Diverse Routing in Networks with Star SRLGs

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ABSTRACT

The notion of *Shared Risk Link Group*, SRLG has been introduced to capture multiple correlated failures in a network. A SRLG is a set of links that fail simultaneously if a given event (risk) occurs. In such multiple failures scenario, the problem of Diverse Routing consists in finding two SRLG-disjoint paths between a pair of nodes. We consider such problem for localized failures, when all the links of a SRLG verify the star property i.e. when they are incident to the same node. We prove that in this case the problem is in general NP-complete and determine some polynomial cases.

Categories and Subject Descriptors

G.2.2 [**Discrete Mathematics**]: Graph theory—*Graph algorithms, Network problems*; F.2.0 [**Analysis of Algorithms and Problem Complexity**]: General

Keywords

Diverse routing, SRLG

Motivations. To ensure reliable communications, many protection schemes have been proposed. One of the most used, called *dedicated path protection*, consists in computing for each demand both a working and a protection path. A general requirement is that these paths have to be diversely routed, so that at least one path can survive a single failure in the network. This method works well in a single link failure scenario, as it consists in finding two edge-disjoint paths between a pair of nodes. This is a well-known problem in graph theory for which there exist efficient polynomial algorithms. However, the problem of finding two diversely routed paths between a pair of nodes becomes much more difficult in case of multiple correlated link failures that can be captured by the notion of *SRLG (Shared Risk Link Group)*.

Group). In fact, a SRLG is a set of network links that fail simultaneously if a given event (i.e. risk) occurs. The scope of this concept is very broad. For example it can correspond to a set of fiber links of an optical backbone network that are physically buried at the same location and so that could be cut simultaneously (i.e. backhoe or JCB fade). It can also represent links that are located in the same seismic area, or radio links in access and backhaul networks subject to localized environmental conditions affecting signal transmission, or traffic jam propagation in road networks. In practice the failures are localized and very common SRLGs are *star SRLGs* or SRLGs verifying the *star property* [7] (coincident SRLGs in [5]). Under this property, a link can be affected by more than one risk but all links of any SRLG share an end-point. Such failure scenarios can correspond to risks arising in router nodes like card failures or to the cut of a conduit containing links issued from a node (see Figure 1).
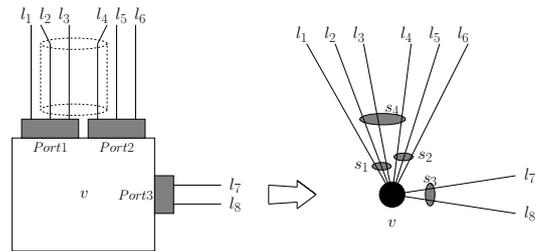

Figure 1: An example of localized risks. Link l_4, for instance, shares risk s_2, corresponding to Port 2 failure, with l_5 and l_6 and shares risk s_4, corresponding to the conduit cut, with links l_2 and l_3.

Related work. In the context of SRLG, basic network connectivity problems have been proven much more difficult to address than their usual counterparts. For instance, the problem of finding a "SRLG-shortest" st-path that is a path from node s to node t having the minimum number of risks has been proven NP-hard and hard to approximate in general (see [2]). However, the problem has been proven to be polynomial in two generic practical cases corresponding to localized failures: when all risks verify the star property [3] and when risks are of *span 1*; i.e. when a link is affected by at most one risk and links sharing a given risk form a connected component [2].

The *diverse routing* problem has been proven NP-complete in general (see [1] for references) and many heuristics have been proposed. The problem has been proven to be polynomial in some specific cases of localized failures: when SRLGs

have span 1 [2], and in a specific case of SRLG's having the star property [4] (this result also follows from [2]).

The more general case of diverse routing with the star property where there are no restrictions on the number of risks per link has been studied in [7]. The authors claim that the diverse routing problem can be solved in polynomial time. Unfortunately their algorithm is not correct; indeed we exhibit counterexamples showing that in some cases their algorithm concludes to the non existence of two SRLG-disjoint paths although two such paths exist. We have not only disproved the validity of their algorithm, but have also been able to prove that the problem is in fact NP-complete (again, contradicting the supposed polynomiality of the algorithm, unless $P = NP$) and we have also determined some cases where the diverse routing problem is polynomial.
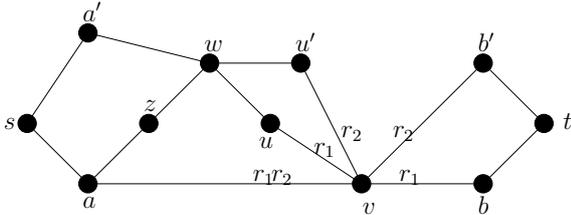

Figure 2: Example

Our precise results. Figure 2 shows a counter example to the algorithm in [7] that can give to the reader a flavor of the difficulty of the problem. In this figure, we have 2 specific risks r_1 and $r_2 \neq r_1$ forming a star in v. Other risks are not represented, we can either suppose there are no other or that they are different from the risks explicitly mentioned and appear on only one link. As vertex v is a cut-vertex any path should contain v. In this case, and to ensure the SRLG-disjointedness, one path should use the subpath u, v, b and the other one should use the subpath u', v, b'. We have two SRLG-disjoint paths $P_1 = \{s, a, z, w, u, v, b, t\}$ and $P_2 = \{s, a', w, u', v, b', t\}$. The algorithm of [7] uses the shortest path $P_a = \{s, a, v, b, t\}$ and then performs a backwards phase which never finds w again. Then the algorithm stops missing the fact that there exist two SRLG-disjoint paths (for details see the full version [1]).

To state precisely our results we use the *colored-graph* model [2, 7, 8], also called *labeled-graph* model. In the colored-graph model, the network is modeled by a graph $G = (V, E)$ and the set of SRLGs by a set of colors $\mathcal{R}$. Each SRLG is modeled by a distinct color and an edge modeling a network link subject to different SRLGs will be assigned as many colors as SRLGs. In such a model, the star property corresponds to the fact that all the edges with the same color are incident to a common vertex and so form a star. In what follows we denote by Δ the maximum degree, and by Δ_c the maximum colored degree (the number of colors incident to a node).

As main results we show that finding 2 color-disjoint paths in a colored graph (i.e. two SRLG-disjoint paths) with the star property is NP-complete and more generally finding $k \geq 2$ color-disjoint paths (for a fixed k) is also NP-complete. This last result holds even if Δ is fixed with $\Delta \geq 6 + k$ or if the number of colors per arc is at most 2 or the maximum number of arcs having the same color is at most 2; these values correspond to real networks parameters.

On the positive side, we show that the diverse routing problem can be solved in polynomial time in particular subcases which are relevant in practice. Namely, we solve the problem when the input graph is a DAG, when the number of SRLGs is bounded by a constant or when the maximum degree $\Delta \leq 4$. The algorithm for DAGs requires $O((|V|\Delta)^{2k})$ time where k is the number of color-disjoint paths. The algorithm for fixed $|\mathcal{R}|$ and the one for $\Delta \leq 4$ require $O(|V| + |E|)$ time.

Finally we also consider the problem of finding the maximum number of color-disjoint paths in a colored graph. This problem has been shown to be NP-hard in [6]. We prove that it is also NP-hard in colored graphs with the star property even if the number of colors per arc is at most 3 or the maximum number of arcs having the same color is at most 2. Moreover, unless $P = NP$, such problem cannot be approximated within $O(|V|)$ even if either the number of colors per arc is fixed or if the maximum number of arcs having the same color is fixed. Such inapproximability results hold also when the graph is a Directed Acyclic Graph (DAG).

These results, along with the polynomial cases, give an almost complete characterization of the problem of finding color-disjoint paths in networks with the star property.

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