned the lane reservation for buses only; Zagorianakos [18] stressed on analyzing the importance and advantage of lane reservation strategy for the 2004 Athens Olympics; In France, one of the lanes of A1 between Charles De Gaulle Airport and St Denis is recently dedicated to taxis and buses only between 7am-10am. It is worthwhile to point out that the studies above put emphasis on lane reservation only at the strategic level. At the tactical level, Yingfeng Wu et al. [19] firstly considered the lane reservation problem in time constrained transportation network. They presented the mathematical model for the problem, and proposed a heuristic to obtain near optimal solutions. Yunfei Fang et al. [20] proposed an exact algorithm based on cut-and-solve for the lane reservation problem with multiple origin-destination pairs. Then they [21] extended a capacitated lane reservation problem. However, their works don’t respect to the risk of transportation. In our work, the lane reservation strategy is applied to the hazardous materials transportation, where risk is the primary ingredient that separates hazardous materials transportation problems from other transportation problems.

It is well known that the likelihood of a traffic accident varies with road structure (e.g. the number of lanes and the use of lanes) [2]. The risk of hazardous materials transportation though reservation lanes has much little impact on the population and environment. Nevertheless, lanes reservation will certainly impact the normal traffic because only special shipment can pass through the reserved lanes. Thus, decision-maker (like a governmental planner, a regulator and/or manager of a shipper) should try to optimally catch a compromise between the level of risk and the impact on the normal traffic due to applying lane reservation strategy during the hazardous materials transportation. For the depictions above, it is understandable that the hazardous materials transportation problem based on lane reservation is a multi-objective optimization problem.

The contribution of this paper is that we address a quite new hazardous materials transportation problem based on lane reservation, which is to decide lanes to be reserved on the existing transportation network and select each hazardous materials shipment path from the reserved lanes. We first present a multi-objective integer linear programming model for the problem with multiple types of hazardous materials and multiple origin-destination pairs. We then apply the ε-constraint method to solving the proposed problem. The remainder of the paper is organized as follows. In Section II, a formal problem description is given and the mathematical model is developed, respectively. In Section III the ε-constraint method is adopted to solve the multi-objective model. Computational results on sets of randomly generated instances are reported to demonstrate the effectiveness of the proposed approach in Section IV. Finally, Section V gives the conclusion.

II. THE MODEL

A. Problem Description

The regional network for hazardous materials transportation, \( G = (V, A) \), consists of a number of nodes \( V = \{1, \ldots, N\} \). The nodes may be the origins generating a great quantity of hazardous materials, towns with lots of population or the destinations denoting disposal plants. Nodes are connected to each other by means of transportation routes \( A = \{(i, j), i, j \in V\} \). An arc \((i, j)\) denotes a transportation route from node \(i\) to node \(j\). Kinds of hazardous materials must be carried from the origins \(O = \{o_1, o_2, \ldots, o_{|O|}\}\) to its corresponding destination nodes \(D = \{d_1, d_2, \ldots, d_{|D|}\}\). We call the shipment of hazardous material beginning at origin node \(o_i\) as shipment \(w\).

The problem is to decide lanes to be reserved on the transportation network and select each hazardous materials shipment path from the reserved lanes to ensure that each shipment can be finished from its source to destination within a given travel time and the given risk threshold value of each reserved lane, meanwhile optimally come to a compromise between the total impact on the normal traffic and the total hazardous materials transportation risk.

B. Assumptions and Notation

The following assumptions are given:

- Note that any transport network can be defined so as to satisfy that the probability of the hazardous material accident on an arc is constant and directly proportional to distance of the arc.
- Note that each of the hazardous materials accidents happens independently.
- It is assumed that there are at least two lanes on a road.

The following sets are given:

\( G = (V, A) \) directed transportation network
$O = \{o_1, o_2, \ldots, o_{n_o}\}$ origin nodes
$D = \{d_1, d_2, \ldots, d_{n_d}\}$ destination nodes

Let us define:
$TS_{ij}$: the travel time on a reserved lane of arc $(i, j)$
$K_{ij}$: the number of lanes on arc $(i, j)$
$C_{ij}$: the cost of a reserved lane on arc $(i, j)$
$T_c$: the deadline to accomplish shipment $w$
$PT_w$: the threshold value of the probability of the hazardous material accidents on arc $(i, j)$

$PS_w^{ij}$: the probability of hazardous material accident during the shipment $w$ on a reserved lane of arc $(i, j)$
$E_{ij}$: the population exposure in the bandwidth along arc $(i, j)$

There are two decision variables below.

$x_{ij} = \begin{cases} 
1 & \text{if there is a reserved lane on arc } (i, j) \in A \text{ and the shipment } w \text{ passes the arc} \\
0 & \text{otherwise} 
\end{cases}
$ (1)

$y_{ij} = \begin{cases} 
1 & \text{if there is a reserved lane on arc } (i, j) \in A \\
0 & \text{otherwise} 
\end{cases}
$ (2)

C. Formulation

The mathematical formulations for the various objectives and constraints can be constructed as follows. It is a multi-objective integer linear programming model.

Minimize $f_1 = \sum_{(i,j) \in A} C_{ij} x_{ij}$

Minimize $f_2 = \sum_{w} \sum_{(i,j) \in A} E_{ij} PS_w^{ij} x_{ij}$

Subject to

$\sum_{j} x_{ij} = 1, \forall w \in W, o_w \in O, \forall (o_w, j) \in A$ (3)

$\sum_{i} x_{ij} = 1, \forall w \in W, d_w \in D, \forall (i, d_w) \in A$ (4)

$\sum_{w} x_{ij}^{W} = \sum_{w} x_{ij}^{W}, \forall w \in W, \forall i \neq o_w, d_w, \forall (i, j) \in A$ (5)

$x_{ij}^{W} \leq y_{ij}, \forall (i, j) \in A, \forall w \in W$ (6)

$\sum_{w} PS_w^{ij} y_{ij} \leq PT_{ij}, \forall (i, j) \in A, \forall w \in W$ (7)

$\sum_{(i,j) \in A} TS_{ij} x_{ij} \leq T_w, \forall (i, j) \in A, \forall w \in W$ (8)

$x_{ij} \in \{0,1\}, \forall (i, j) \in A$ (9)

$y_{ij} \in \{0,1\}, \forall (i, j) \in A$ (10)

We define two main objectives for the problem. The objective $1$ is to minimize the total impact on the normal traffic. The impact of reserved lanes from node $i$ to node $j$ on the normal traffic can be translated into the lane reservation cost $C_{ij}$, which is defined by $T_{ij}$ according to [19]. The objective $2$ is to minimize the total hazardous materials transportation risk associated with a unit flow of each shipment. As point out by Erkut and Verter [22], researchers have not come to a consensus on how to model the risk associated with hazardous materials transportation. In quantitative risk assessment, risk is defined as the product of the probability of hazardous materials accident and its consequence. One of the simple methods for risk is to that multiplying the hazardous materials accident probability by population exposure. Risk assessment is a quite important task but it is out of the aim of our work. Therefore, in this study, our proposed model uses this method as a surrogate for risk measure. How to formulate the risk for entire path between an origin and a destination can be found in [22].

We introduce the following constraints which have to be satisfied to design an appropriate transportation network. Constraints (3) and (4) state that there is only one path for each shipment $w$ starting from the origin node $o_w$ or arriving at the destination node $d_w$. Constraint (5) ensures the flow conservation. Constraint (6) represents that there is a reserved lane in the path of one shipment if and only if this lane is reserved. Constraint (7) specifies that the risk caused by all the shipment pass arc $(i, j)$ can not exceed the risk threshold value of the corresponding arc. Constraint (8) guarantees that the total travel time for shipment $w$ does not exceed the deadline $T_w$. Constraints (9) and (10) specify that sign restrictions on the variables.

III. THE EPSILON-CONSTRAINT METHOD FOR THE PROPOSED MULTI-OBJECTIVE MODEL

Without loss of generality, multi-objective optimization can be formulated as follows:

$$\min \{f_1(x), f_2(x), \ldots, f_k(x)\}$$

$$X \in S$$

where $k$ is the number of objective function and $S$ is the feasible solution space. A vector $x^* \in S$ is said to be weakly Pareto optimal or a weakly efficient solution for the multi-objective problem if and only if there is no $x \in S$ such that $f_i(x) < f_i(x^*)$ for all $i \in \{1, \ldots, k\}$. A vector $x^* \in S$ is said to be Pareto optimal or a strict efficient solution for the multi-objective problem if and only if there is no $x \in S$ such that $f_i(x) \leq f_i(x^*)$ for all $i \in \{1, \ldots, k\}$ and at least one strict inequality. The image of all the efficient solutions is called Pareto front.

One of the multi-objective optimization techniques is the $\varepsilon$-constraint method (also called the $\varepsilon$-constraint or trade-off method) introduced by Haines et al. [23]. This method is based on minimizing of one of the objective functions and restricting the other objectives within some allowable values $\varepsilon_i$. Hence, a single objective minimization is carried out for the most relevant objective function subject to additional constraints on the remaining objective functions. It generates one point of the Pareto front at a time, and then the levels $\varepsilon_i$ are altered to generate the entire Pareto front. Miettinen in 1999 [24] proved that if it exists, a solution to the $\varepsilon$-constraint formulation is weakly Pareto optimal.

As Section II presented, the proposed multi-objective model for hazardous materials transportation problem based on lane reservation has two competing objective functions subject to a set of constraints. The first objective function is
the cost of the lane reservation within the transportation network, while the second objective function is the total transportation risk on condition that those lanes that all the shipments passed are reserved. That is to say, a reserved lane is a condition of a shipment. Therefore, we can consider the first objective function as the main objective function. Hence, the proposed multi-objective problem can be transformed into the single objective problem with the objective function \( f_1 \) being minimized while the objective function \( f_2 \) being imposed as a constraint to the feasible solution space.

Fig. 1 depicts the flowchart of the proposed \( \varepsilon \)-constraint method, which is explained below.

1) The problem presented in Section II (called Problem P) is transformed into Problem \( P_0 \). In Problem \( P_0 \), the objective is to minimize the 1-th objective function of \( P \), and the 2-th objective function of \( P \) is dealt with as a constraint of \( P_0 \).

\[
P_0: \begin{align*}
\text{Minimize} & \quad \sum_{(i,j) \in A} C_{ij}y_{ij} \\
\text{Subject to} & \quad \sum_{W} \sum_{(i,j) \in A} E_{ij}PS_{ij}^{w}w_{ij} \leq \varepsilon, \\
& \quad \text{and constraints (3)-(10).}
\end{align*}
\]

Where \( \varepsilon \) is an upper bound of the value of the objective function \( f_2 \) and is not necessarily a small value close to zero.

2) A systematic variation of \( \varepsilon \) yields a set of weakly Pareto optimal solutions. However, improper selection of \( \varepsilon \) can result in a formulation with no feasible solution. Note that \( \varepsilon \) is bounded by \([\text{lower limit}, \text{upper limit}] \). We define the limits according to a) and b).

a) for the objective function \( f_1 \), an optimal vector \((y_{ij}^*, x_{ij}^w)\) can be found by solving the following Problem \( P_1 \).

\[
P_1: \begin{align*}
\text{Minimize} & \quad \sum_{(i,j) \in A} C_{ij}y_{ij} \\
\text{Subject to} & \quad \text{constraints (3)-(10).}
\end{align*}
\]

Let \( f_2(x_{ij}^w) \) be the upper limit of \( \varepsilon \), i.e.,

\[
\varepsilon \leq f_2(x_{ij}^w).
\]

b) for the objective function \( f_2 \), an optimal vector \((y_{ij}^*, x_{ij}^w)\) can be found by solving the following Problem \( P_2 \).

\[
P_2: \begin{align*}
\text{Minimize} & \quad \sum_{W} \sum_{(i,j) \in A} E_{ij}PS_{ij}^{w}w_{ij} \\
\text{Subject to} & \quad \text{constraints (3)-(10).}
\end{align*}
\]

And let \( f_2(x_{ij}^w) \) be the lower limit of \( \varepsilon \), i.e.,

\[
\varepsilon \geq f_2(x_{ij}^w).
\]

3) Choose randomly and uniformly \( \varepsilon \) between \( f_2(x_{ij}^w) \) and \( f_2(x_{ij}^w) \). Solve Problem \( P_0 \), obtain a Pareto optimal solutions.

4) Repeat 3) \( n \) times by varying \( \varepsilon \) such that \( f_2(x_{ij}^w) \leq \varepsilon \leq f_2(x_{ij}^w) \) and obtain \( n \) Pareto optimal solutions, which can theoretically form a Pareto front.

**IV. NUMERICAL RESULTS**

The proposed model formulation in Section III was implemented in C. All the computations were done on a HP PC with a Pentium IV Processor 3.0 GHZ. The mixed integer programming (MIP) solver provided by the commercial optimization software package CPLEX (Version 12.1) was applied to solving the integer programming (IP) problems, i.e., Problems \( P_0, P_1 \) and \( P_2 \). CPLEX was run in default setting.

**A. Experimental Setup**

In order to test the proposed model, we design the random test problems as follows. The transportation network was generated based on the random network topology generator introduced by Waxman [25]. In his model, given the number of the nodes, the nodes of the network are randomly and uniformly distributed in the plane and the arcs are added according to probabilities that depend on the distances between the nodes. The probability function to have an arc between two nodes can be found in [25]. We generate the origin-destination (o-d) pairs randomly from the nodes of the network.

All the test instances were generated with a random number stream. Let Uniform \((a, b)\) stand for a uniform distribution between parameters \( a \) and \( b \). We generated several sets of randomly generated test instances using the parameters described below. \( T_{ij} \) and \( PS_{ij}^{w} \) respectively denotes the travel times and the probability of accident of the hazardous material shipment \( w \) on arc \((i, j)\) without reserved lane. \( T_{ij} \) were generated by using the following parameters: \( T_{ij} = T_{i+1,j} = \text{Uniform}(2, 4), T_{ij} = T_{j,i} = \sum_{k=i}^{j-1} T_{k,k+1} \) and the travel times \( T_{ij} \) = \( T_{ij} \ast \text{Uniform}(0.5, 0.8) \). The deadline \( T_{w} \) is set to be distance \((o_{w}, d_{w}) \ast \text{Uniform}(\sqrt{2}, \sqrt{3}) \), where distance \((o_{w}, d_{w}) \) is the shortest travel time from \( o_{w} \) to \( d_{w} \) in a reserved path. Similarly, let \( P_{W}^{w} = P_{W}^{w} = \text{Uniform}(8, 20), \) considering the effects of the number of lanes, truck configuration, population density,
where one unit of risk is $10^7$ events * yr$^{-1}$. Note that $PT_{ij}\approx \sum_{w} P_{ij}^w \cdot \text{Uniform}(0.4, 0.8)$. Lastly, the population exposure in the bandwidth along arc $(i,j)$, $E_{ij}$, is generated by Uniform $(10, 80)$, whose unit is 10; $K_{ij}$, the number of lanes on arc $(i,j)$, can be defined by Uniform $(4, 10)$ in accordance with [26].

### B. Experiment for Randomly Generated Instances

Firstly, let us define the network density as $D = \frac{|A|}{|V|(|V| - 1)}$, where $|V|$ and $|A|$ is the number of nodes and arcs respectively [19]. The maximum number of arcs is $|P|(|P| - 1)$, so the maximal and minimal density is 1 (for complete networks) and 0 respectively. For each problem, we allowed the software to run until it was solved to optimality. Let $n = 20$. Hence, for each test, 20 different values of $\varepsilon$ were generated so that 20 Pareto optimal solutions could be obtained each time. Total computational time means the computational time for obtaining 20 solutions in all, and average computational time is defined as total computational time divided by 20. The detailed computational results are analyzed as follows.

Tables I and II respectively gives the summary result on the random generated instances with $D \leq 0.35$ and $0.35 < D \leq 0.55$. As can be seen from Tables I and II, increases in the number of the nodes, arcs and shipments of the problems lanes led to a steady increase in the number of binary variables, the number of constraints and the average computational time (in CPU seconds) for finding a Pareto optimal solution for the instances. In more detail, both in Tables I and II, given the number of the shipments, the larger the number of nodes the more time spent on finding an optimal solution. Table I represents that given the number of shipment, the time for set 1-5, 6-10, 11-15 and 16-20 raises gradually. Given the number of the nodes, the larger the number of shipment, the more time. In Table I the average time for instance sets 7-10 is more than the counterpart for sets 11-14, respectively. Given the number of nodes and shipments, the larger the number of arcs, the more time. Take instances sets 6-10 with nodes from 30 to 70 and 10 shipments as an example, the average computational time for the bigger network density is more than the counterpart for the smaller one. The number of binary variables, the number of constraints work as the time in the same way, and analyses on them are omitted here.

**Table I:** The summary result on the random generated instances with $D \leq 0.35$

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<th>$W$</th>
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<td>2924</td>
<td>27281</td>
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</tr>
<tr>
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<td>1069</td>
<td>20</td>
<td>3384</td>
<td>35841</td>
<td>1549.593</td>
<td>77.498</td>
</tr>
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</table>

Then, we also have confirmed whether the solutions of a problem could be effective by taking instance set 7 and set 37 as an example. For this test, 20 different values of $\varepsilon$ were generated, and 13 and 19 different Pareto optimal solutions could be obtained in all, as shown in Fig.2 and Fig.3. As these figures indicate, all the solutions seem likely to make up of a curve. Decision-makers can choose one of points as a Pareto optimal solution for the problem according to their preference.
In this paper, we presented a new problem based on the lane reservation strategy in the hazardous materials transportation network. We developed a multi-objective integer programming model for the problem and then apply the \(\varepsilon\)-constraint method to solving the proposed model. Numerical results of a large set of examples showed the effectiveness of the \(\varepsilon\)-constraint method. The \(\varepsilon\)-constraint method can solve the proposed problems within reasonable time and moreover generate candidate solutions that can shape an efficient Pareto front for the multi-objective problem.

Although our problem could be solved optimally by CPLEX, we may have to resort to other algorithms, with which it may be possible to solve even larger problems within shorter time. This may be a future research direction. Another future work may be to extend the proposed mathematical formulation. For example, various objectives of the hazardous materials transportation problem can be implemented in our model with consideration of the risk equity.

**REFERENCES**


