HOSM control under quantization and saturation constraints: Zig-Zag design solutions

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***Abstract*— In many experimental systems, discrete and bounded actuators implement control laws with sampling, quantization and saturation problems. This paper is dedicated to only the last two in the context of the implementation of a super twisting sliding mode control. A new control design, called "Zig-Zag sliding mode control", is proposed. Issues of quantization and saturation problems are respectively investigated directly and implicitly by the proposed control. The main contribution of the proposed method consists in having a faster convergence and well performances even when the saturation of the actuators is decreased up to a certain limit in which other methods fail to converge. Simulation results of the proposed method compared to the results of traditional implementations highlight the well founded Zig-Zag design.**

I. Introduction

It is well known that sliding mode techniques provide very good properties such as robustness against a certain class of perturbations and parametric incertitudes, as well as finite time convergence of the switching function s to zero [1], [2], [3]. The main disadvantage of this technique is a phenomenon called "chattering", which consists of high-frequency oscillations around the sliding surface [4]. This behavior may become difficult, even impossible, the implementation in a certain class of systems, such as mechanical ones. It can also excite non modeled high frequency modes that could destabilize the system under control. Higher order sliding mode techniques retain the excellent properties of classical sliding modes, but minimize the "chattering" effect [1], [5].

Moreover, real systems such as A/D-D/A converters, power electronic converters and actuators in general introduce problems such as discretization, quantization and/or saturation. Because of these limitations, the implementation of continuous control techniques may degrade its performances [6], [7], [8],[9].

In this paper the problems of quantization and saturation are addressed. Implicitly, the sampling frequency is considered fast enough. Some results dealing with this topic for linear systems and nonlinear systems can be found in [7], [10], [11] and [12], [13] respectively. However, for our best knowledge, both quantization and saturation problems are not treated in the case of higher sliding mode control.

From this point of view, a new way to implement the super-twisting algorithm under saturation and quantization constraints is proposed. Our method is presented in the basis of a very simple example which is representative of a wide range of industrial applications.

The present work is organized as follows. The problem statement of this work is presented in Section II. Section III introduces the proposed Zig-Zag sliding mode controller. Simulation results are illustrated in section IV with a comparative study. Finally, some concluding remarks and future researches are drawn in the last section.

II. Problem statement

In order to illustrate the advantage of our method we will consider a system found in a large number of electromechanical applications : an RLE load, driven at first by an ideal actuator, and by a real industrial actuator then. Its state space model is as follows:

$$\frac{di}{dt} = -\frac{R\,i}{L} - \frac{E}{L} + \frac{u}{L} \tag{1}$$

This model could represent, for example, the armature circuit of a DC electric motor (see for example [2]). It is well known that the current of such a load can be controlled by a super-twisting controller. In fact, the following switching function

$$s = i_{ref} - i \tag{2}$$

has relative degree one, which is deduced owing to the explicit presence of the control u in its first time derivative:

$$\dot{s} = \frac{ds}{dt} = \frac{di_{ref}}{dt} + \frac{R\,i}{L} + \frac{E}{L} - \frac{u}{L} \tag{3}$$

Under this condition, the control may be performed by a super-twisting algorithm [1], which is described by the following equations:

$$u = u_1 + u_2 \tag{4}$$

where:

$$\dot{u}_1 = \alpha sign(s) \tag{5}$$

and

$$u_2 = \ \lambda |s|^{\rho}\, sign(s) \tag{6}$$

with $\rho < 1$.

We show in figure 1 the current of an RLE load controlled by a super-twisting algorithm. The parameters used for this simulation are shown in table I and will be taken as reference for all the simulations run in this paper. The reference signal is a sinusoid of $100\,Hz$ and $2A$ of amplitude. The controller parameters and the initial condition were set to achieve convergence at $25\,ms$.

TABLE I
REFERENCE PARAMETERS

	RLE Load
R	1Ω
L	$50mH$
E	$20V$
	ST controller
α	$2\,10^5$
λ	10
ρ	0.5
S_0	20

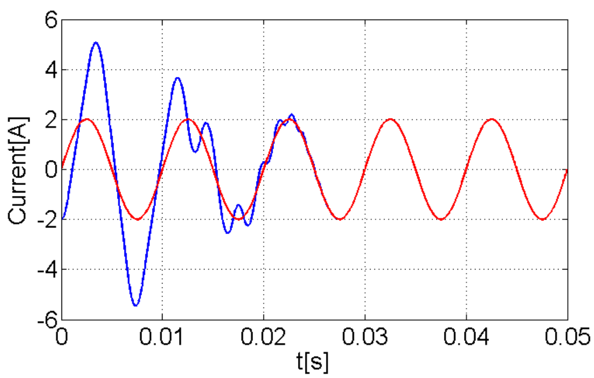

Fig. 1. Super Twisting current control

Unfortunately sources are not ideal. In the case of linear ones we must deal with saturations, but in power applications we usually find sources whose output can take two or more fixed voltage levels, so we have saturation and quantization constraints. This is for example the case of multilevel converters [14], [15]. In this work we will consider systems with linearly quantized symmetric outputs. Such kind of systems may be classified as with odd and even number of levels.

Before the description of the proposed Zig-Zag control, some definitions have to be done.

- We define the *quantization error bound* a as follows:

$$a = \frac{U_{max}}{N-1} \tag{7}$$

where U_{max} and N are the saturation value and the number of levels of the actuator, respectively.

- ***Normalization:*** Assuming that u_{ST} is limited to $\pm U_{max}$, then u_{ST} is normalized with respect to *the quantization error bound* (7):

$$\bar{u}_{ST} = \begin{cases} \frac{u_{ST}}{2a} & \text{if} \quad N \quad \text{is odd} \\ \frac{u_{ST}-a}{2a} & \text{if} \quad N \quad \text{is even} \end{cases}$$

- We recall the *floor* and *ceiling* functions, noted as $\lfloor x \rfloor$ and $\lceil x \rceil$, respectively:

$$\begin{aligned} \lfloor x \rfloor &= \max\{m \,\epsilon\, \mathbb{Z} \,|\, m \leq x\} & (8) \\ \lceil x \rceil &= \min\{\, n \,\epsilon\, \mathbb{Z} \,|\, n \geq x\} & (9) \end{aligned}$$

- At last, we recall the *round* function, based on definitions (8) and (9):

$$\lfloor x \rceil = \begin{cases} \lceil x - 0.5 \rceil & if \quad x < 0 \\ \lfloor x + 0.5 \rfloor & if \quad x \geq 0 \end{cases} \tag{10}$$

u_{ST} is the input given by the Super-Twisting algorithm (see equation equation (4)) and it will be noted that u_{ZZ} the input obtain with the approach proposed in this work. The proposed approach is composed of four steps in order to taking into account quantization and saturation constraints and also to obtain a generic algorithm.

III. Proposed quantization laws (odd and even cases)

A. Systems with odd number of levels

The output of this kind of systems can be expressed as:

$$u_{k_{quantiz}} = \frac{U_{max}}{N-1}\, 2k \tag{11}$$

where k and N are integer values. N is the total number of levels and k is the actual output level; which is between $N_{min} = -\frac{N-1}{2}$ and $N_{max} = \frac{N-1}{2}$. U_{max} is the larger positive output value of the system.

B. Systems with even number of levels

In this case, the output is given by the following expression:

$$u_{k_{quantiz}} = \frac{U_{max}}{N-1}\,(2k+1) \tag{12}$$

Again N is the total number of levels. $k \in \{\mathbb{Z}\}$ is the actual output level. It can take values between $N_{min} = -\frac{N}{2}$ and $N_{max} = \frac{N-1}{2}$.

C. Solution to compute k : Super Twisting Algorithm (odd and even)

A direct quantization of the super twisting algorithm is given as follows

$$\bar{u}_{quantiz} = \begin{cases} \frac{U_{max}}{N-1} 2\lfloor \bar{u}_{ST} \rceil & \text{if} \quad N \quad \text{is odd} \\ \frac{U_{max}}{N-1} (2\lfloor \bar{u}_{ST} \rceil + 1) & \text{if} \quad N \quad \text{is even} \end{cases} \tag{13}$$

It is obvious that this classical quantization solution based on the super twisting algorithm degrades its performances as it will be shown in simulation. To overcome this problem the implementation of a new control design is proposed in the next section.

D. Proposed solution to compute k : Zig-Zag design (odd and even)

A cleaver choice of k is realized hereafter. The computation of such integer will cancel the quantization error by introducing a function sign of the surface s, where s is a switching manifold of the super twisting algorithm. By doing this we introduce a chattering in the quantization error bounds a (7) as it will be shown in simulation.
The proposed algorithm for k is introduced as follows

$$k'_{zz} = \lfloor \bar{u}_{ST} \rceil + 0.5 + 0.5 sign(s). \tag{14}$$

The solution (14) doesn't take the saturation constraints. This fact will be done in the following.

- ***Introduction of saturation constraints:*** The proposed control integer k_{zz} is:

$$k_{zz} = \begin{cases} k'_{zz} & \text{if} \quad N_{min} \leq k'_{zz} \leq N_{max} \\ N_{min} & \text{if} \quad k'_{zz} < N_{min} \\ N_{max} & \text{if} \quad N_{max} < k'_{zz} \end{cases} \tag{15}$$

- ***Denormalization:*** the real output value of the Multilevel System is determined as follows:

$$u_{zz} = \begin{cases} \frac{U_{max}}{N-1} 2k_{zz} & \text{if} \quad N \quad \text{is odd} \\ \frac{U_{max}}{N-1}(2k_{zz}+1) & \text{if} \quad N \quad \text{is even} \end{cases} \tag{16}$$

  where k_{zz} is given by (15) according to the parity of the number of levels.

E. Example of Zig-Zag quantization

Assume two identical systems, one of them controlled by a super twisting algorithm and the other one by a Zig-Zag twisting algorithm. Suppose also that the sliding mode is established in both of them, i.e., $s = 0$. Under these assumptions it is possible to use the concept of *equivalent control* [2], [16], i.e., a fictitious continuous control u_{eq} which forces $s = 0$. By replacing equation (6) in (4), we can rewrite the last one as:

$$u_{ST} = u_1 + \lambda |s|^{\rho}\, sign(s) \tag{17}$$

Given that u_{ST} is continuous and that $s = 0$:

$$u_{ST} = u_1 = u_{eq} \tag{18}$$

Note that u_{eq} is the same for both systems as this does not depend on the technique used but on condition $s = 0$. Suppose that u_{eq} is as shown in figure 2 and that the "quantized actuator" in which the Zig-Zag implementation is performed does not saturate so as the sliding mode is not lost. Now we describe the steps to obtain the Zig-Zag implementation. Performing the *saturation* step does not have any effect as the actuator does not saturate. In the *normalization* step u_{ST} is normalized with respect to the *normalization error bound* as shown in figure

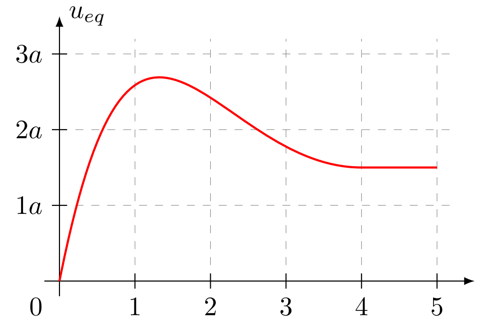

Fig. 2. Equivalent control

3. Using equation (13) a *mapping* into real numbers is accomplished. With this equation, u_{ST} is mapped to the mean value of the levels in which u_{ST} is "contained". This is represented by the blue line shown in figure 3: when u_{ST} is between 0 and 1 the mean value is 0.5, when u_{ST} is between 1 and 2 it is 1.5 and when u_{ST} is between 2 and 3 it is 2.5. Now the last term is taken into

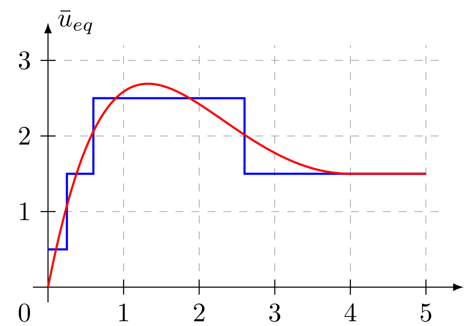

Fig. 3. Mapping to the mean value of levels

account: $\frac{1}{2} sign(s)$. It depends on s and "decides" to what level is assigned k. Finally, the output of the actuator is determined by the *denormalization* step. The Zig-Zag control is shown in figure 4. The green zones represent the high frequency switching given by the sign function.

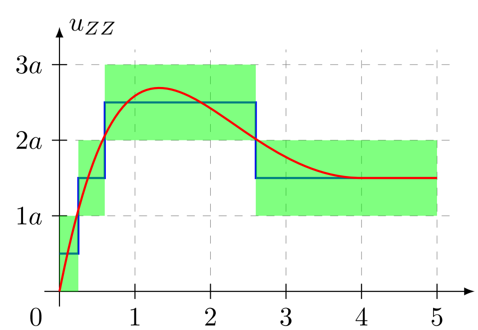

Fig. 4. Zig-Zag implementation

Note that the Zig-Zag control could be *locally* seen as a classical sliding mode control. In fact, by making the change of variables $u' = u_{ZZ} - [u]$ we have:

$$u' = \frac{a}{2} sign(s) \tag{19}$$

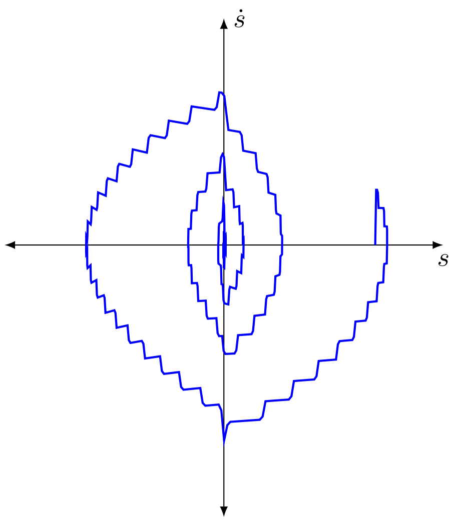

Fig. 5. Phase portrait of a Zig-Zag twisting algorithm

It is shown in figure 5 a typical phase portrait of the Zig-Zag twisting algorithm. This algorithm is called "Zig-Zag twisting" because it converges to the origin "twisting" around it as the *super-twisting* algorithm does, but in a zig-zag manner.

Now, in order to highlight the benefits of zig-zag design in the next sections, it will be compared in simulations to usual implementation methods: multilevel PWM and classical quantization methods.

IV. Comparative study based on simulation results

In this section we will run two simulations in order to compare the zig-zag technique with continuous Super-Twisting, classical quantization and multilevel PWM methods. Saturation is present in the four implementations. The parameters of the load and Super-Twisting controller are those used in section II. The only difference between simulations remains in the parameters of the multilevel converter.

A. Case 1

In this case the number of levels is $N = 5$ and the saturation voltage U_{max} is $130\,V$. Results are shown in figures 6a to 6d. It can be seen that, in this case, convergence is achieved only with the Zig-Zag implementation. Even the Super-Twisting algorithm fails to converge.

B. Case 2

Now, the number of levels is still $N = 5$ but the saturation level is modified to $U_{max} = 150\,V$. Results are shown in figures 7a to 7d.

In this case the four implementations provide convergence of the load current to its reference. As it can be seen, the Zig-Zag solution provides a convergence that is about five times faster.

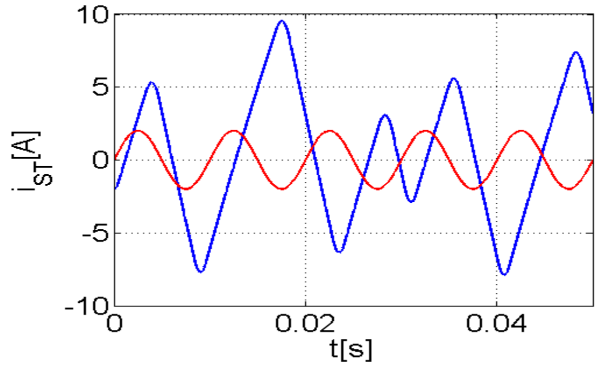

Fig. 6a. Super Twisting implementation

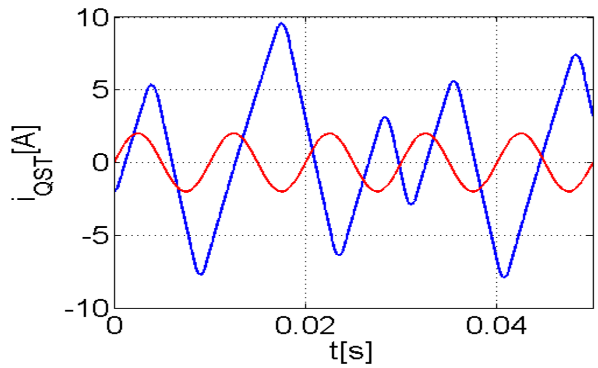

Fig. 6b. Classic quantization implementation

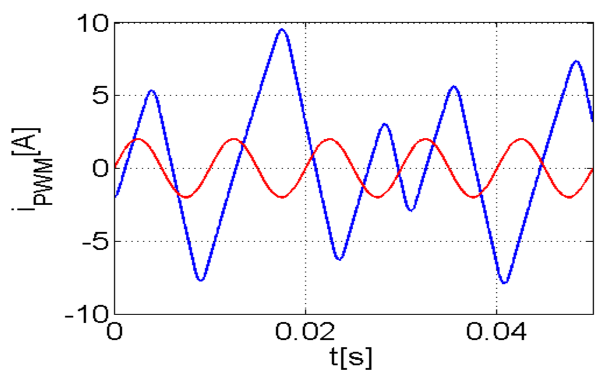

Fig. 6c. Multilevel PWM implementation

V. Conclusion and perspectives

In this paper we have proposed a new way to deal with quantization and saturation problems for the special case of a super-twisting controller. Simulation results highlighted the performances of the proposed method. Our ongoing works will focus on generalizations of zig-zag method to other HOSM algorithms and of proof of convergence in generalized case with analytical conditions.

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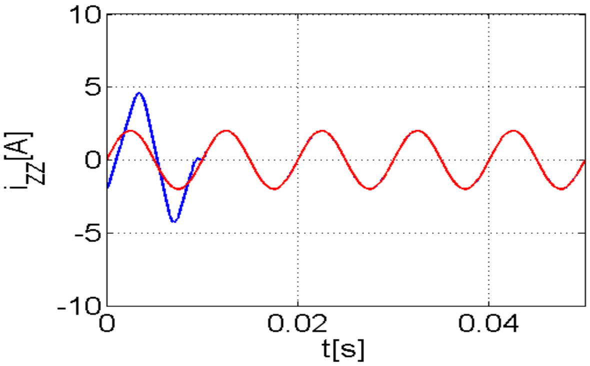

Fig. 6d. Zig-Zag implementation

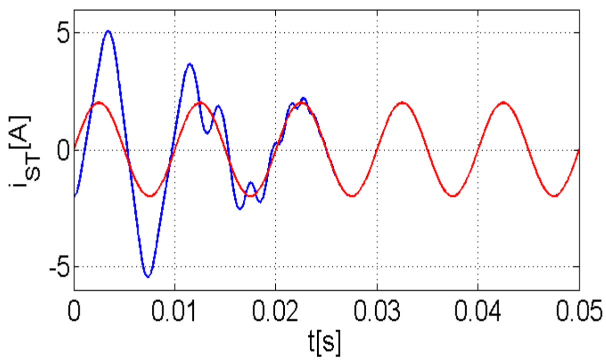

Fig. 7a. Super Twisting implementation

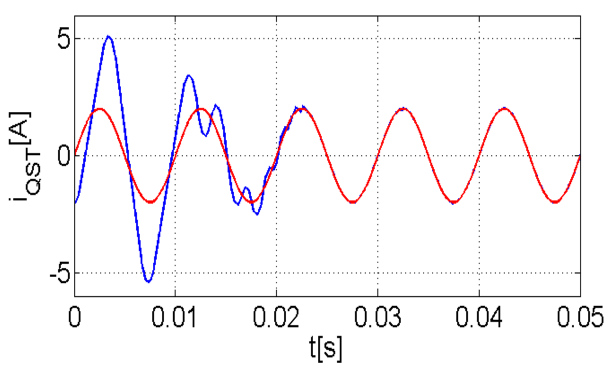

Fig. 7b. Classic quantization implementation

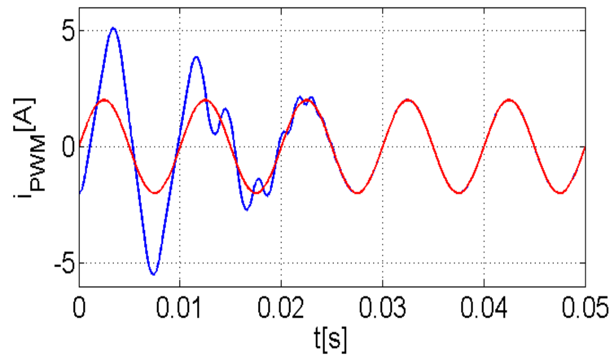

Fig. 7c. Multilevel PWM implementation

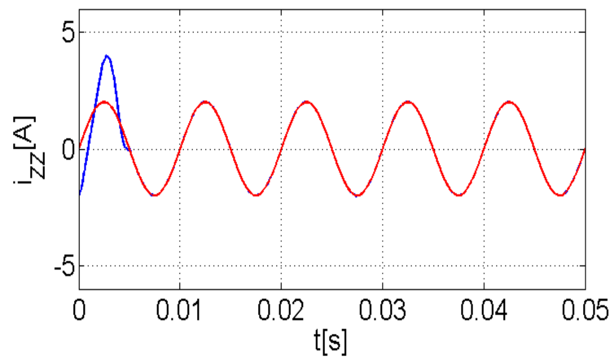

Fig. 7d. Zig-Zag implementation

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